3.3c

24. $y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2) = u^3 - 2 + u^2 - 2u^{-1}$

$y'(u) = 3u^2 + 2u + 2u^{-2}$

30. $y = \dfrac{x+1}{x^3+x-2}$

$$y' = \frac{(x^3+x-2)(1) - (x+1)(3x^2+1)}{(x^3+x-2)^2}$$
$$= \frac{x^3+x-2 - (3x^3+3x^2+x+1)}{(x^3+x-2)^2}$$
$$= \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

34. $g(t) = \dfrac{t-\sqrt{t}}{t^{\frac{1}{3}}} = \dfrac{t}{t^{\frac{1}{3}}} - \dfrac{t^{\frac{1}{2}}}{t^{\frac{1}{3}}} = t^{\frac{2}{3}} - t^{\frac{1}{6}}$

$g'(t) = \frac{2}{3}t^{-\frac{1}{3}} - \frac{1}{6}t^{-\frac{5}{6}}$

36. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2} = A + Bx^{-1} + Cx^{-2}$

$y' = -Bx^{-2} - 2Cx^{-3}$

40. $y = \dfrac{u^6 - 2u^3 + 5}{u^2} = \dfrac{u^6}{u^2} - \dfrac{2u^3}{u^2} + \dfrac{5}{u^2} = u^4 - 2u + 5u^{-2}$

$y' = 4u^3 - 2 - 10u^{-3}$

55. $y = \frac{3x+1}{x^2+1}$ point $(1,2)$

$$y' = \frac{(x^2+1)(3) - (3x+1)(2x)}{(x^2+1)^2}$$

$$= \frac{3x^2+3-6x^2-2x}{(x^2+1)^2}$$

$$= \frac{-3x^2-2x+3}{(x^2+1)^2}$$

at $x=1$ $y' = \frac{-3(1)^2-2(1)+3}{(1^2+1)^2} = \frac{-2}{4} = \frac{-1}{2}$

tangent line:

at $x=1$ $y' = \frac{-1}{2}$ pt $(1,2)$

$$y-2 = \frac{-1}{2}(x-1)$$

$$y = \frac{-1}{2}x + \frac{5}{2}$$

normal line:

$m=2$ pt $(1,2)$

$$y-2=2(x-1)$$

$$y=2x$$

60. $f(x) = \frac{1}{3-x}$

$f'(x) = \frac{(3-x)(0) - 1(-1)}{(3-x)^2} = \frac{1}{(3-x)^2} = \frac{1}{9-6x+x^2}$

$f''(x) = \frac{(9-6x+x^2)(0) - 1(-6+2x)}{[(3-x)^2]^2} = \frac{6-2x}{(3-x)^4}$

$= \frac{2(3-x)}{(3-x)^4}$

$= \frac{2}{(3-x)^3}$

63. $f(5)=1 \quad f'(5)=6 \quad g(5)=-3 \quad g'(5)=2$

ⓐ $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = 1(2) + (-3)(6) = -16$

ⓑ $\left(\frac{f}{g}\right)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{-3(6)-(1)(2)}{(-3)^2} = \frac{-20}{9}$

ⓒ $\left(\frac{g}{f}\right)'(5) = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \frac{1(2)-(-3)(6)}{(1)^2} = 20$

64. $f(2)=-3 \quad g(2)=4 \quad f'(2)=-2 \quad g'(2)=7$

ⓐ $h(x) = 5f(x) - 4g(x)$

$h'(x) = 5f'(x) - 4g'(x)$

$h'(2) = 5f'(2) - 4g'(2) = 5(-2) - 4(7) = -38$

ⓑ $h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(2) = f(2)g'(2) + g(2)f'(2) = (-3)(7) + (4)(-2) = -29$

64 cont'd

ⓒ $h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(4)(-2)-(-3)(7)}{4^2} = \frac{13}{16}$

ⓓ $h(x) = \frac{g(x)}{1+f(x)}$

$h'(x) = \frac{(1+f(x))g'(x) - g(x)[0+f'(x)]}{[1+f(x)]^2}$

$h'(2) = \frac{[1+f(2)]g'(2) - g(2)f'(2)}{[1+f(2)]^2} = \frac{(1-3)(7)-4(-2)}{(1-3)^2} = \frac{-3}{2}$

65. $f(x) = \sqrt{x}\, g(x)$ $\quad g(4)=8 \quad g'(4)=7$

$f(x) = x^{\frac{1}{2}} g(x)$

$f'(x) = x^{\frac{1}{2}} g'(x) + g(x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$f'(4) = 4^{\frac{1}{2}} g'(4) + g(4)\left(\frac{1}{2}\right)(4)^{-\frac{1}{2}} = 2(7) + 8\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 16$

66. $h(2) = 4 \quad h'(2) = -3$

find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$

First find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)$ then evaluate when $x=2$

[NOTE: be careful with notation]

$\frac{d}{dx}\left(\frac{h(x)}{x}\right) = \frac{xh'(x) - h(x)\cdot 1}{x^2} = \frac{xh'(x) - h(x)}{x^2}$

