

$$26. f(t) = t^4 - 5t$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{[(a+h)^4 - 5(a+h)] - (a^4 - 5a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^4 + 4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5a - 5h - a^4 + 5a}{h} \\ &= \lim_{h \rightarrow 0} \frac{4a^3h + 6a^2h^2 + 4ah^3 + h^4 - 5h}{h} \\ &= \lim_{h \rightarrow 0} (4a^3 + 6a^2h + 4ah^2 + h^3 - 5) \\ &= 4a^3 - 5 \end{aligned}$$

$$28. f(x) = \frac{x^2 + 1}{x - 2}$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{(a+h)^2 + 1}{(a+h) - 2} - \frac{a^2 + 1}{a - 2}}{h} \\ &= \lim_{h \rightarrow 0} \left( \frac{a^2 + 2ah + h^2 + 1}{a+h-2} - \frac{a^2 + 1}{a-2} \right) \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{(a^2 + 2ah + h^2 + 1)(a-2) - (a^2 + 1)(a+h-2)}{(a+h-2)(a-2)} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^3 - 2a^2 + 2a^2h - 4ah + ah^2 - 2h^2 + a - 2 - (a^3 + a^2h - 2a^2 + a + h - 2)}{h(a+h-2)(a-2)} \\ &= \lim_{h \rightarrow 0} \frac{a^2h - 4ah + ah^2 - 2h^2 - h}{h(a+h-2)(a-2)} \\ &= \lim_{h \rightarrow 0} \frac{a^2 - 4a + ah - 2h - 1}{(a+h-2)(a-2)} \\ &= \frac{a^2 - 4a - 1}{(a-2)^2} \end{aligned}$$

$$30. f(x) = \sqrt{3x+1}$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{3(a+h)+1} - \sqrt{3a+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{3a+3h+1} - \sqrt{3a+1}}{h} \cdot \frac{\sqrt{3a+3h+1} + \sqrt{3a+1}}{\sqrt{3a+3h+1} + \sqrt{3a+1}} \\ &= \lim_{h \rightarrow 0} \frac{(3a+3h+1) - (3a+1)}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\ &= \lim_{h \rightarrow 0} \frac{3}{\sqrt{3a+3h+1} + \sqrt{3a+1}} \\ &= \frac{3}{2\sqrt{3a+1}} \end{aligned}$$

$$32. \lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

compare to  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

$$a = 16$$

$$f(x) = \sqrt[4]{x}$$

$$34. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan(x) - 1}{x - \frac{\pi}{4}}$$

compare to  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$$a = \frac{\pi}{4}$$

$$f(x) = \tan(x)$$

$$36. \lim_{t \rightarrow 1} \frac{t^4 + t - 2}{t - 1}$$

compare to  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$$a = 1$$

$$f(t) = t^4 + t$$

$$43. C(x) = 5000 + 10x + 0.05x^2$$

(units) production level	avg rate of change (\$/unit)
(i) 100 to 105	$\frac{C(105) - C(100)}{105 - 100} = 20.25$
(ii) 100 to 101	$\frac{C(101) - C(100)}{101 - 100} = 20.05$

$$(b) C'(100) = \lim_{h \rightarrow 0} \frac{C(100+h) - C(100)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5000 + 10(100+h) + 0.05(100+h)^2 - 6500}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5000 + 1000 + 10h + 0.05(10000 + 200h + h^2) - 6500}{h}$$

$$= \lim_{h \rightarrow 0} \frac{20h + 0.05h^2}{h}$$

$$= \lim_{h \rightarrow 0} (20 + 0.05h)$$

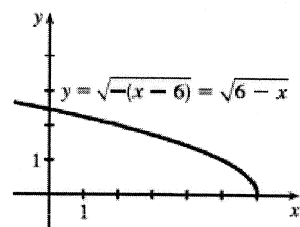
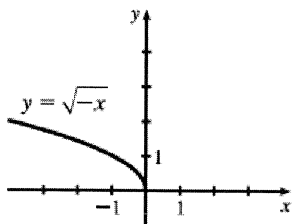
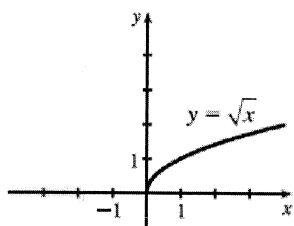
$$= 20$$

$$C'(100) = \$20/\text{unit}$$

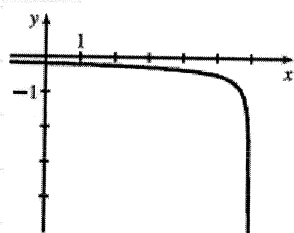
45. (a)  $f'(x)$  is the rate of change of the production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.

(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17/ounce. So the cost of producing the 800th (or 801st) ounce is about \$17.

