# Test 11B AP Statistics Name:

**Directions:** *Work on these sheets. Tables and formulas appear on a separate sheet.*

**Part 1: Multiple Choice.** *Circle the letter corresponding to the best answer.*

**1.** You want to compute a 90% confidence interval for the mean of a population with unknown population standard deviation. The sample size is 30. The value of $t^*$ you would use for this interval is
(a) 1.96
(b) 1.645
(c) 1.699
(d) .90
(e) 1.311
(f) None of the above

**2.** A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2). The margin of error of this interval is
(a) 95%
(b) 5
(c) 2.5
(d) 10
(e) The answer cannot be determined from the information given.

**3.** The effect of acid rain upon the yield of crops is of concern in many places. In order to determine baseline yields, a sample of 13 fields was selected, and the yield of barley ($g/400m^2$) was determined. The output from SAS appears below:

| | | | | | QUANTILES (DEF=4) | | | EXTREMES | |
|---|---|---|---|---|---|---|---|---|---|
| N | 13 | SUM WGTS | 13 | 100% MAX | 392 | 99% | 392 | LOW | HIGH |
| MEAN | 220.231 | SUM | 2863 | 75% Q3 | 234 | 95% | 392 | 161 | 225 |
| STD DEV | 58.5721 | VAR | 3430.69 | 50% MED | 221 | 90% | 330 | 168 | 232 |
| SKEW | 2.21591 | KURT | 6.61979 | 25% Q1 | 174 | 10% | 163 | 169 | 236 |
| USS | 671689 | CSS | 41168.3 | 0% MIN | 161 | 5% | 161 | 179 | 239 |
| CV | 26.5958 | STD MEAN | 16.245 | | | 1% | 161 | 205 | 392 |

A 95% confidence interval for the mean yield is:
(a) $220.2 \pm 1.96(58.6)$
(b) $220.2 \pm 1.96(16.2)$
(c) $220.2 \pm 2.18(58.6)$
(d) $220.2 \pm 2.18(16.2)$
(e) $220.2 \pm 2.16(16.2)$

**4.** To use the two-sample $t$ procedure to perform a significance test on the difference between two means, we assume
(a) The populations' standard deviations are known
(b) The samples from each population are independent
(c) The distributions are exactly normal in each population
(d) The sample sizes are large
(e) All of the above

**5.** We wish to test if a new feed increases the mean weight gain compared to an old feed. At the conclusion of the experiment it was found that the new feed gave a 10 kg bigger gain than the old feed. A two-sample $t$-test with the proper one-sided alternative was done and the resulting $P$-value was 0.082. This means:

(a) There is an 8.2% chance the null hypothesis is true.
(b) There was only a 8.2% chance of observing an increase greater than 10 kg (assuming the null hypothesis was true).
(c) There was only an 8.2% chance of observing an increase greater than 10 kg (assuming the null hypothesis was false).
(d) There is an 8.2% chance the alternate hypothesis is true.
(e) There is only an 8.2% chance of getting a 10 kg. increase.

**6.** The water diet requires one to drink two cups of water every half hour from when one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weights (in pounds) are

| Person | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Weight before the diet | 180 | 125 | 240 | 150 |
| Weight after six weeks | 170 | 130 | 215 | 152 |

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet – weight after six weeks on the diet) is normally distributed with mean $\mu$. To determine if the diet leads to weight loss, we test the hypotheses

$$H_0: \mu = 0, H_a: \mu > 0.$$

Based on these data we conclude that

(a) We would not reject $H_0$ at significance level 0.10.
(b) We would reject $H_0$ at significance level 0.10 but not at 0.05.
(c) We would reject $H_0$ at significance level 0.05 but not at 0.01.
(d) We would reject $H_0$ at significance level 0.01.
(e) The sample size is too small to allow use of the $t$ procedures.

