3. What does the Squeeze Theorem say about $\lim_{x\to 7} f(x)$ if $\lim_{x\to 7} l(x) = \lim_{x\to 7} u(x) = 6$ and $f(x)$, $u(x)$, and $l(x)$ are related as in Figure 8? The inequality $f(x) \le u(x)$ is not satisfied for all x. Does this affect the validity of your conclusion?

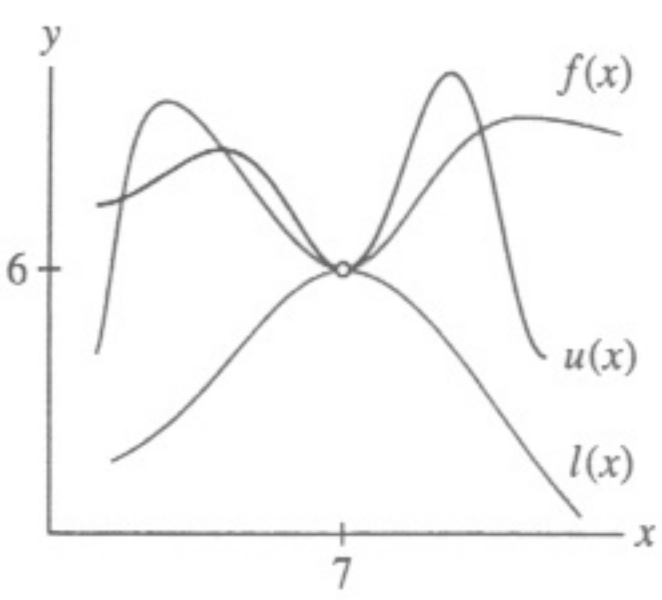


FIGURE 8

4. Determine $\lim_{x\to 0} f(x)$ assuming that $\cos x \le f(x) \le 1$.

5. State whether the inequality provides sufficient information to determine $\lim_{x\to 1} f(x)$, and if so, find the limit.

(a) $4x - 5 \le f(x) \le x^2$

(b) $2x - 1 \le f(x) \le x^2$

(c) $4x - x^2 \le f(x) \le x^2 + 2$

6. GU Plot the graphs of $u(x) = 1 + \left|x - \frac{\pi}{2}\right|$ and $l(x) = \sin x$ on the same set of axes. What can you say about $\lim_{x\to \frac{\pi}{2}} f(x)$ if $f(x)$ is squeezed by $l(x)$ and $u(x)$ at $x = \frac{\pi}{2}$?

In Exercises 7–16, evaluate using the Squeeze Theorem.

7. $\lim_{x\to 0} x^2 \cos \frac{1}{x}$

8. $\lim_{x\to 0} x \sin \frac{1}{x^2}$

9. $\lim_{x\to 1} (x-1) \sin \frac{\pi}{x-1}$

10. $\lim_{x\to 3} (x^2 - 9)\frac{x-3}{|x-3|}$

11. $\lim_{t\to 0} (2^t - 1) \cos \frac{1}{t}$

12. $\lim_{x\to 0+} \sqrt{x}\, e^{\cos(\pi/x)}$

13. $\lim_{t\to 2} (t^2 - 4) \cos \frac{1}{t-2}$

14. $\lim_{x\to 0} \tan x \cos\left(\sin \frac{1}{x}\right)$

15. $\lim_{\theta\to \frac{\pi}{2}} \cos\theta \cos(\tan\theta)$

16. $\lim_{t\to 0+} \sin t \tan^{-1}(\ln t)$

In Exercises 17–26, evaluate using Theorem 2 as necessary.

17. $\lim_{x\to 0} \frac{\tan x}{x}$

18. $\lim_{x\to 0} \frac{\sin x \sec x}{x}$

19. $\lim_{t\to 0} \frac{\sqrt{t^3+9}\,\sin t}{t}$

20. $\lim_{t\to 0} \frac{\sin^2 t}{t}$

21. $\lim_{x\to 0} \frac{x^2}{\sin^2 x}$

22. $\lim_{t\to \frac{\pi}{2}} \frac{1-\cos t}{t}$

23. $\lim_{\theta\to 0} \frac{\sec\theta - 1}{\theta}$

24. $\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin\theta}$

25. $\lim_{t\to \frac{\pi}{4}} \frac{\sin t}{t}$

26. $\lim_{t\to 0} \frac{\cos t - \cos^2 t}{t}$

27. Let $L = \lim_{x\to 0} \frac{\sin 14x}{x}$.

(a) Show, by letting $\theta = 14x$, that $L = \lim_{\theta\to 0} 14\frac{\sin\theta}{\theta}$.

