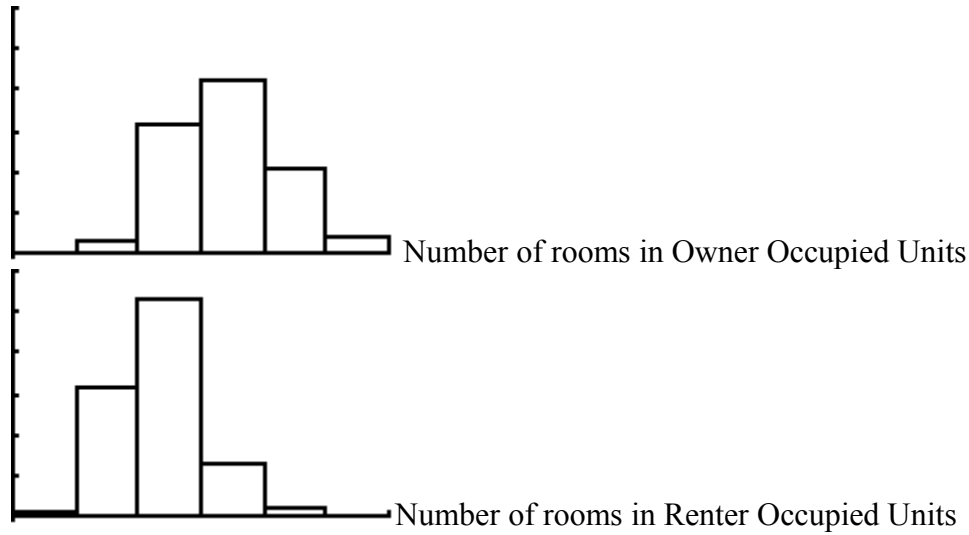


7.4

The histograms of the two distributions are shown below using the same scale for comparison.



From these histograms we can see that the rooms from Owned Units are fairly symmetrically distributed while the rooms from Rented Units are slightly skewed to the right. The average of the distribution for the Rented Units is clearly lower than the average for the Owned Units. We do not have measures of spread, but from the appearance of the histograms, I think the Owner Units is a bit more varied.

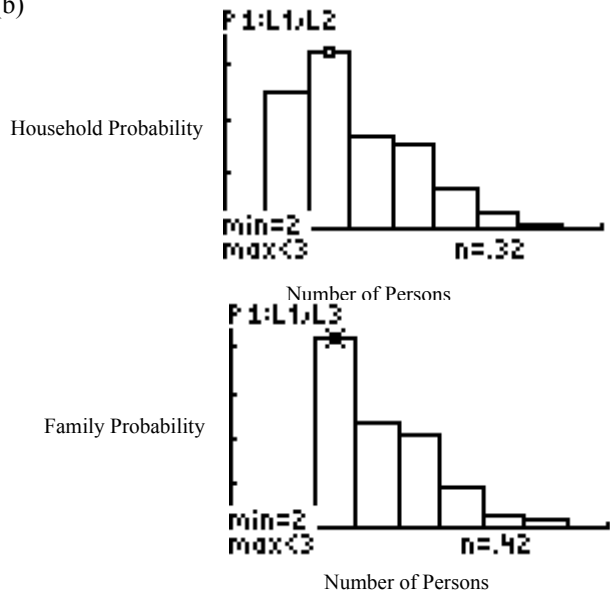
7.8

- (a) $P(\hat{p} \geq 0.45) = \text{normalcdf}(0.45, 1E99, 0.4, 0.023) = 0.01485$
- (b) $P(\hat{p} < 0.35) = \text{normalcdf}(-1E99, 0.35, 0.4, 0.023) = 0.01485$
- (c) $P(0.35 \leq \hat{p} < 0.45) = 1 - 2(0.01485) = 0.9703$

7.10

(a) Sum of Household Probabilities = 1, and Sum of Family Probabilities = 1. Since all probabilities are non-negative and between 0 and 1 inclusive, these are valid probability distributions.

(b)



The Size of Family Distribution is much more skewed right than the Household distribution. The mean household size is 2.6 with standard deviation 1.4 and the mean family size is 3.14 with standard deviation 1.25. So it appears that in general, family sizes are a bit larger than household sizes.

7.22

If X represents the grade point, the mean grade point is $\mu_X = 2.25$, which is a "C".

