

UPPER AND LOWER BOUNDS

A *positive* integer, a , is the upper bound of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-a)$ results in a polynomial function with *nonnegative* coefficients and remainder.

A *negative* integer, b , is the lower bound of the real zeros of a polynomial function $f(x)$ if $f(x) \div (x-b)$ results in a polynomial function that has coefficients and remainder with alternating signs.

****Note:** A coefficient of 0 may be positive or negative as needed to fit the pattern for upper or lower bound.

$$\text{LOWER BOUND} \leq \text{REAL ZEROS} \leq \text{UPPER BOUND}$$

Ex 1: $f(x) = 2x^3 - x^2 - 8x + 4$

x	2	-1	-8	4
1	2	1	-7	-3
2	2	3	-2	0
3	2	5	7	25
-1	2	-3	-5	9
-2	2	-5	2	0

zero 2
upper bound 3
lower bound -2

$-2 \leq \text{real zeros} \leq 3$

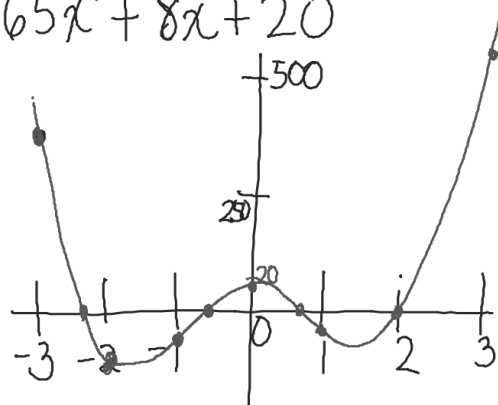
$y\text{int} = (0, 4)$

$\frac{p}{q} = \frac{\pm(1, 2, 4)}{\pm(1, 2)} = \pm\left(1, \frac{1}{2}, 2, 4\right)$

possible rational zero = $\left(\frac{1}{2}\right)$

$$f(x) = 12x^4 + 4x^3 - 65x^2 + 8x + 20$$

x	12	4	-65	8	20	
1	12	16	-49	-41	-21	
zero	2	12	28	-9	-10	0
u.B.	3	12	40	55	173	539
-1	12	-8	-57	65	-45	(0,20)
-2	12	-20	-25	58	-96	
LB	-3	12	-32	31	-85	275



$-3 \leq \text{real zeros} \leq 3$

possible rational zeros

$$\frac{p}{q} = \frac{\pm(1, 2, 4, 5, 10, 20)}{\pm(1, 2, 3, 4, 6, 12)}$$

$$\pm \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{2}{3}, \frac{5}{6}, \frac{5}{12} \right)$$