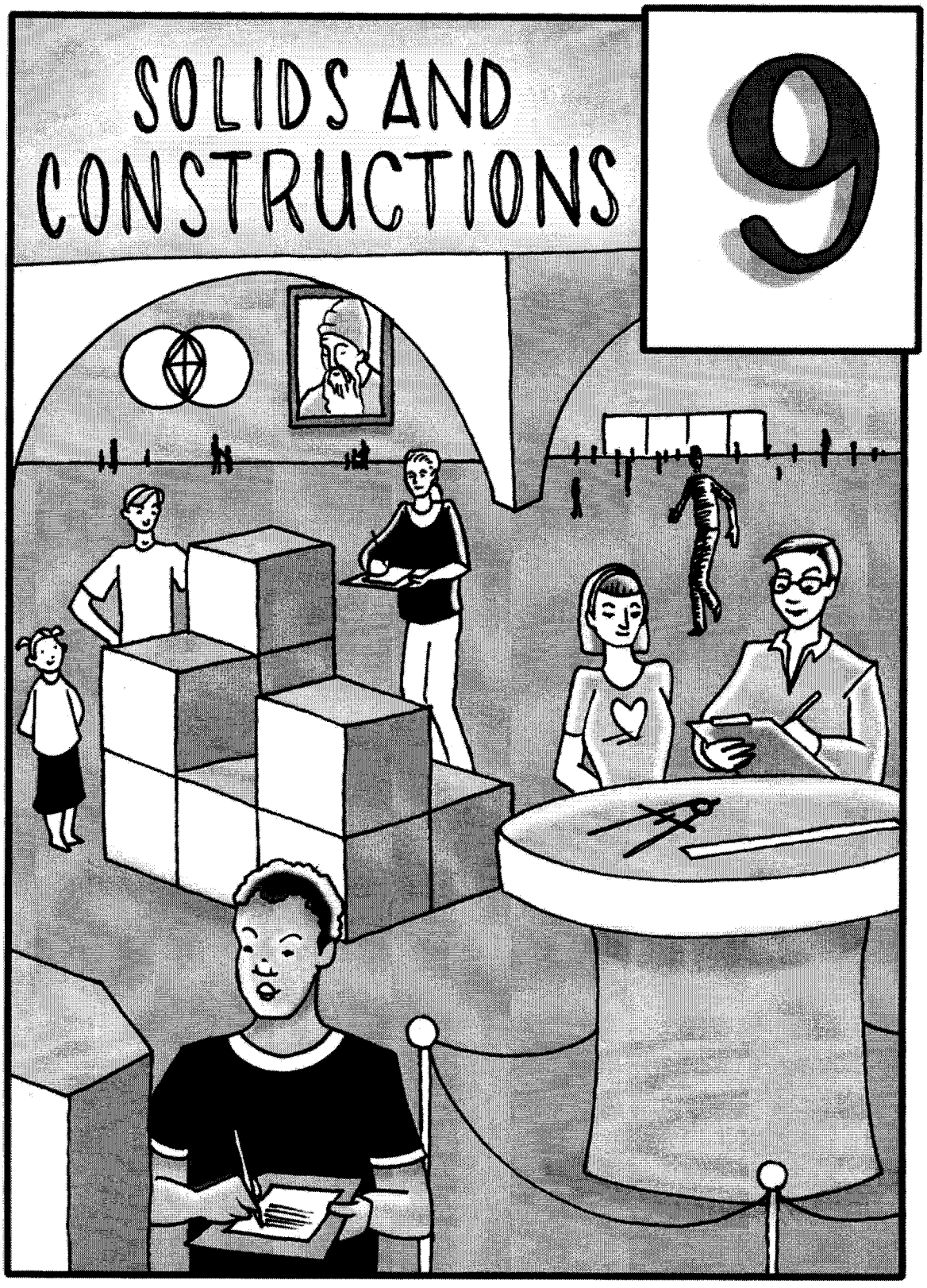
SOLIDS AND CONSTRUCTIONS

9



CHAPTER 9 Solids and Constructions

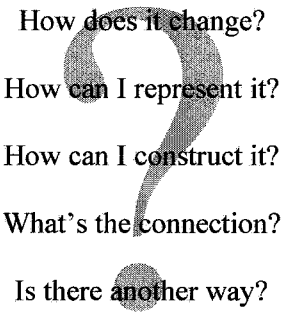
In your study of geometry so far, you have focused your attention on two-dimensional shapes. You have **investigated** the special properties of triangles, parallelograms, regular polygons and circles, and have developed tools to help you describe and analyze those shapes. For example, you have tools to find an interior angle of a regular hexagon, to calculate the length of the hypotenuse of a right triangle, and to measure the perimeter of a triangle or the area of a circle.

In Section 9.1, you will turn your focus to three-dimensional shapes (called **solids**), such as cubes and cylinders. You will learn several ways to represent three-dimensional solids and develop methods to measure their volume and surface area.

Then, in Section 9.2, you will learn how to use special tools to construct accurate diagrams of two-dimensional shapes and geometric relationships. During this **investigation**, you will revisit many of the geometric conjectures and theorems that you have developed so far.

Guiding Questions

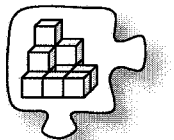
Think about these questions throughout this chapter:



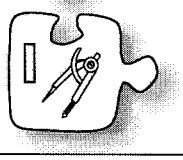
In this chapter, you will learn:

- How to find the surface area and volume of three-dimensional solids, such as prisms and cylinders.
- How to represent a three-dimensional solid with a mat plan, a net, and side and top views.
- How the volume changes when a three-dimensional solid is enlarged proportionally.
- How to construct familiar geometric shapes (such as a rhombus and a regular hexagon) using construction tools such as tracing paper, a compass and straightedge, and a dynamic geometry tool.

