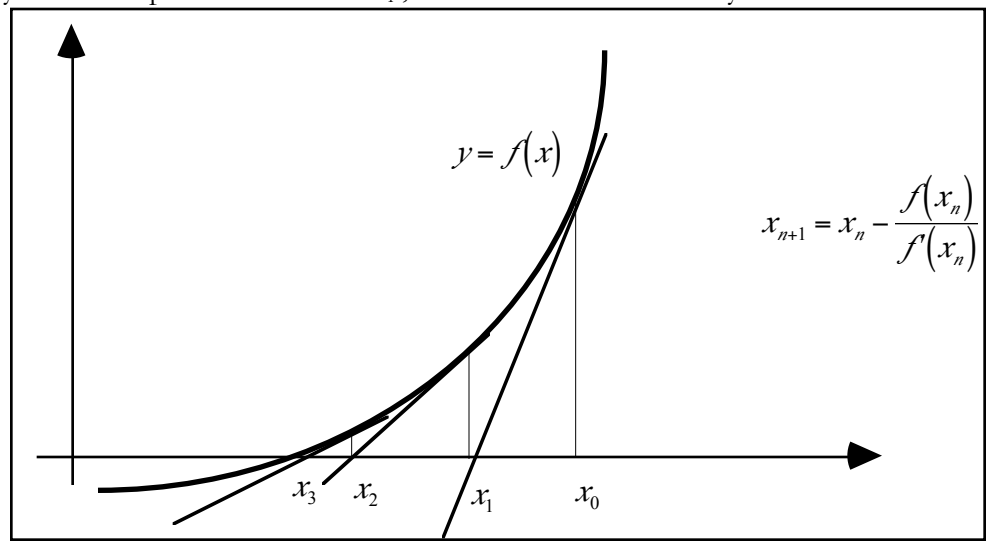
Newton's Method of Roots - Classwork

The concept of Newton's method of finding roots is based on making an initial guess x_0 of the root of the function $f(x)$. From there, you come up with a series of x_i , which will be successively closer to the root of $f(x)$.



Sample problems - do by hand. You may use your calculators only for basic operations. Complete two iterations of Newton's method using the indicated initial guess.

1. $f(x) = x^2 - 2 \qquad x_1 = 1$

x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$

2. $f(x) = x^3 - x^2 - 2x - 2 \qquad x_1 = 2$

x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$

1. On all calculators, place the function which you wish the solution to in Y1.
2. Place the derivative of the function in Y2.
3. It is best to set your MODE to FLOAT giving the maximum amount of accuracy. However, if you are asked to apply Newton until two approximations differ by less than d decimal places, you may wish to set your MODE to d decimal places first.
4. Make your initial Guess and store it as X. Then type X-Y1/Y2 STO X. You will get the first iteration.
5. Press ENTER and you will get successive iterations.
6. To do another problem, simply repeat the steps above. You can use 2nd ENTER so you do not have to type the command on line 4.

3. Find the roots of $f(x) = 5x^2 - 4x - 7$

4. Find the smallest positive root of $f(x) = \sin x - x^2 + 1$

Warning: Do not depend on this routine on the A.P. exam. They usually give you a problem to calculate a root of a function through one or two iterations where calculators are not allowed. You must know the formula and how to apply it algebraically. Newton's method fails if the derivative of the function is zero at your original guess.

Newton's Method of Roots - Homework

In the following exercises, use Newton's Method by hand to find the first iteration in approximating the zeros and continue the process with the calculator until 2 successive approximations differ by less than .001.

1. $f(x) = x^3 + x + 3$ (Initial Guess = 1)

2. $f(x) = x^5 + x + 3$ (Initial Guess = 1)

3. $f(x) = x^2 - \dfrac{1}{x-1}$ (Initial Guess = 3)

4. $f(x) = x^4 - 10x^2 - 7$ (Initial Guess = 3)

5. $f(x) = 2x + \sin(x+1)$ (Initial Guess $= -\dfrac{\pi}{6}$)

6. $f(x) = x^3 - \cos x + 2$ (Initial Guess = 3)

7. $f(x) = \sin x$ has a root at $x = 0$. Suppose your initial guess is 1.3. Show that Newton's method does not work here and formulate a reason why it does not. Look at the picture visually in order to understand what is happening.

8) Newton's method can be used to determine square roots. For $x = \sqrt{a}$, use the equation $f(x) = x^2 - a$. Use this method to find $\sqrt{7}$ and $\sqrt{214}$. Use this method to find $\sqrt[5]{5}$

9. Show that Newton's method fails to converge for the function $f(x) = x^{1/3}$ using $x_1 = .1$