	Students	Adults	Total
T-Shirts	0.267	0.383	0.65
Sweatshirts	0.117	0.233	0.35
Total	0.384	0.616	1

The marginal relative frequencies, recorded in the row and column labeled Total, are found by adding the joint relative frequencies in each row and column.

8. Possible answer: First find the total number of individuals in the survey or study by adding all the values in the two-way table. Next find the joint relative frequencies by dividing each cell in the two way table by the total number of individuals in the survey. Record these results in a new table. Finally, add the rows and columns in this new table to find the marginal relative frequencies.

9a. The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of customers that were surveyed. There were 118 customers surveyed, so each value is divided by 118.

	Satisfied	Dissatisfied	Total
Team 1	0.17	0.07	0.24
Team 2	0.29	0.1	0.39
Team 3	0.29	0.08	0.37
Total	0.75	0.25	1

b. To determine the probability that a customer will be satisfied after working with a particular team, take the number of the customers that were satisfied after working with the team and divide by the total number of customers that worked with the team.
P(Being satisfied with Team 1) $= 20 \div 28 = 0.71$
P(Being satisfied with Team 2) $= 34 \div 46 = 0.74$
P(Being satisfied with Team 3) $= 34 \div 44 = 0.77$

c. Team C has the highest rate of customer satisfaction.

10. The value is always equal to 1. It represents the portion of the people in the survey who are in the survey, so it must equal 1.

11. Maria has made an error. If you add up all the joint relative frequencies in her table the sum is 1.1. The sum should be 1. The sum of the joint relative frequencies in Brennan's table is 1.

12. 0.48 is a little less than half. A little less than half of the 107 brownies and muffins is around 50.

13a. The joint relative frequencies are determined by dividing the value in each cell in the table by the total number of questions asked. There were 120 questions asked, so each value should be divided by 120.

		Work less than 5 miles from home?		
		Yes	No	Total
Use new system?	Yes	0.2	0.27	0.47
	No	0.37	0.17	0.54
	Total	0.57	0.44	1

b. P(Works close to home would use the new system) $= 24 \div 68 = 0.35$

c. P(Would use the system lives far from home) $= 32 \div 56 = 0.57$

TEST PREP

14. B; there were $9 + 14 = 23$ teachers polled.

15. Use a table to find the joint and marginal relative frequencies:

	Yes	No	Total
Junior	0.223	0.313	0.536
Senior	0.167	0.298	0.464
Total	0.390	0.610	1

Inspecting the table shows that C is correct.

16. $0.25 + x = 0.3$
$x = 0.05$

CHALLENGE AND EXTEND

17. To find the marginal relative frequencies, add the rows and columns in the joint relative frequency table.

	Yes	No	Total
Children	0.125	0.1	0.225
Teenagers	0.725	0.05	0.775
Total	0.85	0.15	1

Marginal relative frequencies are in the row and column labeled Total.

18. P(teenager purchasing a ticket book) $=$ $0.725 \div 0.775 = 0.94 = 94\%$

19. According to the table 12.5% of the fair attendees are children who will buy a ticket at the entrance. Because 12.5% of the 80 teenagers and children who attend the fair equals 10, 10 children will buy a ticket at the entrance.

20. According to the table, 10% of the fair attendees are children who did not buy a ticket at the entrance. According to the problem, 10% of the total fair goers is equal to 12. 10% of 120 is 12, so there are 120 children and teenagers attending the fair.

21. The total of the marginal relative frequencies is always 1, so $1 - 1 = 0$.

22. The maximum is 1. It can't be higher because that would represent more than 100% of the data.

7-5 COMPOUND EVENTS

CHECK IT OUT!