7.26

The standard deviation of the grade points is $\sigma_X = 1.178$.

7.33

We will conduct a simulation. Assign digits 1-100 such that 1 to 35 represent “Tony gets a hit” and 36-100 represent “Tony does NOT get a hit.” Generate 300 random integers between 1 and 100 and record the occurrence of runs of 6 or more numbers 36 or above.

Trial number	Runs of 6 or more “no hits”? (1 = yes, 0 = no)
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

The above simulation represents 10 seasons of 300 at bats per season. In each of these 10 seasons we observed every time that there were one or more runs of 6 “no hits in a row”. This indicates that it is VERY LIKELY to occur that Tony Gwyn will go for 6 or more at bats in a season without getting a hit. We should not be surprised that this happens. It is consistent with his 35% long-term average.

7.36

Let X represent the time the first reaction, so we have $\mu_x=40$ min, and $\sigma_x=2$ min.

Let Y represent the time the second reaction, so we have $\mu_y=25$ min, and $\sigma_y=1$ min.

The total time, T , for the process takes , so that $T=X+Y+5$ So, $\mu_T=40+25+5=70$ min.

7.41

If X represents Fred's score and Y represents Leona's, then

$$X - Y \sim N(0, \sqrt{2^2 + 2^2})$$

So

$$\begin{aligned} P(|X - Y| \geq 5) &= P(X - Y \leq -5 \text{ or } X - Y \geq 5) \\ &= 2 * \text{normalcdf}(-1E99, -5, 0, \sqrt{8}) = 0.0771 \end{aligned}$$

7.46

(a) $\mu_X = 550^\circ \text{C}$, and $\sigma_X = 5.7008^\circ \text{C}$

(b) $\mu_{X-550} = 0^\circ \text{C}$, and $\sigma_{X-550} = 5.7008^\circ \text{C}$

(c) $\mu_Y = 1022^\circ \text{F}$, and $\sigma_Y = 10.262^\circ \text{C}$

7.47

Given the probability distribution we could assign random digits 0-99 as follows

Digits 0-9: The Temperature is 540°C

Digits 10-34: The Temperature is 545°C

Digits 35-64: The Temperature is 550°C

Digits 65-89: The Temperature is 555°C

Digits 90-99: The Temperature is 560°C

I generated 100 random integers from 0 thru 99, ordered them and got the following simulated data table:

Temp ($^\circ \text{C}$)	Freq
540	12
545	27
550	34
555	23
560	4

$\bar{x} = 549$, and $s_X = 5.27$. I came pretty close with $n = 100$ repetitions of the experiment.

7.42

X	\$1690	\$975	\$562.50
P(X)	0.25	0.5	0.25

$$P(X > \$1000) = 0.25$$

$\mu_X = \$1050.625$ (Note that it is good to invest over the long term (5% gain over the long term... etc.)

7.48

Let X represent the torque applied by the capping machine, and let Y represent the strength (maximum torque limit) of the cap. We are given that $X \sim N(7 \text{ in-lbs}, 0.9 \text{ in-lbs})$ and that $Y \sim N(10 \text{ in-lbs}, 1.2 \text{ in-lbs})$.

(a) It is reasonable to assume that X and Y are independent because the probability that Y takes on a certain value does not change knowing a value of X . That is the machine and the process that makes the caps operate independently and have nothing to do with each other.

(b) $P(X \geq Y) = P(X - Y \geq 0) = \text{normalcdf}(0, 1E99, -3, 1.5) = 0.0227$

7.50

(a) If T represents total resistance, T is normally distributed with $\mu_T = 100 + 250 = 350$ ohms, and $\sigma_T = \sqrt{(2.5)^2 + (2.8)^2} \approx 3.7537$ ohms.

(b) $P(345 < T < 355) = \text{normalcdf}(345, 355, 350, 3.7537) = 0.817$

7.55

Age at Death	21	22	23	24	25	>26
Profit, X	-99750	-99500	-99250	-99000	-98750	1250
Probability, $P(X)$	0.00183	0.00186	0.00189	0.00191	0.00193	0.99058

7.56

If we insure many thousands of 21-year-old men, we will collect, in our first year millions (or billions) of dollars. If some of them die, we will still have lots of money after paying the \$100000 for those who died.