(b) Compute L.

28. Evaluate $\lim_{h\to 0} \frac{\sin 9h}{\sin 7h}$. *Hint:* $\frac{\sin 9h}{\sin 7h} = \left(\frac{9}{7}\right)\left(\frac{\sin 9h}{9h}\right)\left(\frac{7h}{\sin 7h}\right)$.

In Exercises 29–48, evaluate the limit.

29. $\lim_{h\to 0} \frac{\sin 9h}{h}$

30. $\lim_{h\to 0} \frac{\sin 4h}{4h}$

31. $\lim_{h\to 0} \frac{\sin h}{5h}$

32. $\lim_{x\to \frac{\pi}{6}} \frac{x}{\sin 3x}$

33. $\lim_{\theta\to 0} \frac{\sin 7\theta}{\sin 3\theta}$

34. $\lim_{x\to 0} \frac{\tan 4x}{9x}$

35. $\lim_{x\to 0} x \csc 25x$

36. $\lim_{t\to 0} \frac{\tan 4t}{t \sec t}$

37. $\lim_{h\to 0} \frac{\sin 2h \sin 3h}{h^2}$

38. $\lim_{z\to 0} \frac{\sin(z/3)}{\sin z}$

39. $\lim_{\theta\to 0} \frac{\sin(-3\theta)}{\sin(4\theta)}$

40. $\lim_{x\to 0} \frac{\tan 4x}{\tan 9x}$

41. $\lim_{t\to 0} \frac{\csc 8t}{\csc 4t}$

42. $\lim_{x\to 0} \frac{\sin 5x \sin 2x}{\sin 3x \sin 5x}$

43. $\lim_{x\to 0} \frac{\sin 3x \sin 2x}{x \sin 5x}$

44. $\lim_{h\to 0} \frac{1-\cos 2h}{h}$

45. $\lim_{h\to 0} \frac{\sin(2h)(1-\cos h)}{h^2}$

46. $\lim_{t\to 0} \frac{1-\cos 2t}{\sin^2 3t}$

47. $\lim_{\theta\to 0} \frac{\cos 2\theta - \cos\theta}{\theta}$

48. $\lim_{h\to \frac{\pi}{2}} \frac{1-\cos 3h}{h}$

49. Calculate $\lim_{x\to 0-} \frac{\sin x}{|x|}$.

50. Use the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ to evaluate the limit $\lim_{\theta\to 0} \frac{\sin 3\theta - 3\sin\theta}{\theta^3}$.

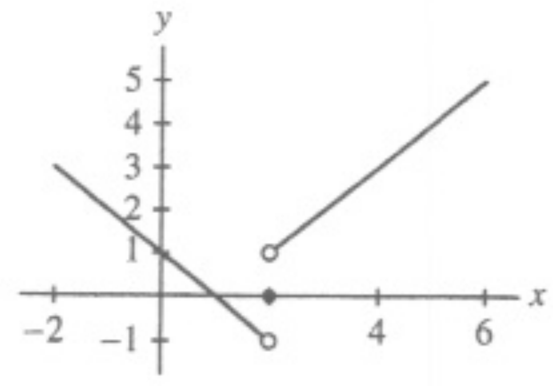
51. Prove the following result stated in Theorem 2:

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = 0 \qquad \boxed{7}$$

Hint: $\frac{1-\cos\theta}{\theta} = \frac{1}{1+\cos\theta} \cdot \frac{1-\cos^2\theta}{\theta}$.

52. GU Investigate $\lim_{h\to 0} \frac{1-\cos h}{h^2}$ numerically (and graphically if you have a graphing utility). Then prove that the limit is equal to $\frac{1}{2}$. *Hint:* See the hint for Exercise 51.

