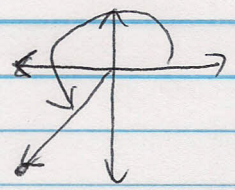
Review Solutions 6.3-6.5

5) $-1-i$ $\quad r = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{-1}\right) = \frac{\pi}{4}$



So, $\theta = \frac{5\pi}{4}$

$-1-i = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

6) $4+5i$ $\quad \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

7) $4+5i$ $\quad \sqrt{4^2+5^2} = \sqrt{41}$

8) $(-2,1)$ to $(2,5)$ $\quad \langle 2-(-2), 5-1\rangle = \langle 4,4\rangle$

9) a] $2u+v$

$2\langle -3,-3\rangle + \langle 4,2\rangle = \langle -6,-6\rangle + \langle 4,2\rangle = \langle -2,-4\rangle$

b] $u \cdot v = \langle -3,-3\rangle \cdot \langle 4,2\rangle = -12 + -6 = -18$

c] $u \cdot v \cdot w = -18\langle 1,-4\rangle = \langle -18, 72\rangle$

d] $3u - 2w = 3\langle -3,-3\rangle - 2\langle 1,-4\rangle$

$= \langle -9,-9\rangle - \langle 2,-8\rangle = \langle -9-2, -9+8\rangle$

$= \langle -11,-1\rangle$

10) $z \cdot w = 6 \cdot 3\left[\cos\left(\frac{5\pi}{7}+\frac{\pi}{2}\right) + i\sin\left(\frac{5\pi}{7}+\frac{\pi}{2}\right)\right]$

$= 18\left[\cos\left(\frac{10+7}{14}\right)\pi + i\sin\left(\frac{10+7}{14}\right)\pi\right]$

$= 18\left[\cos\left(\frac{17\pi}{14}\right) + i\sin\left(\frac{17\pi}{14}\right)\right]$

$$\frac{z}{w} = \frac{6}{3}\left[\cos\left(\frac{10-7}{14}\right)\pi + i\sin\left(\frac{10-7}{14}\right)\pi\right]$$

$$= 2\left[\cos\left(\frac{3\pi}{14}\right) + i\sin\left(\frac{3\pi}{14}\right)\right]$$

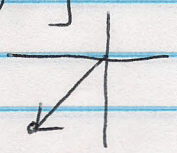
11) a] $3\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\right]$

$$= 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

b] $3\left(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \frac{-3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

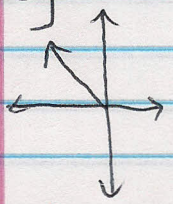
12) a] $-1-i$ $\quad r = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$ $\quad \theta = \tan^{-1}\left(\frac{-1}{-1}\right) = 45 = \frac{\pi}{4}$

3rd Quad: $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$



$$= \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

b] $-3+4i$ $\quad r = \sqrt{9+16} = 5$ $\quad \theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.13°$



2nd Quad: $180 - 53.13 = 126.87°$

$$= 5\left(\cos 126.87° + i\sin 126.87°\right)$$

13) initial pt: (x_1, y_1) $\quad$ terminal pt: $(7,-1)$

$$\langle 7-x_1, -1-y_1\rangle = \langle 6,-3\rangle$$

$7-x=6$ $\quad -1-y_1=-3$

$-x=-1$ $\quad -y_1=-2$

$x=1$ $\quad y_1=2$ $\quad (1,2)$

14) Use De Moivre's: $\left(\sqrt{3}\right)^6\left[\cos\left(\frac{\pi}{7}\cdot 6\right)+i\sin\left(\frac{\pi}{7}\cdot 6\right)\right]$

a] $= 27\left(\cos\frac{6\pi}{7} + i\sin\frac{6\pi}{7}\right)$

b] Distribute 27 in a]. Use calc: $-24.326 + 11.715i$ in radians

15) Rewrite in trig: $-\sqrt{3}-i$ $r=\sqrt{3+1}=2$

$\theta=\tan\left(\frac{-1}{-\sqrt{3}}\right)=30^\circ$

3rd Quad: $180+30=210^\circ$

$\left(-\sqrt{3}-i\right)^5=\left[2(\cos 210+i\sin 210)\right]^5$

$=2^5(\cos 210\cdot 5+i\sin 210\cdot 5)$

$=32(\cos 1050+i\sin 1050)$ $\quad 1050-720=330^\circ$

$=32(\cos 330^\circ+i\sin 330^\circ)$

$=32\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)=16\sqrt{3}-16i$

16) $(3+4i)(2-5i)=6-15i+8i-20i^2$ FOIL

$=26-7i$

17) $\frac{3+4i}{2-5i}\frac{(2+5i)}{(2+5i)}=\frac{6+15i+8i+20i^2}{4-25i^2}=\frac{-14+23i}{29}$

18) a] $\frac{7\pi}{6}-\frac{2\pi}{3}=\frac{7\pi}{6}-\frac{4\pi}{6}=\frac{3\pi}{6}=\frac{\pi}{2}$

b] $u=\langle -2,-3\rangle$ $v=\langle 5,2\rangle$

$\cos\theta=\frac{u\cdot v}{\|u\|\|v\|}$ $\quad \|u\|=\sqrt{4+9}=\sqrt{13}$ $\quad \|v\|=\sqrt{25+4}=\sqrt{29}$

$\cos\theta=\frac{-2(5)+(-3)(2)}{\sqrt{13}\cdot\sqrt{29}}=\frac{-16}{\sqrt{377}}$

$\theta=\cos^{-1}\left(\frac{-16}{\sqrt{377}}\right)=145.49^\circ$

19) slope of $u=\frac{45}{15}=3$ slope of $v=\frac{12}{-5}$ neither

b] slope of $u=\frac{4}{8}=\frac{1}{2}$ slope of $v=\frac{-1}{-2}=\frac{1}{2}$ parallel