$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} = \frac{2h'(2) - h(2)}{2^2} = \frac{2(-3)-4}{4} = \frac{-5}{2}$

67. $u(x) = f(x)\,g(x)$ $\qquad v(x) = \dfrac{f(x)}{g(x)}$

ⓐ find $u'(1)$

$$u'(x) = f(x)\,g'(x) + g(x)\,f'(x)$$

$$u'(1) = f(1)\,g'(1) + g(1)\,f'(1) = 2(-1) + (1)(2) = 0$$

ⓑ find $v'(5)$

$$v'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$$

$$v'(5) = \frac{g(5)\,f'(5) - f(5)\,g'(5)}{[g(5)]^2} = \frac{2(-\frac{1}{3}) - 3(\frac{2}{3})}{(2)^2} = \frac{-2}{3}$$

68. $P(x) = F(x)\,G(x)$ $\qquad Q(x) = \dfrac{F(x)}{G(x)}$

ⓐ find $P'(2)$

$$P'(x) = F(x)\,G'(x) + G(x)\,F'(x)$$

$$P'(2) = F(2)\,G'(2) + G(2)\,F'(2) = 3(\tfrac{1}{2}) + 2(0) = \frac{3}{2}$$

ⓑ find $Q'(7)$

$$Q'(x) = \frac{G(x)\,F'(x) - F(x)\,G'(x)}{[G(x)]^2}$$

$$Q'(7) = \frac{G(7)\,F'(7) - F(7)\,G'(7)}{[G(7)]^2} = \frac{(1)(\frac{1}{4}) - 5(\frac{-2}{3})}{(1)^2} = \frac{43}{12}$$

69 ⓐ $y = xg(x)$

$y' = xg'(x) + g(x)\cdot 1 = xg'(x) + g(x)$

ⓑ $y = \dfrac{x}{g(x)}$

$y' = \dfrac{g(x)\cdot 1 - xg'(x)}{[g(x)]^2} = \dfrac{g(x) - xg'(x)}{[g(x)]^2}$

ⓒ $y = \dfrac{g(x)}{x}$

$y' = \dfrac{xg'(x) - g(x)\cdot 1}{x^2} = \dfrac{xg'(x) - g(x)}{x^2}$

70. ⓐ $y = x^2 f(x)$

$y' = x^2 f'(x) + f(x)\cdot(2x) = x^2 f'(x) + 2x f(x)$

ⓑ $y = \dfrac{f(x)}{x^2}$

$y' = \dfrac{x^2 f'(x) - f(x)\cdot(2x)}{(x^2)^2} = \dfrac{x f'(x) - 2f(x)}{x^3}$

ⓒ $y = \dfrac{x^2}{f(x)}$

$y' = \dfrac{f(x)\cdot(2x) - x^2 f'(x)}{[f(x)]^2} = \dfrac{2x f(x) - x^2 f'(x)}{[f(x)]^2}$

70. cont'd (d) $y = \dfrac{1 + x f(x)}{\sqrt{x}} = \dfrac{1 + x f(x)}{x^{\frac{1}{2}}}$

$$y' = \frac{x^{\frac{1}{2}} \cdot \frac{d}{dx}[1 + x f(x)] - [1 + x f(x)] \frac{d}{dx}(x^{\frac{1}{2}})}{(x^{\frac{1}{2}})^2}$$

$$= \frac{x^{\frac{1}{2}}[0 + x f'(x) + f(x) \cdot 1] - [1 + x f(x)] \cdot \frac{1}{2} x^{-\frac{1}{2}}}{x}$$

$$= \frac{x^{\frac{3}{2}} f'(x) + x^{\frac{1}{2}} f(x) - \frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} x^{\frac{1}{2}} f(x)}{x}$$

$$= \frac{x^{\frac{3}{2}} f'(x) + \frac{1}{2} x^{\frac{1}{2}} f(x) - \frac{1}{2} x^{-\frac{1}{2}}}{x}$$

$$= \frac{\frac{1}{2} x^{-\frac{1}{2}} [2x^2 f'(x) + x f(x) - 1]}{x}$$

$$= \frac{1}{2} x^{-\frac{3}{2}} [2x^2 f'(x) + x f(x) - 1]$$

72. $f(x) = x^3 + 3x^2 + x + 3$

$f'(x) = 3x^2 + 6x + 1$

$f(x)$ has a horizontal tangent when $f'(x) = 0$

$$3x^2 + 6x + 1 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} = \frac{-6 \pm 2\sqrt{6}}{6} = \frac{-3 \pm \sqrt{6}}{3}$$

74. parallel to $y = 1 + 3x$

need to find when $y' = 3$

$y = x\sqrt{x} = x \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$

$3 = \frac{3}{2}x^{\frac{1}{2}}$

$2 = x^{\frac{1}{2}}$

$x = 4$

when $x = 4$ $y = 4\sqrt{4} = 8$

tangent line:

when $x = 4$ $y' = \frac{3}{2}(4)^{\frac{1}{2}} = \frac{3}{2}(2) = 3$ pt $(4, 8)$

$y - 8 = 3(x - 4)$

$y = 3x - 4$