53. The function f is neither left- nor right-continuous at $x = 2$.

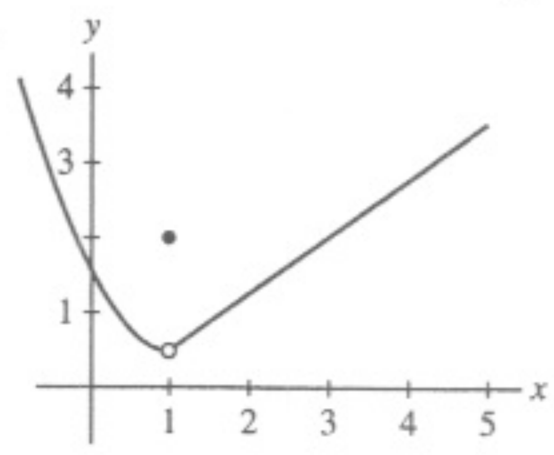


55. $\lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} (x+4) = 8 \neq 10 = f(4)$

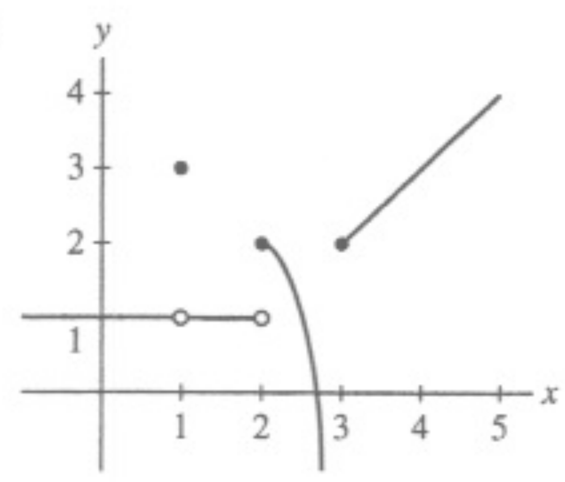
57. $c = \frac{5}{3}$ **59.** $a = 2$ and $b = 1$

61. **(a)** No **(b)** $g(1) = -\frac{\pi}{2}$

63.



65.

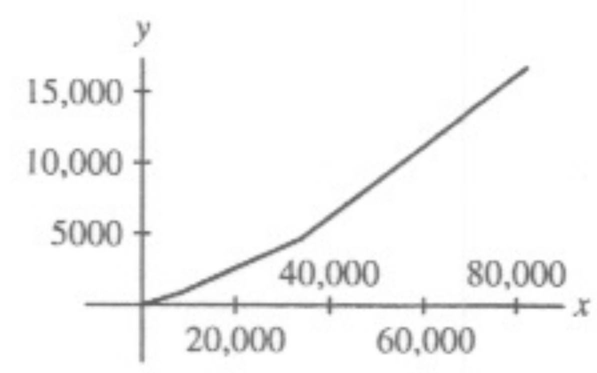


67. -6 **69.** $\frac{1}{3}$ **71.** -1 **73.** $\frac{1}{32}$ **75.** 27 **77.** 1000 **79.** $\frac{\pi}{2}$

81. No. Take $f(x) = -x^{-1}$ and $g(x) = x^{-1}$

83. $f(x) = |g(x)|$ is a composition of the continuous functions $g(x)$ and $|x|$

85. No.



87. $f(x) = 3$ and $g(x) = [x]$

Section 2.5 Preliminary Questions

1. $\frac{x^2-1}{\sqrt{x+3}-2}$

2. **(a)** $f(x) = \frac{x^2-1}{x-1}$ **(b)** $f(x) = \frac{x^2-1}{x-1}$ **(c)** $f(x) = \frac{1}{x}$

3. The "simplify and plug-in" strategy is based on simplifying a function which is indeterminate to a continuous function. Once the simplification has been made, the limit of the remaining continuous function is obtained by evaluation.

