

4. The null hypothesis will be rejected at  $\alpha=0.05$  if the test statistic exceeds:
- (a) 3.84
  - (b) 5.99
  - (c) 7.81
  - (d) 9.49
  - (e) 14.07
5. The approximate  $P$ -value is:
- (a) Between 0.100 and 0.900
  - (b) Between 0.050 and 0.100
  - (c) Between 0.025 and 0.050
  - (d) Between 0.010 and 0.025
  - (e) Between 0.005 and 0.010

**Part 2: Free Response**

*Answer completely, but be concise. Write sequentially and show all steps.*

6. A recent estimate by a large distributor of gasoline claims that 60% of all cars stopping at their service stations chose unleaded gas and that super unleaded and regular were each selected 20% of the time. In order to check the validity of these proportions, a study was conducted of cars stopping at the distributor's service stations in a large city. The results were as follows:

		Gasoline Selected	
Regular	Unleaded	Super Unleaded	
51	261	88	

Carry out a significance test of the distributor's claim.

WE WILL TEST THE HYPOTHESES

$$H_0: P_1 = .6, P_2 = .2, P_3 = .2$$

$H_A$ : AT LEAST ONE OF THESE PROPORTIONS IS DIFFERENT, WHERE

$P_1$  = THE PROPORTION OF CARS CHOOSING UNLEADED GAS,  $P_2$  = THE PROPORTION CHOOSING SUPER UNLEADED, AND  $P_3$  = THE PROPORTION CHOOSING REGULAR.

UNDER THE NULL HYPOTHESIS, THE EXPECTED COUNTS WOULD BE

REG	UNLEADED	SUPER UNLEADED
80	240	80

SINCE ALL EXPECTED COUNTS  $> 5$ ,

WE PROCEED WITH THE CHI-SQUARE GOODNESS OF FIT TEST.

$$\chi^2 = 13.15$$

$P(\chi^2(2) > 13.15) = 0.0014$ . SINCE THE PROBABILITY OF OBSERVING COUNTS LIKE THE ONES ABOVE (ASSUMING THE NULL HYPOTHESIS IS TRUE) IS ~~SEE~~ SO LOW, WE CONCLUDE THE DISTRIBUTION OF THE PROPORTIONS OF THE TYPES OF GAS SELECTED IS PROBABLY DIFFERENT THAN THE DISTRIBUTOR'S CLAIM.

7. A study was performed to examine the personal goals of children in grades 4, 5, and 6. A random sample of students was selected from grades 4, 5, and 6 from schools in Georgia. The students received a questionnaire regarding achieving personal goals. They were asked what they would most like to do at school: make good grades, be good at sports, or be popular. Results are presented in the table below by the sex of the child.

	Boys	Girls	
Make good grades	96	295	391
Be popular	32	45	77
Be good at sports	94	40	134
	222	380	602

- (a) Which type of chi-square procedure is appropriate in this setting? Justify your answer.

SINCE WE HAVE ONE RANDOM SAMPLE WHERE THE RESPONDENTS ARE CATEGORIZED BY TWO VARIABLES, WE SHOULD USE A CHI-SQUARE TEST OF ASSOCIATION; TO TEST THE INDEPENDENCE OF THE TWO VARIABLES "GENDER" AND "GOALS" AMONG 4, 5, 6 GRADERS.

- (b) Carry out the inference procedure you selected in part (a).

OUR HYPOTHESES:  $H_0$ : "GENDER" AND "GOALS" ARE INDEPENDENT FOR GEORGIAN 4, 5, AND 6 GRADERS.

$H_A$ : "GENDER" AND "GOALS" ARE NOT INDEPENDENT.

ASSUME OUR "RANDOM SAMPLE" IS AN SRS, WHERE  $n < \frac{1}{10}$  (ALL 4, 5, 6 GRADERS)

THE EXPECTED COUNTS FOR THE TWO-WAY TABLE ARE... (ALL  $> 5$ )

	B	G
GOOD GRADES	144.189	246.81
POPULAR	28.395	48.604
SPORTS	49.415	84.585

$$\chi^2 = 89.966$$

$$P(\chi^2 > \chi^2) = 2.9 \times 10^{-20}$$

SINCE WE HAVE A VERY LOW P-VALUE, WE REJECT  $H_0$ .

THERE IS VERY STRONG EVIDENCE THAT "GENDER" AND "GOALS" ARE ASSOCIATED FOR THESE GEORGIAN GRADE SCHOOLS.

8. Can you increase the response rate for a mail survey by contacting the respondents before they receive the survey? A study designed to address this question compared three groups of subjects. The first group received a preliminary letter about the survey, the second group was phoned, and the third received no preliminary contact. A positive response was defined as returning the survey within two weeks. Here are the counts:

Response	Intervention			
	Letter	Phone call	None	
Yes	171	146	118	435
No	220	68	455	743
Total	391	214	573	1178

- (a) For each intervention find the proportion of positive responses.

PROPORTIONS OF POSITIVE RESPONSES:

LETTER	PHONE	NONE
0.437	0.682	0.206

- (b) Translate the question of interest into appropriate null and alternative hypotheses for this problem.

$H_0$ : THE PROPORTIONS OF POSITIVE RESPONSES IS THE SAME FOR EACH TYPE OF INTERVENTION (LETTER, PHONE, OR NONE)

$H_A$ : AT LEAST ONE OF THE PROPORTIONS IS NOT THE SAME.

- (c) Give the test statistic, degrees of freedom, and the P-value for the significance test. What do you conclude?

$$\chi^2 = 163.413$$

$$df = 2$$

$$P\text{-VALUE} = 3.28 \times 10^{-34}$$

WE HAVE VERY SIGNIFICANT EVIDENCE THAT THE PROPORTIONS OF POSITIVE RESPONSES ARE NOT THE SAME FOR EACH GROUP.

I pledge that I have neither given nor received aid on this test. \_\_\_\_\_