## Part 2: Free Response

*Answer completely, but be concise. Write sequentially and show all steps.*

**7.** The level of dissolved oxygen in a river is an important indicator of the water's ability to support aquatic life. You collect water samples at 15 randomly chosen locations along a stream and measure the dissolved oxygen. Here are your results in milligrams per liter:

4.53, 5.04, 3.29, 5.23, 4.13, 5.50, 4.83, 4.40, 5.42, 6.38, 4.01, 4.66, 2.87, 5.73, 5.55

Construct a 95% confidence interval for the mean dissolved oxygen level for this stream. Follow the Inference Toolbox.

In a study of the effectiveness of weight-loss programs, 47 subjects who were at least 20% overweight took part in a group support program for 10 weeks. Private weighings determined each subject's weight at the beginning of the program and 6 months after the program's end. The matched pairs $t$ test was used to assess the significance of the average weight loss. The paper reporting the study said, "The subjects lost a significant amount of weight over time, $t(46) = 4.68$, $P < 0.01$."

**8.** Why was the matched-pairs $t$ statistic appropriate?

**9.** Explain to someone who knows no statistics but is interested in weight-loss programs what the practical conclusion is.

**10.** The paper follows the tradition of reporting significance only at fixed levels such as $\alpha = 0.01$. In fact the results are more significant than "$P < 0.01$" suggests. What can you say about the $P$-value of the $t$ test?

A study of iron deficiency among infants compared samples of infants following different feeding regimens. One group contained breast-fed infants, while the children in another group were fed a standard baby formula without any iron supplements. Here are the results on blood hemoglobin levels at 12 months of age.

| Group | n | $\bar{x}$ | s |
|---|---|---|---|
| Breast-fed | 23 | 13.3 | 1.7 |
| Formula | 19 | 12.4 | 1.8 |

**11.** Is there significant evidence that the mean hemoglobin level is higher among breast-fed babies? Give appropriate statistical evidence to support your conclusion.

**12.** Construct a 95% confidence interval for the mean difference in hemoglobin level between the two populations of infants. Interpret your interval in the context of this problem.

*I pledge that I have neither given nor received aid on this test.* _____

M.C. ANSWERS: 1(c), 2(b), 3(d), 4(b), 5(b), 6 (a)

**Part 2: Free Response**

*Answer completely, but be concise. Write sequentially and show all steps.*

7. The level of dissolved oxygen in a river is an important indicator of the water's ability to support aquatic life. You collect water samples at 15 randomly chosen locations along a stream and measure the dissolved oxygen. Here are your results in milligrams per liter:

4.53, 5.04, 3.29, 5.23, 4.13, 5.50, 4.83, 4.40, 5.42, 6.38, 4.01, 4.66, 2.87, 5.73, 5.55

Construct a 95% confidence interval for the mean dissolved oxygen level for this stream. Follow the Inference Toolbox.

WE CONSTRUCT A 95% C.I. FOR $\mu$, THE MEAN DISSOLVED $O_2$ FOR THIS STREAM.

- WE ARE GIVEN THAT THE SAMPLE IS RANDOM.
- THE HISTOGRAM AND BOXPLOT OF THE DATA ARE VERY SYMMETRIC WITH NO OUTLIERS, AND THE NORMAL P-P IS QUITE LINEAR, SO WE BELIEVE THE DATA COME FROM A NORMAL DISTRIBUTION.
- SINCE $15 < \frac{1}{10}$(ALL POSSIBLE SAMPLES), WE PROCEED.

THE TI-83 95% T-INTERVAL: (4.251, 5.292) mg/l. WE ARE 95% CONFIDENT THE TRUE MEAN DISSOLVED $O_2$ IS IN THIS INTERVAL.

In a study of the effectiveness of weight-loss programs, 47 subjects who were at least 20% overweight took part in a group support program for 10 weeks. Private weighings determined each subject's weight at the beginning of the program and 6 months after the program's end. The matched pairs $t$ test was used to assess the significance of the average weight loss. The paper reporting the study said, "The subjects lost a significant amount of weight over time, $t(46) = 4.68$, $P < 0.01$."

