9. f and g are cont and $f(3)=5$
and $\lim_{x\to 3}[2f(x)-g(x)]=4$

Since f and g are cont we know
$\lim_{x\to 3}[2f(x)-g(x)] = 2f(3)-g(3)$

so $2f(3)-g(3)=4$
$2(5)-g(3)=4$
$g(3)=6$

(*) 23. $R(x)=x^2+\sqrt{2x-1}$

$h(x)=x^2$ is a polynomial so h is cont on $(-\infty,\infty)$
$g(x)=\sqrt{x}$ is a root function so g is cont on $[0,\infty)$
$K(x)=2x-1$ is a polynomial so K is cont on $(-\infty,\infty)$

$\star \rightarrow$ $f(x)=\sqrt{2x-1}$
$f(x)=g(K(x))$ is cont on the domain of f which is $[\frac{1}{2},\infty)$

Thus $R(x)=h(x)+f(x)=x^2+\sqrt{2x-1}$ is cont on $[\frac{1}{2},\infty)$

(*) 24. $h(x) = \dfrac{\sin(x)}{x+1}$

$g(x) = \sin(x)$ is a trig function so g is cont on $(-\infty, \infty)$

$f(x) = x+1$ is a polynomial so f is cont on $(-\infty, \infty)$

Then $h(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sin(x)}{x+1}$

☆ → is cont on $(-\infty, -1) \cup (-1, \infty)$ since $x+1 \neq 0$

25. $h(x) = \cos(1-x^2)$

$f(x) = \cos(x)$ is a trig function so f is cont on $(-\infty, \infty)$

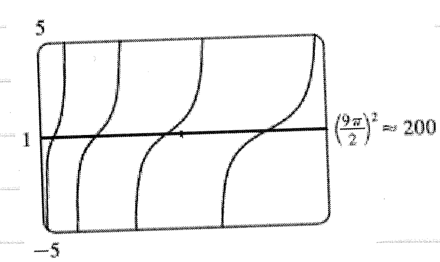
$g(x) = 1-x^2$ is a polynomial so g is cont on $(-\infty, \infty)$

Then $h(x) = f(g(x)) = \cos(1-x^2)$ is cont on $(-\infty, \infty)$

30. $y = \tan(\sqrt{x})$ Let $f(x) = \tan(\sqrt{x})$

$g(x) = \tan(x)$ is a trig function so it is cont on its domain which is $\mathbb{R}$ except $x = \frac{\pi}{2} + \pi n$ (n = integer)

$h(x) = \sqrt{x}$ is a root function so h is cont on its domain which is $[0, \infty)$



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30. cont'd Then $f(x) = g(h(x)) = \tan(\sqrt{x})$ is discontinuous when
$x < 0$ and when $\sqrt{x} = \frac{\pi}{2} + \pi n$
$x = (\frac{\pi}{2} + \pi n)^2$
n: nonnegative integer

32. $\lim_{x \to \pi} \sin(x + \sin x)$

Let $f(x) = \sin(x + \sin x)$
$g(x) = \sin(x)$ is a trig function so
g is cont on $(-\infty, \infty)$
$h(x) = x$ is a polynomial so h is cont on $(-\infty, \infty)$

Then $f(x) = g(h(x) + g(x)) = \sin(x + \sin x)$
is cont on $(-\infty, \infty)$

Then $\lim_{x \to \pi} f(x) = f(\pi) = \sin(\pi + \sin \pi) = \sin(\pi) = 0$

34. $\lim_{x \to 2} (x^3 - 3x + 1)^{-3} = \lim_{x \to 2} \frac{1}{(x^3 - 3x + 1)^3}$

$f(x) = \frac{1}{(x^3 - 3x + 1)^3}$ is a rational function
So f is cont on its domain.
Since $[(2)^3 - 3(2) + 1]^3 \neq 0$ then $x = 2$ is in the domain of f. Thus f is cont at $x = 2$.

Then $\lim_{x \to 2} f(x) = f(2) = 3^{-3} = \frac{1}{27}$

36. $f(x) = \begin{cases} \sin x & \text{if } x < \frac{\pi}{4} \\ \cos x & \text{if } x \geq \frac{\pi}{4} \end{cases}$

$g(x) = \sin(x)$ and $h(x) = \cos(x)$ are trig functions so we know f is cont on $(-\infty, \frac{\pi}{4}) \cup (\frac{\pi}{4}, \infty)$.