Chapter Outline

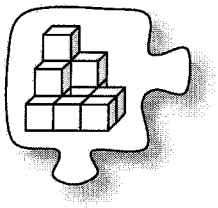


Section 9.1 This section is devoted to the study of three-dimensional solids and their measurement. You will also learn to use a variety of methods to represent the shapes of solids.



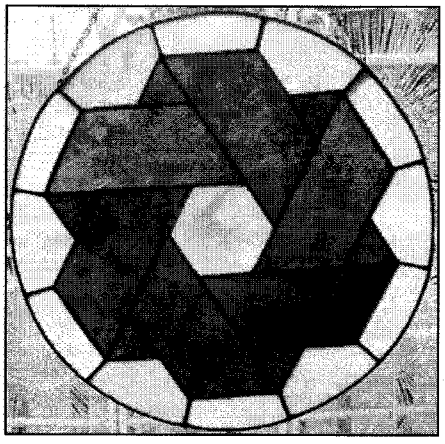
Section 9.2 This section will introduce you to the study of constructing geometric shapes and relationships. For example, you will learn how to construct a perpendicular bisector using only a compass and a straightedge.

9.1.1 How can I build it?



Three-Dimensional Solids

With your knowledge of polygons and circles, you are able to create and explore new, interesting shapes and make elaborate designs such as the one shown in the stained glass window at right. However, in the physical world, the objects we encounter every day are three-dimensional. In other words, physical objects cannot exist entirely on a flat surface, such as a tabletop.



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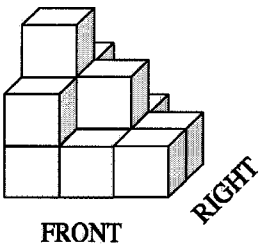
To understand the shapes that you encounter daily, you will need to learn more about how three-dimensional shapes (called **solids**) can be created, described, and measured.

As you work with your team today, be especially careful to explain to your teammates how you "see" each solid. Remember that spatial **visualization** takes time and effort, so be patient with your teammates and help everyone understand how each solid is built.

9-1. Using blocks provided by your teacher, work with your team to build the three-dimensional solid at right. Assume that blocks cannot hover in midair. That is, if a block is on the second level, assume that it has a block below it to prop it up.



a. Is there more than one arrangement of blocks that could look like the solid drawn at right? Why or why not?

b. To avoid confusion, a **mat plan** can be used to show how the blocks are arranged in the solid. The number in each square represents the number of the blocks stacked in that location if you are looking from above. For example, in the lower right-hand corner, the solid is only 1 block tall, so there is a "1" in the corresponding corner of its mat plan. .

2	1	0
3	2	1
2	1	1

RIGHT

FRONT

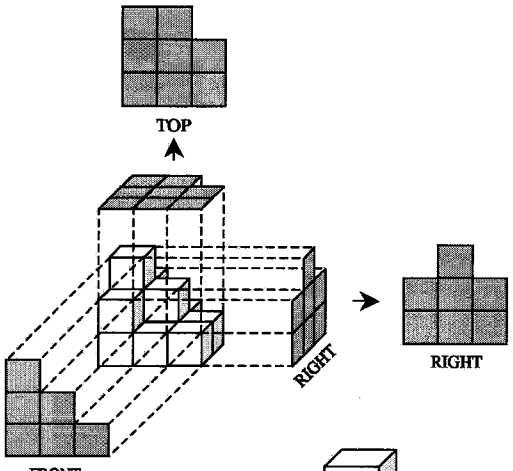
Mat Plan

Verify that the solid your team built matches the solid represented in the mat plan above.

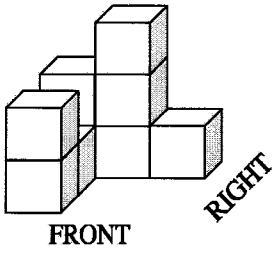
c. What is the **volume** of the solid? That is, if each block represents a "cubic unit," how many blocks (cubic units) make up this solid?

9-2. Another way to represent a three-dimensional solid is by its **side** and **top views**.

For example, the solid from problem 9-1 can also be represented by a top, front, and right-hand view, as shown at right. Each view shows *all* of the blocks that are visible when looking directly at the solid from that direction.



Examine the diagram of blocks below. On graph paper, draw the front, right, and top views of this solid. Assume that there are no hidden blocks.



9-3. For each of the mat plans below:

- Build the three-dimensional solid with the blocks provided by your teacher.
- Find the volume of the solid in cubic units.
- Draw the front, right, and top views of the solid on a piece of graph paper.

a.

0	3	0	
2	3	1	RIGHT
0	2	0	

FRONT

b.

0	2	1	
0	3	0	RIGHT
3	2	1	

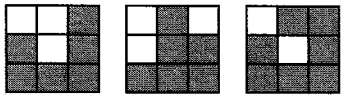
FRONT

c.

1	1	3	
2	1	2	RIGHT
0	0	1	

FRONT

9-4. Meagan built a shape with blocks and then drew the views shown at right.

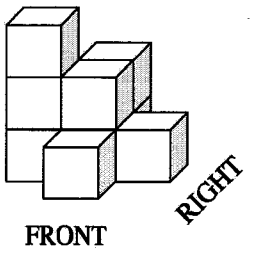


a. Build Meagan's shape using blocks provided by your teacher. Use as few blocks as possible.