1a. Each student can only vote once.

b. $P(\text{votes for Kline} \cup \text{voted for Vila})$
$= P(\text{votes for Kline}) + P(\text{voted for Vila})$
$= 20\% + 55\% = 75\%$

2a. $P(\text{king} \cup \text{heart})$
$= P(\text{king}) + P(\text{heart}) - P(\text{king} \cap \text{heart})$
$= \frac{4}{52} - \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$

b. $P(\text{red} \cup \text{face})$
$= P(\text{red}) + P(\text{face}) - P(\text{red} \cap \text{face})$
$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{8}{13}$

3. $61 - 28 = 33$ people got a hair styling and a manicure.
$P(\text{hair styling} \cup \text{manicure})$
$= P(\text{hair styling}) + P(\text{manicure})$
$- P(\text{hair styling} \cap \text{manicure})$
$= \frac{96}{160} + \frac{61}{160} - \frac{33}{160} = \frac{124}{160} = \frac{31}{40}$
The probability that a customer had a hair styling or a manicure is $\frac{31}{40}$.

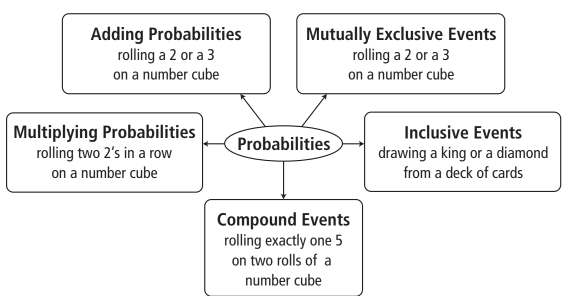
4. $P(\text{all choose different}) = \frac{{}_{62}P_5}{62^5}$
$= \frac{62 \cdot 61 \cdot 60 \cdot 59 \cdot 58}{62 \cdot 62 \cdot 62 \cdot 62 \cdot 62}$
≈ 0.8476
$P(\text{at least 2 choose same})$
$= 1 - P(\text{all choose different})$
$= 1 - 0.8476 \approx 0.152393394$
The probability that at least 2 customers bought the same style is 0.1524 or 15.24%.

THINK AND DISCUSS

1. If events A and B are mutually exclusive, $P(A \cap B) = 0$.
So, $P(A \cup B) = P(A) + P(B) - 0 = P(A) + P(B)$.

2. February 29 occurs only once every 4 years, and March 13 occurs once every year. You are more likely to share a birthday with someone if you were born on March 13.

3.



EXERCISES

GUIDED PRACTICE

1. inclusive events

2. A marble is either black or red.

3. $P(\text{red} \cup \text{blue})$
$= P(\text{red}) + P(\text{blue})$
$= \frac{13}{25} + \frac{2}{25} = \frac{3}{5}$

4. The car cannot turn both left and right;
$P(\text{left} \cup \text{right})$
$= P(\text{left}) + P(\text{right})$
$= 0.1 + 0.2 = 0.3$

5. $P(\text{greater than 5} \cup \text{odd})$
$= P(\text{greater than 5}) + P(\text{odd})$
$- P(\text{greater than 5} \cap \text{odd})$
$= \frac{5}{10} + \frac{5}{10} - \frac{2}{10} = \frac{4}{5}$

6. $P(8 \cup \text{less than 5})$
$= P(8) + P(\text{less than 5})$
$= \frac{1}{10} + \frac{4}{10} = \frac{1}{2}$

7. Pat least 1 even
$= 1 - P$odd $\cap$ odd
$= 1 - \frac{5}{10} \cdot \frac{4}{9} = \frac{7}{9}$

8. $400 \div 2 = 200$ students have a college degree and were married.
$P(\text{degree} \cup \text{married})$
$= P(\text{degree}) + P(\text{married}) - P(\text{degree} \cap \text{married})$
$= \frac{400}{650} + \frac{310}{650} - \frac{200}{650} = \frac{51}{65}$

9. $400 \div 2 = 200$ students have a college degree and were not married.
$650 - 310 = 340$ students were not married.
$P(\text{degree} \cup \text{not married})$
$= P(\text{degree}) + P(\text{not married})$
$- P(\text{degree} \cap \text{not married})$
$= \frac{400}{650} + \frac{340}{650} - \frac{200}{650} = \frac{54}{65}$

