8-4 Significance of Experimental Results

Going Deeper

Essential question: *In an experiment, when is the difference between the control group and treatment group likely to be caused by the treatment?*

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COMMON CORE **Standards for Mathematical Content**

CC.9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*

Suppose you flipped a coin 20 times. Even if the coin were fair, you would not necessarily get exactly 10 heads and 10 tails. But what if you got 15 heads and 5 tails, or 20 heads and no tails? You might start to think that the coin was not a fair coin, after all.

Hypothesis testing is used to determine whether the difference in two groups is likely to be caused by chance.

For example, when tossing a coin 20 times, 11 heads and 9 tails is likely to occur if the coin is fair, but if you tossed 19 heads and 1 tail, you could say it was not likely to be a fair coin.

However, that outcome, while unlikely, is still possible. Hypothesis testing cannot prove that a coin is unfair – it is still possible for a coin to come up with 19 heads by chance, it is just very unlikely. Therefore, you can only say how likely or unlikely a coin is to be biased.

Hypothesis testing begins with an assumption called the *null hypothesis. The* **null hypothesis** states that there is no difference between the two groups being tested. The purpose of hypothesis testing is to use experimental data to test the viability of the null hypothesis.

Helpful Hint

The word *null means* "zero," so the *null* hypothesis is that the difference between the two groups is zero.

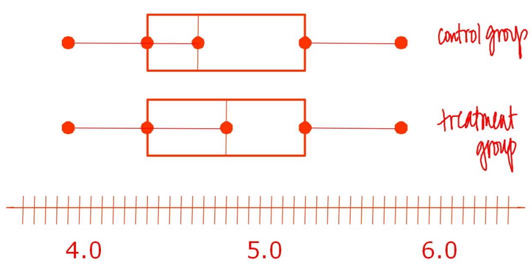
A researcher is testing whether a certain medication for raising glucose levels is more effective at higher doses. In a random trial, fasting glucose levels of 5 patients being treated at a normal dose (Group A) and 5 patients being treated at a high dose (Group B) were recorded. The glucose levels in mmol/L are shown below.

A	5.4	5.7	4.8	4.3	4.6
B	5.5	5.1	4.2	5.9	4.9

A. State the null hypothesis for the experiment.

The glucose levels of the drug will be the same for the control group (A) and the treatment group (B).

B. Compare the results for the control group and the treatment group. Do you think that the researcher has enough evidence to reject the null hypothesis?



There is a small difference in the two groups that is likely to be caused by chance. If anything, the treatment group actually shows a tendency toward higher glucose levels. The researcher cannot reject the null hypothesis, which means that the medication is probably just as effective at the normal dose as it is at the high dose.

A teacher wants to know if students in her morning class do better on a test than students in her afternoon class. She compares the test scores of 10 randomly chosen students in each class.

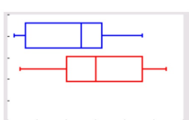
Morning class: 76,81,71, 80,88,66,79,67,85,68

Afternoon class: 80,91,74,92,80,80,88,67,75,78

a. State the null hypothesis.

The students in the morning class will have the same test scores as the students in the afternoon class

b. Compare the results of the two groups. Does the teacher have enough evidence to reject the null hypothesis?



Yes; there is a large difference in the test scores of the two classes. The teacher does have enough evidence to reject the null hypothesis, so she can conclude that students in her afternoon class perform better on tests.

For each experiment, state the null hypothesis.

A potential growth agent is sprayed on the leaves of 12 emerging ferns twice a week for a month. Another 12 emerging ferns are not sprayed with the growth agent. The mean stalk lengths of the two groups of ferns are compared after a month.

The null hypothesis is that the mean stalk lengths of the two groups of ferns will be about the same.

Suppose in part A that the treated ferns had a mean stalk length that is twice the mean stalk length of the untreated ferns. Should the researcher reject the null hypothesis? Does the experimental result prove that the growth agent works? Explain.

Yes, the researcher should reject the null hypothesis because the difference between the control group and the treatment group is so dramatic. No, the result does not prove that the growth agent works; it only provides evidence that rejecting the assumption that the growth agent has no effect is reasonable.

For each experiment, state the null hypothesis.

Ten people with colds are treated with a new formula for an existing brand of cold medicine. Ten other people with colds are treated with the original formula. The mean recovery times for the two groups are compared.

The null hypothesis is that the mean recovery times for the two groups will be about the same.

Suppose in part B that the mean recovery time for both groups is 5 days. Should the researcher reject the null hypothesis? Does the experimental result prove that the new formula is no more effective than the original formula? Explain.

No, the researcher should not reject the null hypothesis because there is no difference between the control group and the treatment group. No, the result does not prove that the new formula is no more effective than the old; it only supports the reasonableness of the assumption that the new formula is no more effective.

In the U.S. legal system, a defendant is assumed innocent until guilt is proved beyond a reasonable doubt. How is this situation like rejecting a null hypothesis?

A null hypothesis (like innocence) is assumed to be true unless the experimental evidence (like evidence of guilt) allows you to reject the null hypothesis (innocence).

Hypothesis testing can be used to compare the mean from a sample to the mean of a population. If the sample contains at least 30 individuals, you can use the *z-test*. Suppose that the population mean is estimated to be μ, and a random sample has n individuals ($n \geq 30$). To find the z-value of a statistic, you need to know the sample mean $\bar{x}$ and standard deviation σ. The *z-value* is found using the following formula:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

sample mean ($\bar{x}$); pop. mean (μ); standard deviation (σ); number of individuals in study (n)

The null hypothesis is that there is no difference in the two groups. If the sample mean is close to the population mean, then the z-value is close to 0. If the z-value is too large, you can reject the null hypothesis.

One common measure used in z-tests is known as a 95% confidence level:

- If $|z| > 1.96$, then you can reject the null hypothesis with 95% certainty.
- If $|z| < 1.96$, then you do not have enough evidence to reject the null hypothesis.

A test prep company claims that its private tutoring can boost scores to an average of 2000. In a random sample of 49 students who were privately tutored, the average was 1910, with a standard deviation of 150. Is there enough evidence to reject the claim?

$$z = \frac{1910 - 2000}{\frac{150}{\sqrt{49}}} = \frac{-90}{\frac{150}{7}} = -4.2$$

The z–value is –4.2, and | z | > 1.96. So, there is enough evidence to reject the null hypothesis. You can say with 95% confidence that the company's claim about private tutoring is false.

A tax preparer claims an average refund of \$3000. In a random sample of 40 clients, the average refund was \$2600, and the standard deviation was \$300. Is there enough evidence to reject his claim?

$$z = \frac{2600 - 3000}{\frac{300}{\sqrt{40}}} \approx -8.43$$

Calculate the Z-value: $\frac{2600-3000}{\frac{300}{\sqrt{40}}}$

$\approx \frac{-400}{47.43} \approx -8.43$

The z–value is –8.43, and | z | > 1.96. So, there is enough evidence to reject the claim of the tax preparer.

To disprove a previous study that claims that college graduates make an average salary of \$46,000, a researcher records the salaries of 50 graduates and finds that the sample mean is \$43,000, with a standard deviation of \$4,500. What is the z-value, and can she reject the null hypothesis?

$$\frac{43000 - 46000}{\frac{4500}{\sqrt{50}}} = -4.71$$

-4.71: yes can reject null hypothesis