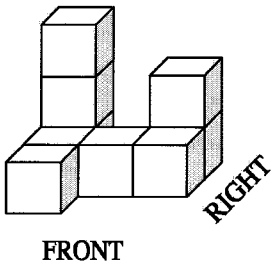
b. What is the volume of Meagan's shape?

9-5. Draw a mat plan for each of the following solids. There may be more than one possible answer! Then find the possible volumes of each.

a.

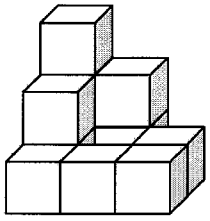


b.



9-6. During this lesson, you have found the volume of several three-dimensional solids. However, what *is* volume? What does it measure? Write a Learning Log entry describing volume. Add at least one example. Title this entry "Volume of a Three-Dimensional Shape" and include today's date.

9-7. **Examine** the solid at right.



a. On your paper, draw a possible mat plan for this solid.

b. Find the volume of this solid.

9-8. Assume that two figures, A and B, are similar.

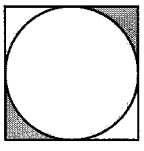
a. If the linear scale factor is $\frac{2}{5}$, then what is the ratio of the areas of A and B?

b. If the ratio of the perimeters of A and B is 14:1, what is the ratio of the areas?

c. If the area of A is 81 times that of B, what is the ratio of the perimeters?

9-9. Find the area of a regular decagon with perimeter 100 units. Show all work.

9-10. The diagram at right shows a circle inscribed in a square. Find the area of the shaded region if the side length of the square is 6 meters.



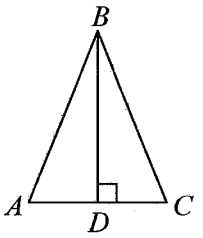
9-11. Solve each system of equations below. Write your solution in the form (x, y). Check your solution.

a. $3x - y = 14$
$x = 2y + 8$

b. $x = 2y + 2$
$x = -y - 10$

c. $16x - y = -4$
$2x + y = 13$

9-12. **Multiple Choice:** What information would you need to know about the diagram at right in order to prove that $\triangle ABD \cong \triangle CBD$ by SAS $\cong$?



a. $\overline{AD} \cong \overline{CD}$

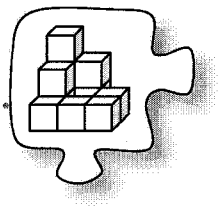
b. $\overline{AB} \cong \overline{CB}$

c. $\measuredangle A \cong \measuredangle C$

d. $\measuredangle ABD \cong \measuredangle CBD$

e. None of these

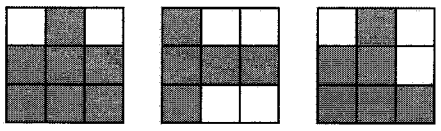
9.1.2 How can I measure it?



Volume and Surface Area of Prisms

Today you will continue to study three-dimensional solids and will practice representing a solid using a mat plan and its side and top views. You will also learn a new way to represent a three-dimensional object, called a **net**. As you work today, you will learn about a special set of solids called **prisms** and will study how to find the surface area and volume of a prism.

9-13. The front, top, and right-hand views of Heidi's solid are shown at right.



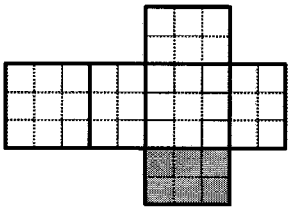
a. Build Heidi's solid using blocks provided by your teacher. Use the smallest number of blocks possible. What is the volume of her solid?

b. Draw a mat plan for Heidi's solid. Be sure to indicate where the front and right sides are located.

c. Oh no! Heidi accidentally dropped her entire solid into a bucket of paint! What is the **surface area** of her solid? That is, what is the area that is now covered in paint?

9-14. So far, you have studied three ways to represent a solid: a three-dimensional drawing, a mat plan, and its side and top views.

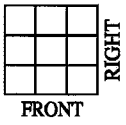
Another way to represent a three-dimensional solid is with a **net**, such as the one shown at right. When folded, a net will form the three-dimensional solid it represents.



a. With your team, predict what the three-dimensional solid formed by this net will look like. Assume the shaded squares make up the base (or bottom) of the solid.

b. Obtain a Lesson 9.1.2 Resource Page and scissors from your teacher and cut out the net. Fold along the solid lines to create the three-dimensional solid. Did the result confirm your prediction from part (a)?

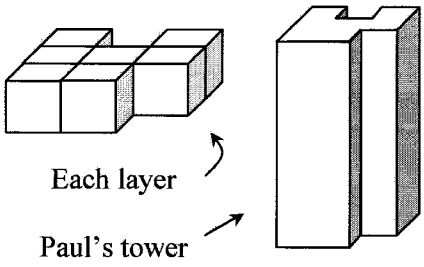
c. Now build the shape with blocks and complete the mat plan at right for this solid.



d. What is the volume of this solid? How did you get your answer?

e. What is the surface area of the solid? How did you find your answer? Be prepared to share any shortcuts with the class.

9-15. Paul built a tower by stacking six identical layers of the shape at right on top of each other.



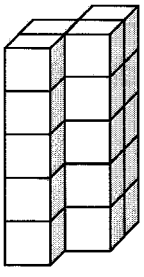
a. What is the volume of his tower? How can you tell without building the shape?

b. What is the surface area of his tower?

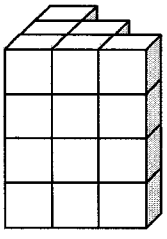
c. Paul's tower is an example of a **prism** because it is a solid and two of its faces (called **bases**) are congruent and parallel. A prism must also have sides that connect the bases (called **lateral faces**). Each lateral face must be a parallelogram.

