

The following exercises are counting problems like those from the last four sections. They test your ability to choose the appropriate counting technique.

**A**

1. Three identical door prizes are to be given to three lucky people in a crowd of 100. In how many ways can this be done?
2. The license plates in a certain state consist of 3 letters followed by 3 nonzero digits. How many such license plates are possible?
3. How many 4-digit numbers **(a)** contain no 0's? **(b)** contain no 1's? **(c)** begin with an even digit and end with an odd digit?
4. A student must take four final exams, scheduled by computer, during the morning and afternoon testing periods on Monday through Friday of exam week. If the order of the student's four exams is important, in how many ways can a computer schedule the exams?
5. **a.** A railway has 30 stations. On each ticket, the departure station and the destination station are printed. How many different tickets are possible?  
**b.** If a ticket can be used in either direction between two stations, how many different tickets are needed?
6. In how many ways can the letters of each of the following words be arranged?
  - a. RADISH
  - b. SQUASH
  - c. TOMATO



7. There are 3 roads from town A to town B, 5 roads from town B to town C, and 4 roads from town C to town D. How many ways are there to go from A to D via B and C? How many different round trips are possible?
8. A teacher must pick 3 high school students from a class of 30 to prepare and serve food at the junior high school picnic. How many choices are possible?
9. All students at John Jay High School must take at least one of the school's three science courses: biology, chemistry, and physics. In order to project the future enrollment in these courses, the principal sent questionnaires to 297 junior high school students and got these results:

132 intend to take biology and chemistry.  
 107 intend to take chemistry and physics.  
 88 intend to take biology and physics.  
 43 intend to take only biology.  
 55 intend to take only chemistry.  
 38 intend to take only physics.

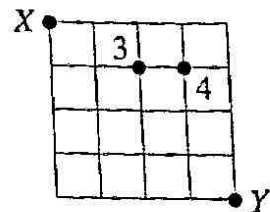
How many students intend to take all three science courses?

10. A radio show plays the top 10 musical hits of the previous week. During the first full week of last January, these 10 hits were chosen from 70 possibilities, and they were played in order of increasing popularity. How many possible orders were there? *Note:* Since calculators cannot evaluate large factorials, use the approximation given by the formula  $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ .

- B** 11. The locks on the gymnasium lockers have dials with numbers from 0 to 39. Each locker combination consists of 3 numbers.
- a. Pythagoras remembers that his numbers are 8, 15, and 17, but he can't remember in which order they appear. If he can try one possibility every 10 s, what is the maximum amount of time that it would take him to find the right combination?
  - b. Hypatia cannot remember any of her combination numbers, but she does remember that exactly two of them are the same. If she can try one possibility every 10 s, what is the maximum amount of time that it would take her to find the right combination?
12. In how many ways can 8 jackets of different styles be hung:
    - a. on a straight bar?
    - b. on a circular rack?
  13. A town council consists of 8 members including the mayor.
    - a. How many different committees of 4 can be chosen from this council?
    - b. How many of these committees include the mayor?
    - c. How many do not include the mayor?
    - d. Verify that the answer to part (a) is the sum of the answers to part (b) and part (c).
  14. Repeat Exercise 13 if the council has 9 members including the mayor.
  15. If you have a \$1 bill, a \$5 bill, a \$10 bill, and a \$20 bill, how many different sums of money can you make using one or more of these bills?

16. **Communications** In Morse code, letters, digits, and various punctuation marks are represented by a sequence of dots and dashes. Sequences can be from 1 unit to 6 units in length. How many such sequences are possible?
17. The Pizza Place offers pepperoni, mushrooms, sausages, onions, anchovies, and peppers as toppings for their regular plain pizza. How many different pizzas can be made?
18. a. How many 4-letter "words" can be formed by using the 8 letters of the word TRIANGLE?  
 b. How many of the "words" formed in part (a) have no vowels?  
 c. How many of the "words" formed in part (a) have at least one vowel?
19. Five boys and five girls stand in a line. How many arrangements are possible (a) if all of the boys stand in succession? (b) if the boys and girls stand alternately?
20. Answer Exercise 19 if the 5 boys and 5 girls stand in a circle instead of a line.
21. How many 5-digit numbers contain at least one 3?

22. In the diagram at the right, all paths go from X toward Y and consist only of south and east steps. For example, there are 3 paths that go from X to the point marked with a 3. Similarly, there are 4 paths that go from X to the point marked with a 4.



- a. Explain how the numbers 3 and 4 are obtained.  
 b. Copy the grid and label each point with the number of paths that go from X to that point.

- C** 23. **Sports** In the World Series, two teams, A and B, play each other until one team has won 4 games. For example, the "word" ABBAAA represents a 6-game series in which team A wins games 1, 4, 5, and 6.
- a. Explain why the number of different 6-game series won by team A is  $\frac{5!}{3! 2!}$ .
- b. Without actually listing the various series between team A and team B, show that there are 70 different sequences of games possible.

24. **Sports** In the tennis championship between players A and B at Wimbledon, the first player to win 3 sets is champion. Find the number of different ways for player A to win the championship.

