Geometry Notes Section 12-7 May 1

Writing Equations of Circles: Completing the Square

Factor each of these trinomials:

$x^2 + 8x + \underline{16}$

$(x+4)(x+4) = (x+4)^2$ $\left(\frac{8}{2}\right)^2$

$x^2 - 10x + 25$

$(x-5)(x-5) = (x-5)^2$ $\left(\frac{-10}{2}\right)^2 = 25$

A **trinomial square** is a trinomial with a factoring pattern like the two trinomials above.

The pattern is $A^2 + 2AB + B^2 = (A+B)^2$ or $A^2 - 2AB + B^2 = (A-B)^2$.

Completing the square (CTS) is a method used to create trinomial squares.

Find the value of *c* that completes the square. Factor the trinomial.

$x^2 + 6x + c = x^2 + 6x + \underline{9} = (x+3)^2$ $\left(\frac{6}{2}\right)^2 = 9$

$y^2 - 20y + c = y^2 - 20y + \underline{100} = (y-10)^2$ $\left(\frac{-20}{2}\right)^2 = 100$

$x^2 + 14x + c = x^2 + 14x + \underline{49} = (x+7)^2$ $\left(\frac{14}{2}\right)^2$

$y^2 - 18y + c$ $81 = y^2 - 18y + 81 = (y-9)^2$

Recall that the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$. If an equation of a circle is not in this form, we can use completing the square to rewrite the equation in this form.

Use CTS to identify the center and radius of the circle.

1. $x^2 + 2x + y^2 - 10y = 10$

$(x^2 + 2x + \underline{1}) + (y^2 - 10y + \underline{25}) = 10 + \underline{1} + \underline{25}$ Use CTS twice and add numbers to right side of equation to balance it

$\left(\frac{2}{2}\right)^2 = 1$ $\left(\frac{-10}{2}\right)^2 = 25$

$(x + \underline{1})^2 + (y - \underline{5})^2 = \underline{36}$ Factor and simplify right side of equation

$x-(-1)$ r^2

Center (-1 , 5) radius = 6

h, k

Use CTS to identify the center and radius of the circle.

2. $x^2 - 12x + y^2 + 22y = -148$

$x^2 - 12x + \underline{36} + y^2 + 22y + \underline{121} = -148 + 36 + 121$

$\left(\frac{-12}{2}\right)^2$ 11^2

$(x-6)^2 + (y+11)^2 = 9$ $C(6, -11)$ $r = 3$

$y-(-11)$ r^2

3. $x^2 + y^2 + 40x + 300 = 0$ $C(-20, 0)$ $r = 10$

$x^2 + 40x + y^2 = -300$

$x^2 + 40x + \underline{400} + y^2 = -300 + \underline{400}$

$(x+20)^2 + (y-0)^2 = 100$

$x-(-20)$