For each of the prisms below, find the volume and surface area.

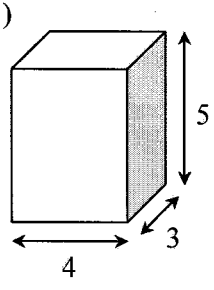
(1)



(2)



(3)



9-16. Heidi created several more solids, represented below. Find the volume of each one.

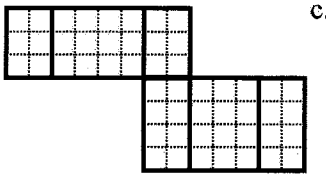
a.

0	3	5
22	10	25
18	15	8
16	12	0

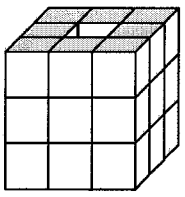
RIGHT

FRONT

b.



c.



9-17. Pilar built a tower by stacking identical layers on top of each other. If her tower used a total of 312 blocks and if the bottom layer has 13 blocks, how tall is her tower? Explain how you know.

9-18. What is the relationship between the area of the base of a prism, its height, and its volume? In a Learning Log entry, summarize how to find the volume of a solid. Be sure to include an example. Title this entry "Finding Volume" and include today's date.

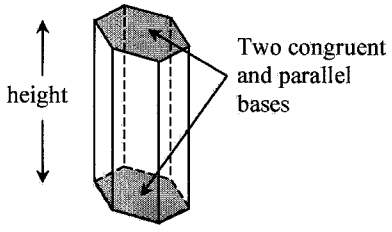
METHODS AND MEANINGS

Polyhedra and Prisms

MATH NOTES

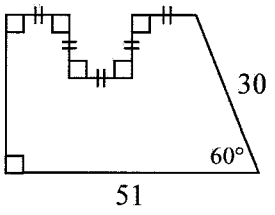
A closed three-dimensional solid that has flat, polygonal faces is called a **polyhedron**. The plural of polyhedron is **polyhedra**. "Poly" is the Greek root for "many," and "hedra" is the Greek root for "faces."

A **prism** is a special type of polyhedron. It must have two congruent, parallel **bases** that are polygons. Also, its **lateral faces** (the faces connecting the bases) are parallelograms formed by connecting the corresponding vertices of the two bases. Note that lateral faces may be any type of parallelogram, such as rectangles, rhombi, or squares.



Review & Preview

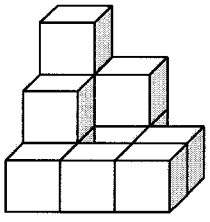
9-19. Mr. Wallis is designing a home. He found the plan for his dream house on the Internet and printed it out on paper.



a. The design of the home is shown at right. If all measurements are in millimeters, find the area of the diagram.

b. Mr. Wallis took his home design to the copier and enlarged it 400%. What is the area of the diagram now? Show how you know.

9-20. At right is the solid from problem 9-7.



a. On graph paper, draw the front, right, and top views.

b. Find the total surface area of the solid.

9-21. Review what you know about the angles of polygons below.

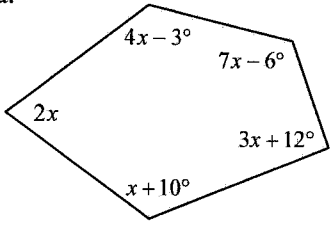
a. If the exterior angle of a polygon is 29°, what is the interior angle?

b. If the interior angle of a polygon is 170°, can it be a regular polygon? Why or why not?

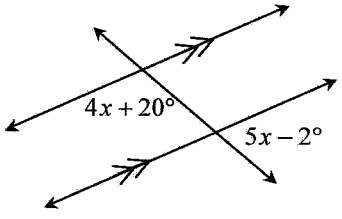
c. Find the sum of the interior angles of a regular 29-gon.

9-22. For each geometric relationship represented below, write and solve an equation for x. Show all work.

a.



b.



9-23. On graph paper, graph $\triangle ABC$ if $A(-3, -4)$, $B(-1, -6)$, and $C(-5, -8)$.

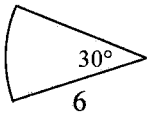
a. What is AB (the length of $\overline{AB}$)?

b. Reflect $\triangle ABC$ across the x-axis to form $\triangle A'B'C'$. What are the coordinates of B'?

c. Rotate $\triangle A'B'C'$ 90° clockwise (↻) about the origin to form $\triangle A''B''C''$. What are the coordinates of C''?

d. Translate $\triangle ABC$ so that $(x, y) \rightarrow (x + 5, y + 1)$. What are the new coordinates of point A?

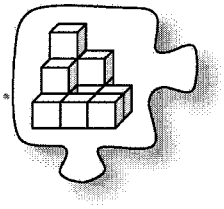
9-24. **Multiple Choice:** Find the perimeter of the sector at right.



a. 12π units b. 3π units c. $6 + 3\pi$ units

d. $12 + \pi$ units e. None of these

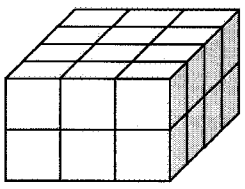
9.1.3 What if the bases are not rectangles?



Prisms and Cylinders

In Lessons 9.1.1 and 9.1.2, you **investigated** volume, surface area, and special three-dimensional solids called prisms. Today you will explore different ways to find the volume and surface area of a prism and a related solid called a cylinder. You will also consider what happens to the volume of a prism or cylinder if it slants to one side or if it is enlarged proportionally.

9-25. **Examine** the three-dimensional solid at right.

