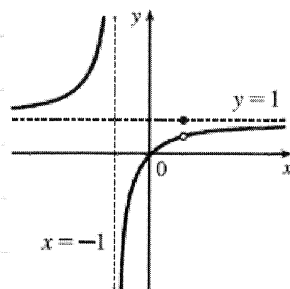


18.

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \quad a = 1$$



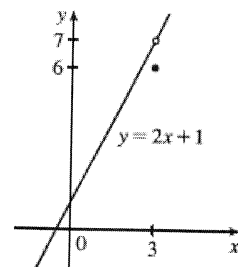
$$f(1) = 1$$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} \\ &= \lim_{x \rightarrow 1} \frac{x(x-1)}{(x+1)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x}{x+1} \\ &= \frac{1}{2} \end{aligned}$$

f is discontinuous at $x = 1$ since
 $\lim_{x \rightarrow 1} f(x) \neq f(1)$

20.

$$f(x) = \begin{cases} \frac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases} \quad a = 3$$



$$f(3) = 6$$

$$\begin{aligned} \lim_{x \rightarrow 3} f(x) &= \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{x-3} \\ &= \lim_{x \rightarrow 3} (2x+1) \\ &= 7 \end{aligned}$$

f is discontinuous at $x = 3$ since
 $\lim_{x \rightarrow 3} f(x) \neq f(3)$

$$21. F(x) = \frac{x}{x^2+5x+6} = \frac{x}{(x+3)(x+2)}$$

F is a rational function so F is cont on its domain which is $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

$$22. G(x) = \sqrt[3]{x}(1+x^3)$$

$f(x) = \sqrt[3]{x}$ is a root function so f is continuous on its domain which is $(-\infty, \infty)$
 $h(x) = 1+x^3$ is a polynomial so h is continuous on $(-\infty, \infty)$

$G(x) = f(x) \cdot h(x)$ so G is cont on $(-\infty, \infty)$

$$27. F(x) = \sqrt{x} \sin(x)$$

$g(x) = \sqrt{x}$ is a root function so g is continuous on its domain which is $[0, \infty)$
 $h(x) = \sin(x)$ is a trig function so h is continuous on its domain which is $(-\infty, \infty)$

$F(x) = g(x) \cdot h(x)$ so F is continuous on $[0, \infty)$

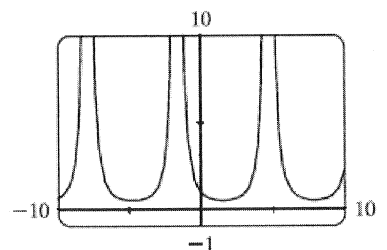
$$29. y = \frac{1}{1+\sin x} \quad \text{let } f(x) = \frac{1}{1+\sin(x)}$$

f is undefined and therefore discontinuous whenever $(1+\sin(x)) = 0$. f is discontinuous

$$\begin{aligned} 1+\sin(x) &= 0 \\ \sin(x) &= -1 \\ x &= \frac{3\pi}{2} \pm 2\pi n \\ n &\text{ is an integer} \end{aligned}$$

When

$$x = \frac{3\pi}{2} \pm 2\pi n$$



31. $\lim_{x \rightarrow 4} \frac{5 + \sqrt{x}}{\sqrt{5+x}}$ let $f(x) = \frac{5 + \sqrt{x}}{\sqrt{5+x}}$

$h(x) = 5$ is a polynomial so h is cont on $(-\infty, \infty)$
 $g(x) = \sqrt{x}$ is a root function so g is cont on its domain which is $[0, \infty)$

$k(x) = \sqrt{5+x}$ is a root function so k is cont on its domain which is $[-5, \infty)$

Then $f(x) = \frac{h(x) \cdot g(x)}{k(x)}$ is cont on $[0, \infty)$

(Note: don't need to worry about dividing by zero because domain of f is $[0, \infty)$)

Since f is cont at $x = 4$

$$\lim_{x \rightarrow 4} f(x) = f(4) = \frac{5 + \sqrt{4}}{\sqrt{5+4}} = \frac{7}{3}$$

33 $\lim_{x \rightarrow \frac{\pi}{4}} x \cos^2(x)$ let $F(x) = x \cos^2(x)$

$g(x) = x$ is a polynomial so g is cont on $(-\infty, \infty)$
 $h(x) = \cos(x)$ is a trig function so h is cont on its domain which is $(-\infty, \infty)$

then $F(x) = g(x) \cdot f(x) \cdot f(x)$ so F is cont on $(-\infty, \infty)$

Since F is cont at $x = \frac{\pi}{4}$ then

$$\lim_{x \rightarrow \frac{\pi}{4}} F(x) = F\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \cos^2\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

35.

