Name Key
AB Calculus
Rotational Volume

1. To enjoy a fresh delicious doughnut, you must first find its volume. In order to model the doughnut mathematically, we will rotate the circle $(x-2)^2 + y^2 = 1$ around the y-axis. This will create the doughnut shape we desire (called a torus) and will enable us to find its volume. Set up an integral to compute the volume of this doughnut by summing up washers perpendicular to the y-axis. Once your integral is set up, use FNINT to compute the volume.

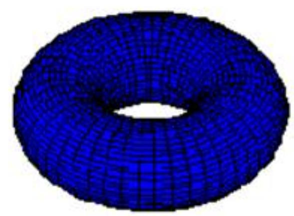
$(x-2)^2 = 1-y^2$

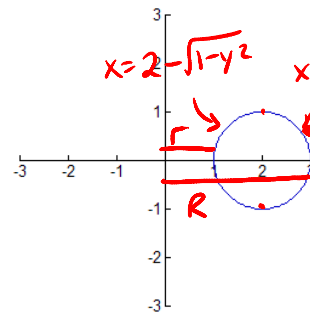
$x = 2 \pm \sqrt{1-y^2}$

$\pi R^2 - \pi r^2$

$$\int_{-1}^{1} \left[\pi\left(2+\sqrt{1-y^2}\right)^2 - \pi\left(2-\sqrt{1-y^2}\right)^2\right] dy$$

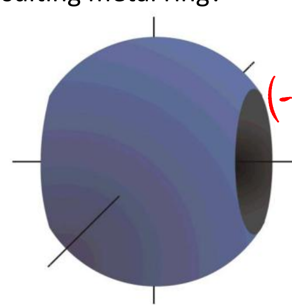
$$= 39.478$$

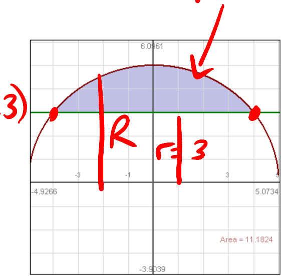




2. A manufacturer drills a hole through the center of a metal sphere of radius 5 inches. The hole has a radius of 3 inches. What is the volume of the resulting metal ring?

outside of sphere is $x^2+y^2=25$





$$\int_{-4}^{4} \left[\pi\left(\sqrt{25-x^2}\right)^2 - \pi(3)^2\right] dx$$

$$\pi\int_{-4}^{4}(25-x^2-9)\,dx = \pi\int_{-4}^{4}(16-x^2)\,dx = \pi\left(16x-\frac{x^3}{3}\right)\Big|_{-4}^{4}$$

$$= \left[\left(64-\frac{64}{3}\right)-\left(-64+\frac{64}{3}\right)\right]\pi = \left(128-\frac{128}{3}\right)\pi = \boxed{\frac{256\pi}{3}}$$