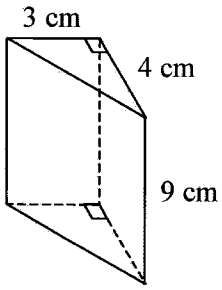


a. On graph paper, draw a net that, when folded, will create this solid.

b. Compare your net with those of your teammates. Is there more than one possible net? Why or why not?

c. Find the surface area and volume of this solid.

9-26. SPECIAL PRISMS



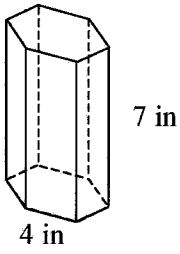
The prism in problem 9-25 is an example of a **rectangular prism**, because its bases are rectangular. Similarly, the prism at right is called a **triangular prism** because the two congruent bases are triangular.

a. Carefully draw the prism at right onto your paper. One way to do this is to draw the two triangular bases first and then to connect the corresponding vertices of the bases. Notice that hidden edges are represented with dashed lines.

b. Find the surface area of the triangular prism. Remember that the surface area includes the areas of all surfaces – the sides and the bases. Carefully organize your work and verify your solution with your teammates.

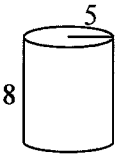
c. Find the volume of the triangular prism. Be prepared to share your team's method with the class.

d. Does your method for finding surface area and volume work on other prisms? For example, what if the bases are hexagonal, like the one shown at right? Work with your team to find the surface area and volume of this hexagonal prism. Assume that the bases are regular hexagons with side length 4 inches.

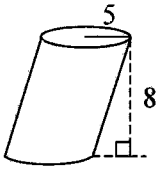


9-27. CYLINDERS

Carter wonders, "*What if the bases are circular?*"



a. Copy the **cylinder** at right onto your paper. Discuss with your team how to find its surface area and volume if the radius of the base is 5 units and the height of the cylinder is 8 units.



b. Now Carter wants to figure out what happens to the volume of a cylinder when it slants as shown in the diagram at right. When the lateral faces of a prism or cylinder are not perpendicular to its base, the solid is referred to as an **oblique** cylinder or prism.

With your team, discuss whether the volume of the cylinder will increase, decrease or stay the same when the prism or cylinder is slanted. Assume that the radius and height of the cylinder do not change. When you agree, explain your answer on your paper. Be sure to provide **reasons** for your statements.

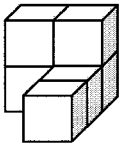
9-28. Hernando needs to replace the hot water tank at his house. He estimates that his family needs a tank that can heat at least 75 gallons of water. His local water tank supplier has a cylindrical model that has a diameter of 2 feet and a height of 3 feet. If 1 gallon of water is approximately 0.1337 cubic feet, determine if the supplier's tank will provide enough water.

METHODS AND MEANINGS

Volume and Total Surface Area of a Solid

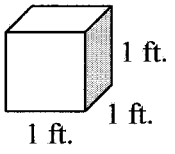
MATH NOTES

Volume measures the size of a three-dimensional space enclosed within an object. It is expressed as the number of 1x1x1 cubes (or parts of cubes) that fit inside a solid.



For example, the solid shown at right has a volume of 6 cubic units.

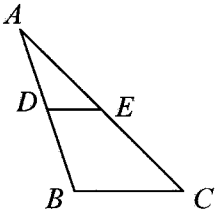
Since volume reflects the number of cubes that fit within a solid, it is measured in **cubic units**. For example, if the dimensions of a solid are measured in feet, then the volume would be measured in cubic feet (a cube with dimensions 1' x 1' x 1').



On the other hand, the **total surface area** of a solid is the area of all of the external faces of the solid. For example, the total surface area of the solid above is 24 square units.

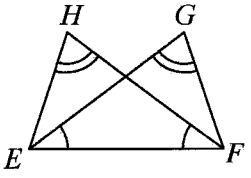
Review & Preview

9-29. In the diagram at right, $\overline{DE}$ is a midsegment of $\triangle ABC$. If the area of $\triangle ABC$ is 96 square units, what is the area of $\triangle ADE$?



9-30. Draw a rectangular prism as neatly as possible on your paper. If the width is 9 cm, the height is 14 cm, and the depth is 7 cm, find the surface area and volume.

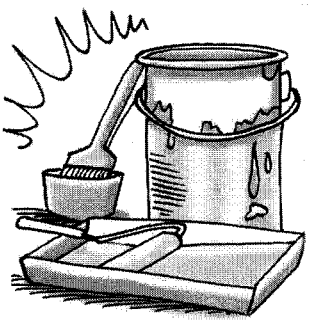
9-31. Are $\triangle EHF$ and $\triangle FGE$ congruent? If so, explain how you know. If not, explain why not.



9-32. Remember that the absolute value of a number is its positive value. For example, $|-5|=5$ and $|5|=5$. Use this understanding to solve the equations below, if possible. If there is no solution, explain how you know.

a. $|x|=6$ b. $|x|=-2$ c. $|x+7|=10$

9-33. Cindy's cylindrical paint bucket has a diameter of 12 inches and a height of 14.5 inches. If 1 gallon ≈ 231 in^3, how many gallons does her paint bucket hold?

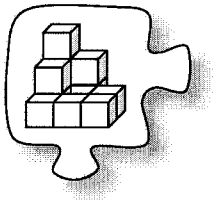


9-34. **Multiple Choice:** Which ratio below is the best approximation of the ratio between the circumference of a circle and its diameter?

a. 2 b. 3

c. 4 d. 6

9.1.4 How does the volume change?

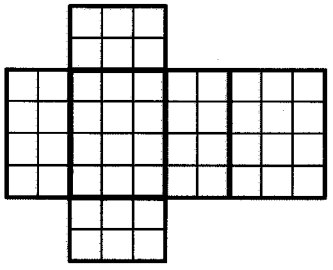


Volumes of Similar Solids

As you continue your study of three-dimensional solids, you will explore how the volume of a solid changes as the solid is enlarged proportionally.