10. $650 - 400 = 250$ students do not have a college degree.
$310 - 200 = 110$ students were not married and do not have a college degree.
$P(\text{no degree} \cup \text{married})$
$= P(\text{no degree}) + P(\text{married})$
$- P(\text{no degree} \cap \text{married})$
$= \frac{250}{650} + \frac{310}{650} - \frac{110}{650} = \frac{9}{13}$

11. $P(\text{all choose different}) = \frac{{}_{8}P_6}{8^6}$
$= \frac{8 \cdot 7}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8}$
≈ 0.08
$P(\text{at least 2 choose same})$
$= 1 - P(\text{all choose different})$
$= 1 - 0.0769 = 0.92$
The probability that at least 2 employees purchased the same drink is 0.92 or 92%.

PRACTICE AND PROBLEM SOLVING

12. The jump rope is either red or green.

13. $P(\text{red} \cup \text{green})$
$= P(\text{red}) + P(\text{green})$
$= \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

14. $P(E \cup G)$
$= P(E) + P(G)$
$= \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

15. $P(E \cup \text{vowel})$
$= P(E) + P(\text{vowel}) - P(E \cap \text{vowel})$
$= \frac{1}{16} + \frac{4}{16} - \frac{1}{16} = \frac{1}{4}$

16. $98 \times \frac{1}{7} = 14$ teachers teach math.
$P(\text{woman} \cup \text{math})$
$= P(\text{woman}) + P(\text{math}) - P(\text{woman} \cap \text{math})$
$= \frac{42}{98} + \frac{14}{98} - \frac{8}{98} = \frac{24}{49}$

17. $98 - 42 = 56$ teachers are men.
$14 - 8 = 6$ math teachers are men.
$P(\text{man} \cup \text{math})$
$= P(\text{man}) + P(\text{math}) - P(\text{man} \cap \text{math})$
$= \frac{56}{95} + \frac{14}{98} - \frac{6}{98} = \frac{32}{49}$

18. $56 - 6 = 50$ teachers are men and don't teach math.
$98 - 14 = 84$ teachers do not teach math.
$P(\text{man} \cup \text{not math})$
$= P(\text{man}) + P(\text{not math}) - P(\text{man} \cap \text{not math})$
$= \frac{56}{98} + \frac{84}{98} - \frac{50}{98} = \frac{45}{49}$

19. $P(\text{no heart}) = \frac{39}{52} = 0.75$
Replacing the card after it is drawn means that the draw of the first card does not affect the draw of the second card. The events are independent.
$P(\text{at least one heart})$
$= 1 - P(\text{no heart})$
$= 1 - 0.75^{13} \approx 0.976$

20. No; possible answer: If event A is rolling a 3 on a number cube and event B is rolling a 4 on a number cube, then, the outcomes 1, 2, 5, and 6 are common to both A' and B'.

21. $P(\text{NBA} \cup CSI)$
$= P(\text{NBA}) + P(CSI)$
$= 0.22 + 0.15 = 0.37$
experimental because it is based on a small sample

22. the percent of schools that offer both music and dance classes

23. The minimum will occur when there is the largest possible intersection between the two events.
largest intersection is 19%
$P(\text{music} \cup \text{drama})$
$= P(\text{music}) + P(\text{drama}) - P(\text{music} \cap \text{drama})$
$= 87 + 19 - 19 = 87\%$
The minimum probability that music or drama are offered is 87%.
The maximum will occur when there is smallest possible intersection between the two events.
smallest intersection is 6%
$P(\text{music} \cup \text{drama})$
$= P(\text{music}) + P(\text{drama}) - P(\text{music} \cap \text{drama})$
$= 87 + 19 - 6 = 100\%$
The maximum probability that music or drama are offered is 100%.