The better way to say this is to calculate the mean profit (the long-run expected value) made on each of thousands of young men. From the table, $\mu_x = \$303.35$. This means that on average we will make \$303 per person. Multiply that by many thousands...

7.57 $\sigma_x = \$9707.57$

7.58

(a)

$$\mu_{.5x+.5x} = .5\mu_x + .5\mu_x = \mu_x = \$303.35$$

$$\sigma_{.5x+.5x} = \sqrt{.25\sigma_x^2 + .25\sigma_x^2} \approx \$6864.29$$

(b) $\mu_{.25x+.25x+.25x+.25x} = .25\mu_x + .25\mu_x + .25\mu_x + .25\mu_x = \mu_x = \303.35

$$\sigma_{.25x+.25x+.25x+.25x} = \sqrt{.0625\sigma_x^2 + .0625\sigma_x^2 + .0625\sigma_x^2 + .0625\sigma_x^2} \approx \$4853.785$$

7.60 (a) $\mu_x = \$3$ million and $\sigma_x = \$2.5199$ million

(b) $\mu_y = 0.9\mu_x - 0.2 = \2.5 million, and $\sigma_y = 0.9\sigma_x = \2.267 million

7.63

Note that $P(\text{Win}) = \frac{1}{4} + \binom{3}{4} \left(\frac{1}{4}\right) = \frac{7}{16}$

So Assign digits from Table B 0000 – 9999 thus: 0000 to 4374 = “Win”, and 4375 to 9999 = “Lose”.

Or Assign digits 1-16 thus: 1-7 = “Win” and 8-16 = “Lose”. I will use my calculator to generate 50 random integers from 1 to 16.

From my simulation, I recorded 27 “Wins” (numbers 7 or less) and 23 losses. So that means I won \$27 but lost \$23, giving me a net of \$4. Or a mean winning of $\$4/50 = \$.08$

The probability distribution of X is

X	\$1.00	-\$1.00
P(X)	0.4375	0.5625

So $\mu_X = -\$0.125$. In my simulation, I ended up winning, but in the long run, I would lose, on average.

7.64 The probability distribution of X:

X	$\mu - \sigma$	$\mu + \sigma$
P(X)	0.5000	0.5000

So

$$\mu_X = .5(\mu - \sigma) + .5(\mu + \sigma) = \mu \quad \text{and} \quad \sigma_X^2 = .5(\mu - \sigma - \mu)^2 + .5(\mu + \sigma - \mu)^2 = \sigma^2$$

7.65

If $\rho = 1$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y = \sigma_X^2 + 2\sigma_X\sigma_Y + \sigma_Y^2 = (\sigma_X + \sigma_Y)^2$$

Then

$$\sigma_{X+Y} = \sqrt{(\sigma_X + \sigma_Y)^2} = \sigma_X + \sigma_Y$$

7.66

Since about 95% of all measurements of the shaft length, X, fall within 2 standard deviations of 11.2 inches,

$$d_1 = 2(0.002) = 0.004 \text{ inches. Similarly, } d_2 = 2(0.001) = 0.002 \text{ inches.}$$

$$\sigma_{X+Y+Z} = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2} \approx 0.002449$$

d is about 2 standard deviation in the distribution of X + Y + Z so d is about $2(0.002449) = .00489$.

The engineer, whose value of $d = 0.008$, is clearly mistaken.

7.67

$\mu_Y = 0 = a + b\mu_X = a + b(1400)$ And $\sigma_Y = 1 = b\sigma_X = b(20)$. Solving this system for a and b, we obtain

$$b = \frac{1}{20} \quad \text{and} \quad a = -70$$

7.68

(a) The possible sums are $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

(b) The possible ways to get a 5 are:

1, 1, 3
 1, 2, 2
 1, 3, 1
 2, 1, 2
 2, 2, 1
 3, 1, 1

So, $P(X=5) = \frac{6}{216}$

(c) Note that the distribution of $X_1 = X_2 = X_3$ is

X_1	1	2	3	4	5	6
$P(X_1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

So $\mu_{X_1} = 3.5$ and $\sigma_{X_1} = 1.70783$

Then $\mu_X = 3(\mu_{X_1}) = 10.5$ and $\sigma_X = \sqrt{3(\sigma_{X_1}^2)} = 2.958$