9-35. HOW DOES THE VOLUME CHANGE?

In Lesson 9.1.3, you began a study of the surface area and volume of similar solids. Today, you will continue that **investigation** in order to generalize about the ratios of similar solids.



a. Describe the solid formed by the net at right. What are its dimensions (its length, width, and height)?

b. Have each team member select a different enlargement ratio from the list below. On graph paper, carefully draw the net of a similar solid using your enlargement ratio. Then cut out your net and build the solid (so that the gridlines end up on the outside the solid) using scissors and tape.

(1) 1 (2) 2 (3) 3 (4) 4

Problem continues on next page →

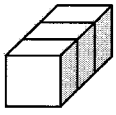
9-35. *Problem continued from previous page.*

c. Find the volume of your solid and compare it to the volume of the original solid. What is the ratio of these volumes? Share the results with your teammates so that each person can complete a table like the one below.

Linear Scale Factor	Original Volume	New Volume	Ratio of Volumes
1			
2			
3			
4			
r			

d. How does the volume change when a three-dimensional solid is enlarged or reduced to create a similar solid? For example, if a solid's length, width, and depth are enlarged by a zoom factor of 10, then how many times bigger does the volume get? What if the solid is enlarged by a zoom factor of r? Explain.

9-36. **Examine** the 1x1x3 solid at right.



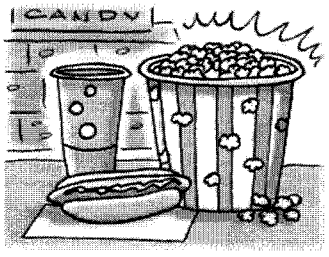
a. Build this solid with blocks provided by your teacher.

b. If this shape is enlarged by a linear scale factor of 2, how wide will the new shape be? How tall? How deep?

c. How many of the 1x1x3 solids would you need to build the enlargement described in part (b) above? Use blocks to prove your answer.

d. What if the 1x1x3 solid is enlarged with a linear scale factor of 3? How many times larger would the volume of the new solid be? Explain how you found your answer.

9-37.



At the movies, Maurice counted the number of kernels of popcorn that filled his tub and found that it had 316 kernels. He decided that next time, he will get an enlarged tub that is similar, but has a linear scale factor of 1.5. How many kernels of popcorn should the enlarged tub hold?

9-38. In your Learning Log, explain how the volume changes when a solid is enlarged proportionally. That is, if a three-dimensional object is enlarged by a zoom factor of 2, by what factor does the volume increase? Title this entry "Volumes of Similar Solids" and include today's date.

Review & Preview

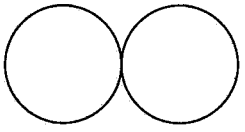
9-39. Koy is inflating a spherical balloon for her brother's birthday party. She has used three full breaths so far and her balloon is only half the width she needs. Assuming that she puts the same amount of air into the balloon with each breath, how many more breaths does she need to finish the task? Explain how you know.

9-40. Draw a cylinder on your paper. Assume the radius of the cylinder is 6 inches and the height is 9 inches.

a. What is the surface area of the cylinder? What is the volume?

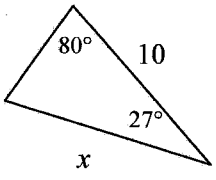
b. If the cylinder is enlarged with a linear scale factor of 3, what is the volume of the enlarged cylinder? How do you know?

9-41. While Katarina was practicing her figure skating, she wondered how far she had traveled. She was skating a "figure 8," which means she starts between two circles and then travels on the boundary of each circle, completing the shape of an "8." If both circles have a radius of 5 feet, how far does she travel when skating one "figure 8"?

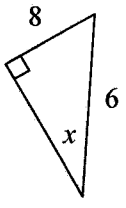


9-42. For each triangle below, solve for x, if possible. If no solution is possible, explain why.

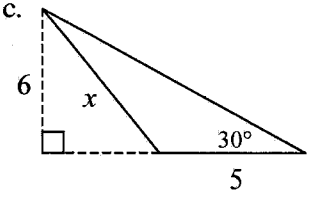
a.



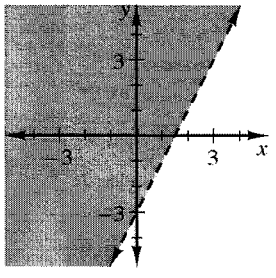
b.



c.



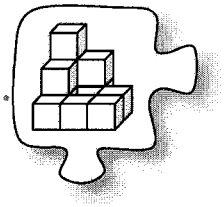
9-43. The graph of the inequality $y > 2x - 3$ is shown at right. On graph paper, graph the inequality $y \leq 2x - 3$. Explain what you changed about the graph.



9-44. **Multiple Choice:** The point $A(-2, 5)$ is rotated 90° counter-clockwise (↺). What are the new coordinates of point A?

a. (2, 5) b. (5, –2) c. (2, –5) d. (–5, –2)

9.1.5 How does the volume change?



Ratios of Similarity

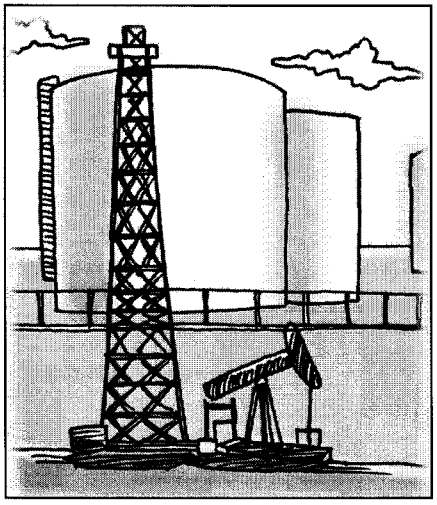
Today, work with your team to analyze the following problems. As you work, think about whether the problem involves volume or area. Also think carefully about how similar solids are related to each other.