$x^3 - x + 1 = 1$

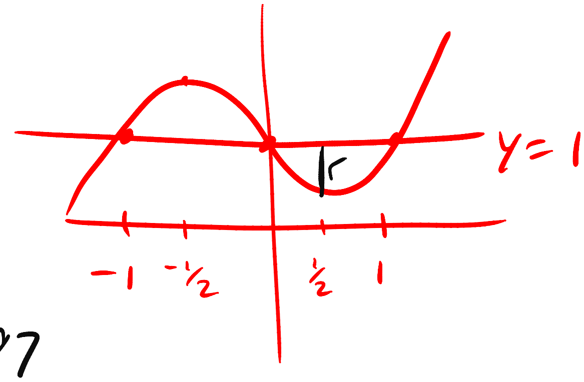
$x^3 - x = 0$

$x(x+1)(x-1) = 0$

$x = 0, 1, -1$

3. Rotate the region bounded by $y = x^3 - x + 1$ about the line $y = 1$. Find the volume.

$-\frac{1}{8} + \frac{1}{2} + 1$



$r = 1 - (x^3 - x + 1)$

$r = -x^3 + x$

$$\int_{-1}^{1} \pi(-x^3 + x)^2\,dx = \frac{16\pi}{105} = .4787$$

$-4 = y^2 - 6y + 4$

$0 = y^2 - 6y + 8$

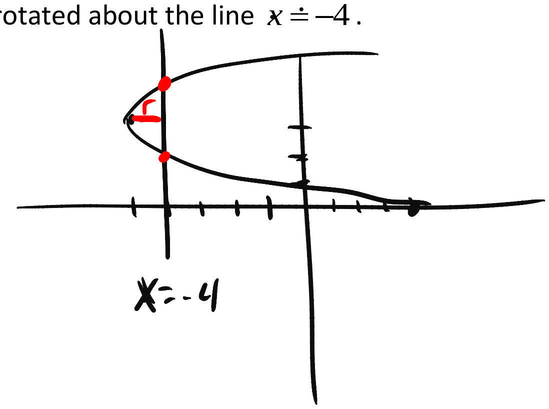
$0 = (y-2)(y-4) \quad y = 2, 4$

4. Find the volume of the region bounded by $x = y^2 - 6y + 4$ rotated about the line $x = -4$.

$x - 4 = y^2 - 6y + 9$
$\quad\ \ +9$

$x + 5 = (y-3)^2$

vertex $(-5, 3)$



$r = (0 - (y^2 - 6y + 4)) - (0 - -4)$

$r = -y^2 + 6y - 4 - 4$

$r = -y^2 + 6y - 8$

$$\int_2^4 \pi(-y^2 + 6y - 8)^2\,dy = \frac{16\pi}{15}$$

$$\pi \int_2^4 (y^4 - 12y^3 + 52y^2 - 96y + 64)\,dy$$

$$\pi\left(\frac{y^5}{5} - 3y^4 + \frac{52y^3}{3} - 48y^2 + 64y\right)\Bigg|_2^4 = \left(\frac{1024}{5} - 768 + \frac{3328}{3} - 768 + 25\right.$$

$$- \left(\frac{32}{5} - 48 + \frac{416}{3} - 192 + 128\right)$$

$$\left(\frac{992}{5} + \frac{2912}{3} - 1168\right)\pi =$$

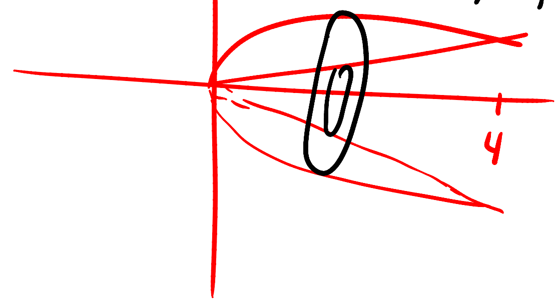
$$\left(\frac{2976 + 14560 - 17520}{15}\right)\pi$$

$$\frac{16\pi}{15}$$

5. Rotate the area between $y=\frac{x}{2}$ and $y=\sqrt{x}$

$\frac{x}{2}=\sqrt{x}$

$\frac{x^2}{4}=x \qquad x^2=4x \qquad x(x-4)=0 \qquad x=0,4$



a) about the x-axis $\quad R=\sqrt{x} \quad r=\frac{x}{2}$

$$\int_0^4 \left(\pi(\sqrt{x})^2 - \pi\left(\frac{x}{2}\right)^2\right)dx$$

$$\pi\int_0^4\left(x-\frac{x^2}{4}\right)dx = \pi\left(\frac{x^2}{2}-\frac{x^3}{12}\right)\Bigg|_0^4 =$$

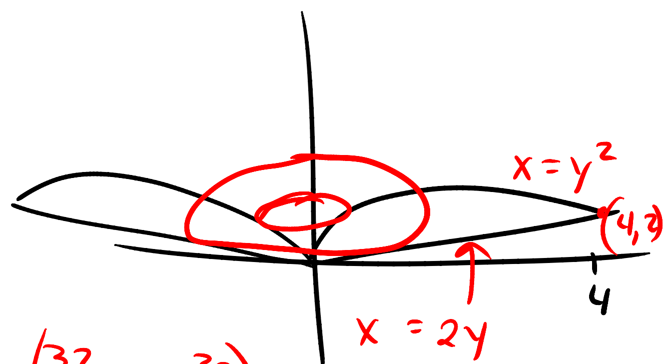
$$\pi\left(8-\frac{16}{3}-0\right) = \frac{8\pi}{3}$$

b) about the y-axis

$R=2y \qquad r=y^2$

$$\int_0^2\left[\pi(2y)^2-\pi\left(y^2\right)^2\right]dy$$

$$\pi\int_0^2\left[4y^2-y^4\right]dy \quad \pi\left(\frac{4y^3}{3}-\frac{y^5}{5}\right)\Bigg|_0^2 = \left(\frac{32}{3}-\frac{32}{5}\right)\pi = \frac{64\pi}{15}$$



$x=\frac{1}{y}$

6. Rotate the region bounded by $y=\frac{1}{x}$, $y=\frac{1}{2}$, and $y=4$ around the line $x=-4$.

$R=\frac{1}{y}+4 \qquad r=4$

$$\int_{1/2}^4\left(\pi\left(\frac{1}{y}+4\right)^2-16\pi\right)dy$$

$$=57.7598$$