28. (a)



(b)



(c)  $f(x) = \sqrt{6-x}$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{6-x-h} - \sqrt{6-x}}{h} \cdot \frac{\sqrt{6-x-h} + \sqrt{6-x}}{\sqrt{6-x-h} + \sqrt{6-x}} \\
 &= \lim_{h \rightarrow 0} \frac{(6-x-h) - (6-x)}{h(\sqrt{6-x-h} + \sqrt{6-x})} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{6-x-h} + \sqrt{6-x})} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{6-x-h} + \sqrt{6-x}} \\
 &= \frac{-1}{2\sqrt{6-x}}
 \end{aligned}$$

$f(x) = \sqrt{6-x}$       $D: (-\infty, 6]$

$f'(x) = \frac{-1}{2\sqrt{6-x}}$       $D: (-\infty, 6)$

(d) double check your graph in (b)  
 matches graph of  $f'(x) = \frac{-1}{2\sqrt{6-x}}$

39.  $a = f, b = f', c = f''$ . We can see this because where  $a$  has a horizontal tangent,  $b = 0$ , and where  $b$  has a horizontal tangent,  $c = 0$ . We can immediately see that  $c$  can be neither  $f$  nor  $f'$ , since at the points where  $c$  has a horizontal tangent, neither  $a$  nor  $b$  is equal to 0.

40. Where  $d$  has horizontal tangents, only  $c$  is 0, so  $d' = c$ .  $c$  has negative tangents for  $x < 0$  and  $b$  is the only graph that is negative for  $x < 0$ , so  $c' = b$ .  $b$  has positive tangents on  $\mathbb{R}$  (except at  $x = 0$ ), and the only graph that is positive on the same domain is  $a$ , so  $b' = a$ . We conclude that  $d = f, c = f', b = f''$ , and  $a = f'''$ .

41. We can immediately see that  $a$  is the graph of the acceleration function, since at the points where  $a$  has a horizontal tangent, neither  $c$  nor  $b$  is equal to 0. Next, we note that  $a = 0$  at the point where  $b$  has a horizontal tangent, so  $b$  must be the graph of the velocity function, and hence,  $b' = a$ . We conclude that  $c$  is the graph of the position function.

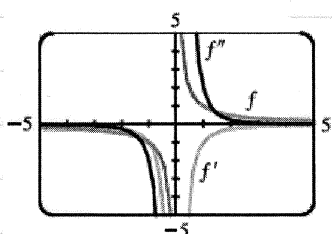
42. Note: jerk is the derivative of acceleration

$a$  must be the jerk since none of the graphs are 0 at its high and low points.  $a$  is 0 where  $b$  has a maximum, so  $b' = a$ .  $b$  is 0 where  $c$  has a maximum, so  $c' = b$ . We conclude that  $d$  is the position function,  $c$  is the velocity,  $b$  is the acceleration, and  $a$  is the jerk.

$$44. f(x) = \frac{1}{x}$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{x(x+h)} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{xh(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$$

$$\begin{aligned} f''(x) &= \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{-1}{(x+h)^2} + \frac{1}{x^2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 + (x+h)^2}{x^2(x+h)^2} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-x^2 + x^2 + 2hx + h^2}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{h(2x+h)}{hx^2(x+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{2x+h}{x^2(x+h)^2} \\ &= \frac{2x}{x^4} \\ &= \frac{2}{x^3} \end{aligned}$$



We see from the graph that our answers are reasonable because the graph of  $f'$  is that of an even function and is negative for all  $x \neq 0$ , and the graph of  $f''$  is that of an odd function (negative for  $x < 0$  and positive for  $x > 0$ ).

$$45. f(x) = 2x^2 - x^3$$

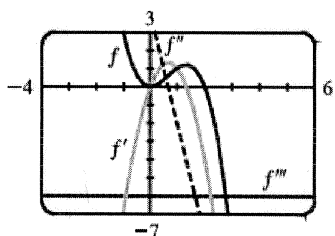
$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - (x+h)^3}{h} - (2x^2 - x^3) \\ &= \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) - (x^3 + 3x^2h + 3xh^2 + h^3) - 2x^2 + x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4x + 2h - 3x^2 - 3xh - h^2)}{h} \\ &= \lim_{h \rightarrow 0} (4x + 2h - 3x^2 - 3xh - h^2) \\ &= 4x - 3x^2 \end{aligned}$$

$$\begin{aligned} f''(x) &= \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h \rightarrow 0} \frac{4(x+h) - 3(x+h)^2}{h} - (4x - 3x^2) \\ &= \lim_{h \rightarrow 0} \frac{4x + 4h - 3(x^2 + 2hx + h^2) - 4x + 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(4 - 6x - 3h)}{h} \\ &= \lim_{h \rightarrow 0} (4 - 6x - 3h) \\ &= 4 - 6x \end{aligned}$$

$$\begin{aligned} f'''(x) &= \lim_{h \rightarrow 0} \frac{f''(x+h) - f''(x)}{h} = \lim_{h \rightarrow 0} \frac{4 - 6(x+h)}{h} - (4 - 6x) \\ &= \lim_{h \rightarrow 0} \frac{4 - 6x - 6h - 4 + 6x}{h} \\ &= \lim_{h \rightarrow 0} \frac{-6h}{h} \\ &= \lim_{h \rightarrow 0} (-6) \\ &= -6 \end{aligned}$$

45  
cont'd

$$\begin{aligned} f^{(4)}(x) &= \lim_{h \rightarrow 0} \frac{f'''(x+h) - f'''(x)}{h} = \lim_{h \rightarrow 0} \frac{-6 - (-6)}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ &= \lim_{h \rightarrow 0} 0 \\ &= 0 \end{aligned}$$



The graphs are consistent with the geometric interpretations of the derivatives because  $f'$  has zeros where  $f$  has a local minimum and a local maximum,  $f''$  has a zero where  $f'$  has a local maximum, and  $f'''$  is a constant function equal to the slope of  $f''$ .