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$$

Let $g(x) = x^2$ since g is a polynomial

g is cont on $(-\infty, 1)$

Let $h(x) = \sqrt{x}$ since h is a root function

h is cont on $(1, \infty)$

Now we need to check that f is cont at $x=1$

$$f(1) = \sqrt{1} = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \sqrt{x} = 1$$

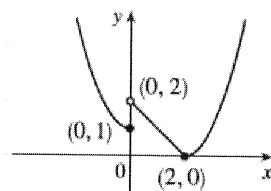
$$\left. \begin{array}{l} \lim_{x \rightarrow 1^-} f(x) = 1 \\ \lim_{x \rightarrow 1^+} f(x) = 1 \end{array} \right\} \lim_{x \rightarrow 1} f(x) = 1$$

f is cont at $x=1$ since $\lim_{x \rightarrow 1} f(x) = f(1)$

Thus f is cont on $(-\infty, \infty)$

37.

$$f(x) = \begin{cases} 1+x^2 & \text{if } x \leq 0 \\ 2-x & \text{if } 0 < x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$



* to start we know...

$$f(0) = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (1+x^2) = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (2-x) = 2$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^-} f(x) = 1 \\ \lim_{x \rightarrow 0^+} f(x) = 2 \end{array} \right\} \lim_{x \rightarrow 0} f(x) \text{ DNE}$$

$$\text{since } \lim_{x \rightarrow 0} f(x) \neq \lim_{x \rightarrow 0} f(x)$$

f is discontinuous at $x=0$ since $\lim_{x \rightarrow 0} f(x) \text{ DNE}$

f is continuous at $x=0$ from the left

$$\text{since } \lim_{x \rightarrow 0^-} f(x) = f(0)$$

* f is cont on $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$ since each piece is a polynomial

37

$$f(2) = 0$$

cont'd

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2-x) = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x-2)^2 = 0$$

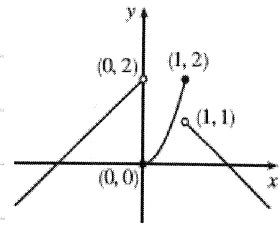
$$\left. \begin{array}{l} \lim_{x \rightarrow 2^-} f(x) = 0 \\ \lim_{x \rightarrow 2^+} f(x) = 0 \end{array} \right\} \lim_{x \rightarrow 2} f(x) = 0$$

f is continuous at $x=2$ since $\lim_{x \rightarrow 2} f(x) = f(2)$

Now we can say f is cont on $(-\infty, 0) \cup (0, \infty)$

39.

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$$



to start we know f is cont on $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$ since each piece of f is a polynomial.

Now look into $x=0$ and $x=1$

$$f(0) = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x+2) = 2$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (2x^2) = 0$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^-} f(x) = 2 \\ \lim_{x \rightarrow 0^+} f(x) = 0 \end{array} \right\} \begin{array}{l} \lim_{x \rightarrow 0} f(x) \text{ DNE} \\ \text{since } \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x) \end{array}$$

f is discontinuous at $x=0$ since $\lim_{x \rightarrow 0} f(x) \text{ DNE}$

f is continuous at $x=0$ from the right since $\lim_{x \rightarrow 0^+} f(x) = f(0)$

$$f(1) = 2$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 2x^2 = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2-x) = 1$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 1^-} f(x) = 2 \\ \lim_{x \rightarrow 1^+} f(x) = 1 \end{array} \right\} \begin{array}{l} \lim_{x \rightarrow 1} f(x) \text{ DNE} \\ \text{since } \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x) \end{array}$$

f is discontinuous at $x=1$ since $\lim_{x \rightarrow 1} f(x) \text{ DNE}$

f is continuous at $x=1$ from the left since $\lim_{x \rightarrow 1^-} f(x) = f(1)$

Now we know f is cont on $(-\infty, 0) \cup [0, 1] \cup (1, \infty)$