9-45. A statue to honor Benjamin Franklin will be placed outside the entry to the Liberty Bell exhibit hall. The designers decide that a smaller, similar version will be placed on a table inside the building. The dimensions of the life-size statue will be four times those of the smaller statue. Planners expect to need 1.5 pints of paint to coat the small statue. They also know that the small statue will weigh 14 pounds.

a. How many pints of paint will be needed to paint the life-size statue?

b. If the small statue is made of the same material as the enlarged statue, then its weight will change just as the volume changes as the statue is enlarged. How much will the life-size statue weigh?

9-46.

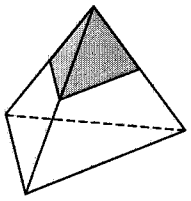


The Blackbird Oil Company is considering the purchase of 20 new jumbo oil storage tanks. The standard model holds 12,000 gallons. Its dimensions are $\frac{4}{5}$ the size of the similarly shaped jumbo model, that is, the ratio of the dimensions is 4:5.

a. How much more storage capacity would the purchase of the twenty jumbo models give Blackbird Oil?

b. If jumbo tanks cost 50% more than standard tanks, which tank is a better buy?

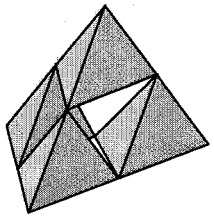
9-47. In problem 7-13, your class constructed a large tetrahedron like the one at right. Assume the dimensions of the shaded tetrahedron at right are half of the dimensions of the similar enlarged tetrahedron.



a. If the volume of the large tetrahedron is 138 in^3, find the volume of the small shaded tetrahedron.

b. Each face of a tetrahedron is an equilateral triangle. If the small shaded tetrahedron has an edge length of 16 cm, find the total surface area of the both tetrahedra.

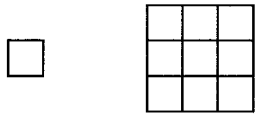
c. Your class tried to construct a tetrahedron using four smaller congruent tetrahedra. However, the result left a gap in the center, as shown in the diagram at right. If the volume of each small shaded tetrahedron is 50 in^3, what is the volume of the gap? Explain how you know.



METHODS AND MEANINGS

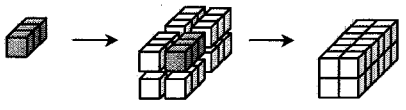
The $r : r^2 : r^3$ Ratios of Similarity

When a two-dimensional figure is enlarged proportionally, its perimeter and area also grow. If the linear scale factor is r, then the perimeter of the figure is enlarged by a factor of r while the area of the figure is enlarged by a factor of r^2. Examine what happens when the square at right is enlarged by a linear scale factor of 3.



$P = 4$ un, $A = 1$ un^2 | $P = 4 \cdot 3 = 12$ un, $A = 1 \cdot 3^2 = 9$ un^2

When a solid is enlarged proportionally, its surface area and volume also grow. If it is enlarged by a linear scale factor of r, then the surface area grows by a factor of r^2 and the volume grows by a factor of r^3. The example at right shows what happens to a solid when it is enlarged by a linear scale factor of 2.



Original solid: SA = 14 un^2, V = 3 un^3 | Width, height, and depth are doubled | **Result**: SA = 56 un^2, V = 24 un^3

Thus, if a solid is enlarged proportionally by a linear scale factor of r, then:

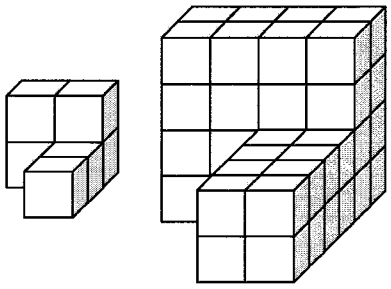
New edge length = $r \cdot$ (corresponding edge length of original solid)

New surface area = $r^2 \cdot$ (original surface area)

New volume = $r^3 \cdot$ (original volume)

Review & Preview

9-48. Consider the two similar solids at right.

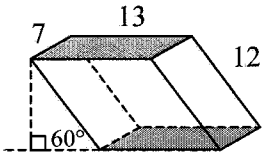


a. What is the linear scale factor?

b. Find the surface area of each solid. What is the ratio of the surface areas? How is this ratio related to the linear scale factor?

c. Now find the volumes of each solid. How are the volumes related? Compare this to the linear scale factor and record your observations.

9-49. Elliot has a modern fish tank that is in the shape of an oblique prism, shown at right. If the slant of the prism makes a 60° angle, find the volume of water the tank can hold. Assume all measurements are in inches.



9-50. Decide if the following statements are true or false. If they are true, explain how you know. If they are false, provide a counterexample.

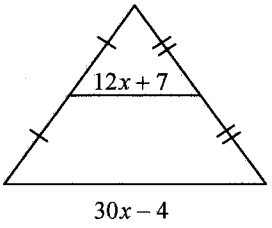
a. If a quadrilateral has two sides that are parallel and two sides that are congruent, then the quadrilateral must be a parallelogram.

b. If the interior angles of a polygon add up to 360°, then the polygon must be a quadrilateral.

c. If a quadrilateral has 3 right angles, then the quadrilateral must be a rectangle.

d. If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be a rhombus.