Section 2.5 Exercises

1. $\lim_{x\to 6} \frac{x^2-36}{x-6} = \lim_{x\to 6} \frac{(x-6)(x+6)}{x-6} = \lim_{x\to 6} (x+6) = 12$

3. 0 **5.** $\frac{1}{14}$ **7.** -1 **9.** $\frac{11}{10}$ **11.** 2 **13.** 1 **15.** 2 **17.** $\frac{1}{8}$

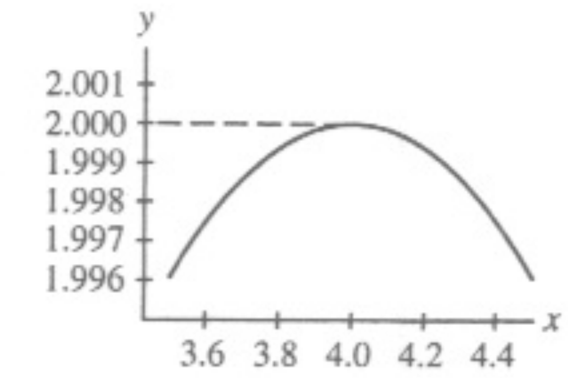
19. $\frac{7}{17}$

21. Limit does not exist.

- As $h \to 0+$, $\frac{\sqrt{h+2}-2}{h} \to -\infty$.
- As $h \to 0-$, $\frac{\sqrt{h+2}-2}{h} \to \infty$.

23. 2 **25.** $\frac{1}{4}$ **27.** 1 **29.** 9 **31.** $\frac{\sqrt{2}}{2}$ **33.** $\frac{1}{2}$

35. $\lim_{x\to 4} f(x) \approx 2.00$; to two decimal places, this matches the value of 2 obtained in Exercise 23.



37. 12 **39.** -1 **41.** $\frac{4}{3}$ **43.** $\frac{1}{4}$ **45.** $2a$ **47.** $-4+5a$ **49.** $4a$

51. $\frac{1}{2\sqrt{a}}$ **53.** $3a^2$ **55.** $c = -1$ and $c = 6$ **57.** $c = 3$ **59.** +

Section 2.6 Preliminary Questions

1. $\lim_{x\to 0} f(x) = 0$; No

2. Assume that for $x \neq c$ (in some open interval containing c),

$$l(x) \le f(x) \le u(x)$$

and that $\lim_{x\to c} l(x) = \lim_{x\to c} u(x) = L$. Then $\lim_{x\to c} f(x)$ exists and

$$\lim_{x\to c} f(x) = L.$$

3. **(a)**

Section 2.6 Exercises

1. For all $x \neq 1$ on the open interval $(0, 2)$ containing $x = 1$, $\ell(x) \le f(x) \le u(x)$. Moreover,

$$\lim_{x\to 1} \ell(x) = \lim_{x\to 1} u(x) = 2.$$

Therefore, by the Squeeze Theorem,

$$\lim_{x\to 1} f(x) = 2.$$

3. $\lim_{x\to 7} f(x) = 6$

5. **(a)** *not* sufficient information **(b)** $\lim_{x\to 1} f(x) = 1$ **(c)** $\lim_{x\to 1} f(x) = 3$

7. $\lim_{x\to 0} x^2 \cos\frac{1}{x} = 0$ **9.** $\lim_{x\to 1} (x-1)\sin\frac{\pi}{x-1} = 0$

11. $\lim_{t\to 0} (2^t - 1)\cos\frac{1}{t} = 0$

13. $\lim_{t\to 2} (t^2-4)\cos\frac{1}{t-2} = 0$

15. $\lim_{\theta\to\frac{\pi}{2}} \cos\theta\cos(\tan\theta) = 0$

17. 1 **19.** 3 **21.** 1 **23.** 0 **25.** $\frac{2\sqrt{2}}{\pi}$ **27.** **(b)** $L = 14$ **29.** 9

31. $\frac{1}{5}$ **33.** $\frac{7}{3}$ **35.** $\frac{1}{25}$ **37.** 6 **39.** $-\frac{3}{4}$ **41.** $\frac{1}{2}$ **43.** $\frac{6}{5}$ **45.** 0

47. 0 **49.** -1 **53.** $-\frac{9}{2}$

55. $\lim_{t\to 0+} \frac{\sqrt{1-\cos t}}{t} = \frac{\sqrt{2}}{2}$; $\lim_{t\to 0-} \frac{\sqrt{1-\cos t}}{t} = -\frac{\sqrt{2}}{2}$

59. **(a)**

x	$c - .01$	$c - .001$	$c + .001$	$c + .01$
$\frac{\sin x - \sin c}{x - c}$	.999983	.99999983	.99999983	.999983

Here $c = 0$ and $\cos c = 1$.