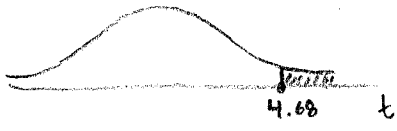
8. Why was the matched-pairs $t$ statistic appropriate?

SINCE THE AMOUNT OF WEIGHT LOST WAS DEPENDENT ON THE STARTING WEIGHT OF EACH SUBJECT, IT IS APPROPRIATE TO RECORD THE DIFFERENCE IN STARTING AND ENDING WEIGHT FOR EACH INDIVIDUAL AND USE THESE DIFFERENCES TO ASSESS AVERAGE WEIGHT LOSS.

9. Explain to someone who knows no statistics but is interested in weight-loss programs what the practical conclusion is.

SINCE THE P-VALUE IS LESS THAN 0.01, THE CONCLUSION IS THAT THE AVERAGE WEIGHT LOSS WOULD MOST LIKELY NOT HAVE OCCURRED SIMPLY DUE TO CHANCE, BUT THAT THE SUPPORT GROUP PROBABLY HELPED. WHAT WE DON'T KNOW IS WHAT THE AVERAGE WEIGHT LOSS WAS APPROXIMATELY.

**10.** The paper follows the tradition of reporting significance only at fixed levels such as $\alpha = 0.01$. In fact the results are more significant than "$P < 0.01$" suggests. What can you say about the $P$-value of the $t$ test? GIVEN THE $t(46) = 4.68$ WE HAVE AN ACTUAL P-VALUE OF




$P = tcdf(4.68, 1E99, 46) = 0.0000128$, SHOWING VERY SIGNIFICANT RESULTS.

A study of iron deficiency among infants compared samples of infants following different feeding regimens. One group contained breast-fed infants, while the children in another group were fed a standard baby formula without any iron supplements. Here are the results on blood hemoglobin levels at 12 months of age.

| Group | n | $\bar{x}$ | s |
|---|---|---|---|
| Breast-fed | 23 | 13.3 | 1.7 |
| Formula | 19 | 12.4 | 1.8 |

**11.** Is there significant evidence that the mean hemoglobin level is higher among breast-fed babies? Give appropriate statistical evidence to support your conclusion.

WE WILL TEST WHETHER $\mu_1$, THE MEAN HEMOGLOBIN LEVEL IN BREAST-FED BABIES, IS HIGHER THAN $\mu_2$, THE MEAN LEVEL IN FORMULA-FED BABIES. $H_0: \mu_1 = \mu_2$, $H_A: \mu_1 > \mu_2$

- WE HOPE THAT BOTH SAMPLES WERE SRS, BUT WE CAN'T VERIFY THIS.
- THE SAMPLES ARE NOT VERY LARGE, BUT WE'LL ASSUME THAT THE DATA HAVE NO APPARENT OUTLIERS OR STRONG SKEWING. $n_1 > 5$ AND $n_2 > 5$, SO WE PROCEED WITH 2-SAMPLE $t$ PROCEDURES.

THE TI-83 2-SAMP T-TEST: $t(37.59) = 1.654$, P-VALUE $= 0.053$

A P-VALUE OF 5.3% COULD BE CONSIDERED SIGNIFICANT BUT NOT VERY SIGNIFICANT. THEREFORE WE DO NOT HAVE VERY SIGNIFICANT EVIDENCE THAT BREAST-FED BABIES HAVE HIGHER HEMOGLOBIN.

**12.** Construct a 95% confidence interval for the mean difference in hemoglobin level between the two populations of infants. Interpret your interval in the context of this problem.

THE 95% CONFIDENCE INTERVAL FOR THE MEAN DIFFERENCE IN HEMOGLOBIN LEVELS $(\mu_1 - \mu_2)$, IS $(-0.2021, 2.0021)$. WE ARE 95% CONFIDENT THAT THE MEAN DIFFERENCE IS IN THIS INTERVAL.

*I pledge that I have neither given nor received aid on this test.* _____