CC Algebra 2H 7-2 Notes April 23

Theoretical and Experimental Probability

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome.** All of the possible outcomes make up the **sample space**. An **event** is an outcome or set of outcomes.

For outcomes that have the same likelihood of occurring, or equally likely outcomes, the **theoretical probability** of an event, *E*, is defined as follows.

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}$$

$0 \le P(E) \le 1$ certainty

Example 1. A red number cube and a blue number cube each numbered 1–6 are rolled. If all numbers on each cube are equally likely, what is the probability that the sum is 6 or 7?

$\underset{Red}{6} \times \underset{Blue}{6} = 36$ outcomes (R, B) $\frac{11}{36}$

Sample space {(3,4) (1,6) (2,5) (1,5) (2,4) (3,3) (4,3) (6,1) (5,2) (5,1) (4,2)}

The sum of all probabilities in the sample space is 1. The **complement** of an event *E* is the set of all outcomes in the sample space that are not in *E*.

The probability of the complement of an event *E* is
$P(\text{ not } E) = 1 - P(E)$

Example 2. Use the number cubes from Example 1. What is the probability that the sum is not 10?

$P(10) = (6,4)\ (4,6)\ (5,5)$ $P(\text{not } 10) = 1 - P(10)$

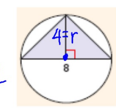
$1 - \frac{3}{36}$

$1 - \frac{1}{12}$

$\frac{11}{12}$

Geometric probability is a form of theoretical probability determined by a ratio of lengths, volumes, or areas.

Example 3. Find the probability that a point inside the figure chosen at random is inside the unshaded region of the circle.



$P = \frac{\odot - \triangle}{\odot} = \frac{\pi(4^2) - 16}{\pi \cdot 4^2} = \frac{16\pi - 16}{(16 \cdot \pi)} \approx .68$

Example 4. A clerk has 4 different letters that need to go in 4 different envelopes. What is the probability that all 4 letters are placed in the correct envelopes?

$\frac{\text{correct envelopes}}{\text{arrangements}}$ $\frac{1}{24}$ ${}_4P_4 = 4!$ OR $4 \cdot 3 \cdot 2 \cdot 1$

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Example 5. Five boys and 3 girls are applying for a summer job. Three applicants will be selected. Find the probability that

a) all 3 are boys $= \frac{5}{28}$

$P(3\text{ boys}) = \frac{3\text{ boys}}{\text{any 3 Kids}}$

$\frac{\frac{5 \cdot 4}{2}}{\frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3 \cdot 2 \cdot 1}}} = \frac{10}{56} = \frac{{}_5C_3}{{}_8C_3}$ $\frac{\frac{5!}{3!2!}}{\frac{8!}{3!5!}}$

b) 2 are girls and 1 is a boy $P(2G + 1B) = \frac{15}{56}$

$= \frac{2G \text{ and } 1B}{{}_8C_3}$

$= \frac{{}_3C_2 \times {}_5C_1}{56} = \frac{\frac{3!}{2!1!} \times 5}{56}$

To find the experimental probability of an event, conduct or observe an experiment for a large number of trials. Then count the number of times the event, *A*, occurs.

The **experimental probability** of the event is found by using the following formula.

$$P(A) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Example 3. The table shows the results of an experiment with a spinner.

Spinner Experiment				
Color	Red	Yellow	Green	Blue
Spins	30	25	12	33

Outcomes

Trials

Total # of Trials = 100

Find the experimental probability that the spinner lands on red or blue.

$P(\text{Red or Blue}) = \frac{30+33}{100}$

$= \frac{63}{100}$

Find the probability that the spinner lands on a color other than green.

$P(\text{not green}) = 1 - P(\text{green})$

$1 - \frac{12}{100}$

$\frac{88}{100} = \frac{22}{25}$