24a. $P(\text{purple}) = \frac{2.25^2}{9^2} = \frac{2.25}{81} = \frac{1}{36}$

b. $P(\text{red}) = \frac{3^2 - 1.5^2}{9^2} = \frac{6.75}{81} = \frac{1}{12}$

c. $P(\text{red} \cup \text{blue})$
$= P(\text{red}) + P(\text{blue}) - P(\text{red} \cap \text{blue})$
$= \frac{1}{9} + \frac{16}{81} - \frac{1}{36} = \frac{91}{324}$

d. $P(\text{yellow})$
$= 1 - P(\text{not yellow})$
$= 1 - \frac{91}{234} = \frac{233}{324}$

25a. Possible answer: The probability that a person is born color-blind *or* male will be greater, because it includes more successful outcomes, such as color-blind females and non-color-blind males.

b. $P(\text{male} \cap \text{color-blind}) = 0.08 \bullet 52 = 4.16\%$
$P(\text{color-blind}) = 0.08 \bullet 52 + 0.005 \bullet 48 = 4.4$
$P(\text{male} \cup \text{color-blind})$
$= P(\text{male}) + P(\text{color-blind}) - P(\text{male} \cap \text{color-blind})$
$= 52 + 4.4 - 4.16 = 52.24\%$

26a. $P(3 \text{ and } 5 \cup 4 \text{ and } 4)$
$= P(3 \text{ and } 5) + P(4 \text{ and } 4)$
$= \frac{2}{36} + \frac{1}{36} = \frac{1}{12}$

b. $P(\text{one } 3) + \text{P}(\text{two } 3)$
$= 1 - P(\text{no 3s})$
$= 1 - \frac{25}{36} = \frac{11}{36}$

Since two 3's is also a small straight,
$\frac{11}{36} - \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$.

27. $P(\text{under 18} \cup \text{owner's property})$
$= P(\text{under 18}) + P(\text{owner's property})$
$- P(\text{under 18} \cap \text{owner's property})$
$0.95 = 0.8 + 0.64 - P(\text{under 18} \cap \text{owner's property})$
$P(\text{under 18} \cap \text{owner's property}) = 0.49$

28a. $P(\text{all male} \cup \text{all female})$
$= P(\text{all male}) + P(\text{all female})$
$= \frac{{}_{10}C_4}{{}_{24}C_4} + \frac{{}_{14}C_4}{{}_{24}C_4} \approx 0.11$

b. At any given point the committee must have at least one man or one woman. The probability is 1.

29a. $P(\text{vowel}) = \frac{42}{100} = 0.42$

b. $P(Y) = \frac{2}{100} = 0.02$

c. $P(\text{vowel or } Y) = \frac{44}{100} = 0.44$;
it is the sum of the probabilities.

30. Possible answer: There are a total of 4 outcomes: $\{\text{HH, HT, TH, TT}\}$. Three of these have at least one heads. The probability is $\frac{3}{4}$.
The event "at least one heads" is the complement of the event "no heads." The probability is
$1 - \left(\frac{1}{2} \bullet \frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$.

TEST PREP

31. D
$P(5 \text{ and } 2 \text{ and } 7)$
$= \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} = \frac{1}{1000}$

32. F
$P(\text{tails}) = \frac{1}{2}$

33. D

$P(5 \cup \text{greater than } 3)$
$= P(5) + P(\text{greater than } 3) - P(5 \cap \text{greater than } 3)$
$= \frac{1}{6} + \frac{3}{6} - \frac{1}{6} = \frac{1}{2}$

34. 1; the complement of an event contains all unfavorable outcomes. The probability of an event or its complement is the probability of all outcomes, 1.

CHALLENGE AND EXTEND

35. $P(\text{at least 2 people share a birthday})$
$= 1 - P(\text{no one shares a birthday})$
$= 1 - \frac{_{365}P_{10}}{365^{10}}$
$= 1 - \left(\frac{1}{365}\right)^{10}\left(\frac{365!}{(365-10)!}\right) \approx 0.12$

36. $P(\text{ferry} \cup \text{train})$
$= P(\text{ferry}) + P(\text{train}) - P(\text{ferry} \cap \text{train})$
$= \frac{47}{162} + \frac{80}{162} - \frac{27}{162} = \frac{50}{81}$