Now we need to check continuity at $x = \frac{\pi}{4}$

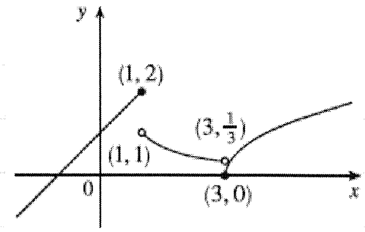
$f(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$$\left.\begin{aligned} \lim_{x \to \frac{\pi}{4}^-} f(x) &= \lim_{x \to \frac{\pi}{4}^-} \sin(x) = \sin(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2} \\ \lim_{x \to \frac{\pi}{4}^+} f(x) &= \lim_{x \to \frac{\pi}{4}^+} \cos(x) = \cos(\tfrac{\pi}{4}) = \tfrac{\sqrt{2}}{2} \end{aligned}\right\} \lim_{x \to \frac{\pi}{4}} f(x) = \frac{\sqrt{2}}{2}$$

since $\lim_{x \to \frac{\pi}{4}} f(x) = f(\frac{\pi}{4})$ f is cont at $x = \frac{\pi}{4}$

Thus f is cont on $(-\infty, \infty)$

38. $f(x) = \begin{cases} x+1 & \text{if } x \le 1 \\ \frac{1}{x} & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \ge 3 \end{cases}$



f is cont on $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$ since these pieces are cont on their domains.

Now need to check continuity at $x=1$ and $x=3$

$f(1) = 2$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (x+1) = 2$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \frac{1}{x} = 1$

$\lim\limits_{x \to 1} f(x)$ DNE since $\lim\limits_{x \to 1^-} f(x) \ne \lim\limits_{x \to 1^+} f(x$

f is discontinuous at $x=1$ since $\lim\limits_{x \to 1} f(x)$ DNE

f is cont at $x=1$ from the left since $\lim\limits_{x \to 1^-} f(x) = f(1)$

$f(3) = 0$

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \frac{1}{x} = \frac{1}{3}$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} \sqrt{x-3} = 0$

$\lim\limits_{x \to 3} f(x)$ DNE since $\lim\limits_{x \to 3^-} f(x) \ne \lim\limits_{x \to 3^+} f(x)$

f is discontinuous at $x=3$ since $\lim\limits_{x \to 3} f(x)$ DNE

f is cont at $x=1$ from the right since $\lim\limits_{x \to 3^+} f(x) = f(3)$

So f is cont on $(-\infty, 1] \cup (1, 3) \cup (3, \infty)$

41. $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$

Since each piece of f is a polynomial
f is cont on $(-\infty, 2) \cup (2, \infty)$.

Now we need f to be cont at $x = 2$

$f(2) = (2)^3 - c(2) = 8 - 2c$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (cx^2 + 2x) = 4c + 4$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^3 - cx) = 8 - 2c$$

to be cont at $x = 2$ $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$

so $4c + 4 = 8 - 2c$

$\therefore c = \frac{2}{3}$

42. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2<x<3 \\ 2x-a+b & \text{if } x \geq 3 \end{cases}$

Each piece of f is cont on its domain so f is cont on $(-\infty,2) \cup (2,3) \cup (3,\infty)$

Need f to be cont at $x=2$ and $x=3$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x+2)(x-2)}{x-2}$$
$$= \lim_{x\to 2^-} (x+2)$$
$$= 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (ax^2-bx+3) = 4a-2b+3$$

For f to be cont at $x=2$ $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

so $4 = 4a-2b+3$
$4a-2b=1$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2-bx+3) = 9a-3b+3$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x-a+b) = 6-a+b$$

For f to be cont at $x=3$ $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^+} f(x)$

so $9a-3b+3 = 6-a+b$
$10a-4b=3$

Now we have ① $4a-2b=1$ and ② $10a-4b=3$ [solve system of equations]

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42 cont'd

$-2(4a-2b) = -2(1) \rightarrow -8a + 4b = -2$

$$\begin{aligned} -8a + 4b &= -2 \\ \underline{10a - 4b} &\underline{= 3} \\ 2a &= 1 \\ a &= \tfrac{1}{2} \\ b &= \tfrac{1}{2} \end{aligned}$$