37. $P(\text{ferry} \cup \text{rental car})$
$= P(\text{ferry}) + P(\text{rental car}) - P(\text{ferry} \cap \text{rental car})$
$= \frac{47}{162} + \frac{94}{162} - \frac{24}{162} = \frac{13}{18}$

38. $P(\text{train} \cap \text{ferry} \cup \text{train} \cap \text{rental car})$
$= P(\text{train} \cap \text{ferry}) + P(\text{train} \cap \text{rental car})$
$- P(\text{train} \cap \text{ferry} \cap \text{train} \cap \text{rental car})$
$= \frac{27}{162} + \frac{19}{162} - \frac{11}{162} = \frac{35}{162}$

39. $P(B \cup C)$
$= P(B) + P(C) - P(B \cap C)$
$= 0.3 + 0.7 - 0.1 = 0.9$

40. $P(A \cup B \cup C)$
$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap C)$
$- P(A \cap B) + P(A \cap B \cap C)$
$= 0.5 + 0.3 + 0.7 - 0.1 - 0.3 - 0.2 + 0.1 = 1$

41. $P(A \cup C)$
$= P(A) + P(C) - P(A \cap C)$
$= 0.5 + 0.7 - 0.3 = 0.9$
$P(B \cap (A \cup C))$
$= P(B) + P(A \cup C) - P(B \cup (A \cup C))$
$= 0.3 + 0.9 - 1 = 0.2$

READY TO GO ON?

1. The total number of customers in the table is 234. Calculate joint relative frequencies by dividing each entry by the total number. Then add each row and column to calculate the marginal relative frequencies.

		Reads *Look Around*		
		Yes	**No**	**Total**
Reads *Super News*	**Yes**	$\frac{62}{234} \approx 0.265$	$\frac{15}{235} \approx 0.064$	0.329
	No	$\frac{21}{234} \approx 0.090$	$\frac{136}{234} \approx 0.581$	0.671
	Total	0.355	0.645	1

2. The marginal relative frequency for the row with the condition "Reads Super News" is about 0.329, or 32.9%. Out of these, about 0.265, or 26.5%, also read Look Around. The conditional relative frequency is $\frac{0.265}{0.329} \approx 0.805$, or about 80.5%.

3. The marginal relative frequency for the column with the condition "Reads Look Around" is about 0.355, or 35.5%. Out of these, about 0.265, or 26.5%, also read Super News. The conditional relative frequency is about $\frac{0.265}{0.355} \approx 0.747$, or about 74.7%. (Note the probabilities should not be rounded until the last step.)

4. $P(\text{even} \cup 1)$
$= P(\text{even}) + P(1)$
$= \frac{15}{30} + \frac{1}{30} = \frac{8}{15}$

5. $P(\text{even} \cup \text{multiple of } 7)$
$= P(\text{even}) + P(\text{multiple of } 7)$
$- P(\text{even} \cap \text{multiple of } 7)$
$= \frac{15}{30} + \frac{4}{30} - \frac{2}{30} = \frac{17}{30}$

6. $85 - 60 = 25$ part-time employees.
$85 - 40 = 45$ not married employees.
$P(\text{part time} \cap \text{not married})$
$= 1 - P(\text{full time} \cup \text{married})$
$= 1 - [P(\text{full time}) + P(\text{married})$
$- P(\text{full time} \cap \text{married})]$
$= 1 - \left[\frac{60}{85} + \frac{40}{85} - \frac{30}{85}\right] = \frac{15}{85}$
$P(\text{part time} \cup \text{not married})$
$= P(\text{part time}) + P(\text{not married})$
$- P(\text{part time} \cap \text{not married})$
$= \frac{25}{85} + \frac{45}{85} - \frac{15}{85} = \frac{11}{17}$

STUDY GUIDE: REVIEW

1. dependent events

2. permutation

3. conditional relative frequency

7-1 PERMUTATIONS AND COMBINATIONS

4. $7 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 7{,}000{,}000$
There are 7,000,000 possible different 7-digit telephone numbers.