9-51. Write and solve an equation based on the geometric relationship shown at right.



9-52. Solve each equation below. Check your solution.

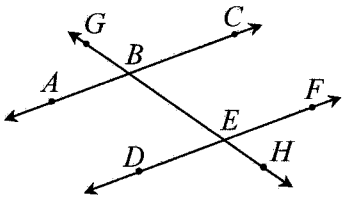
a. $20 - 6(5 + 2x) = 10 - 2x$

b. $2x^2 - 9x - 5 = 0$

c. $\frac{3}{5x-1} = \frac{1}{x+1}$

d. $|2x - 1| = 5$

9-53. **Multiple Choice:** For $\measuredangle ABE \cong \measuredangle BEF$ in the diagram at right, what must be true?

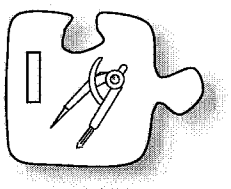


a. $\measuredangle ABE \cong \measuredangle BED$ b. $\measuredangle ABE \cong \measuredangle GBC$

c. $\overline{AC} \parallel \overline{GH}$ d. $\overline{AC} \parallel \overline{DF}$

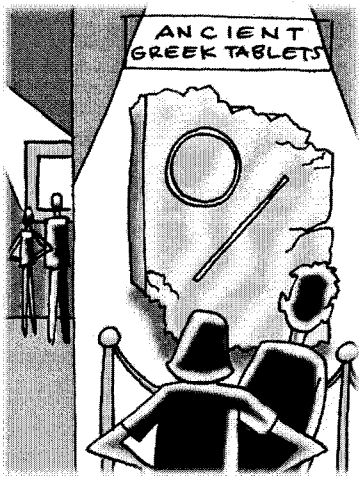
e. None of these.

9.2.1 How can I construct it?



Introduction to Construction

So far in this course, you have used tools such as rulers, tracing paper, templates, protractors, and even computers to draw geometric relationships and shapes. But how did ancient mathematicians accurately construct shapes such as squares or equilateral triangles without these types of tools?



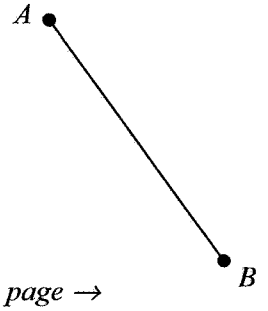
Today you will start by exploring how to construct several geometric relationships and figures with tracing paper. You will then **investigate** how to construct geometric shapes with tools called a compass and a straightedge, much like the ancient Greeks did over 2000 years ago. As you study these forms of **construction**, you will not only learn about new geometric tools, but also gain a deeper understanding of some of the special geometric relationships and shapes you have studied so far in this course.

9-54. CONSTRUCTING WITH TRACING PAPER

To start this focus on construction, you will begin with a familiar tool: tracing paper. Obtain several sheets of tracing paper and a straightedge from your teacher. Note: A straightedge is *not* a ruler. It does not have any markings or measurements on it. A 3x5 index card makes a good straightedge.

a. Starting with a smooth, square piece of tracing paper, find a way to create parallel lines (or creases). Make sure the lines are *exactly* parallel. Be ready to share with the class how you accomplished this.

b. On a new piece of tracing paper, trace line segment $\overline{AB}$ at right. Use your straightedge for accuracy. Can you fold the tracing paper so that the resulting crease not only finds the midpoint of $\overline{AB}$ but also is perpendicular to $\overline{AB}$? (This is called a **perpendicular bisector**.) Again, be ready to share your method.



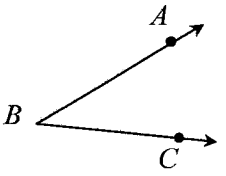
Problem continues on next page →

9-54. *Problem continued from previous page.*

c. On the perpendicular bisector from part (b) above, choose a point C and then connect $\overline{AC}$ and $\overline{BC}$ to form $\triangle ABC$. What type of triangle did you construct? Use your geometry knowledge to **justify** your answer.

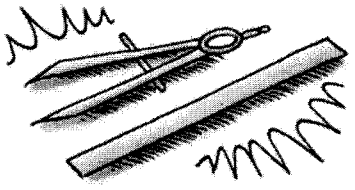
d. In part (b), you figured out how to use tracing paper and a straightedge to construct a line that bisects another line. How can you construct an angle bisector?

On a piece of tracing paper, trace $\measuredangle ABC$ at right. Construct an angle bisector. That is, find $\overrightarrow{BD}$ so that $\measuredangle ABD \cong \measuredangle CBD$.



9-55. CONSTRUCTING WITH A COMPASS AND A STRAIGHTEDGE

Producing a geometric shape with a compass and a straightedge is another form of **construction**. Obtain a Lesson 9.2.1 Resource Page from your teacher (or download from www.cpm.org) and explore what types of shapes you can construct using these tools.



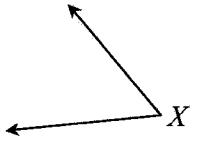
a. Find point C on the resource page. Use your compass to construct two circles with different radii that have a center at point C. (Note: Circles that have the same center are called **concentric** circles.)

b. With tracing paper, copying a line segment means just putting the tracing paper over the line and tracing it. But how can you copy a line using only a compass and a straightedge?

On the resource page, find $\overline{AB}$. Next to $\overline{AB}$, use your straightedge to draw a new line. With your team, decide how to use the compass to mark off two points (C and D) so that $\overline{AB} \cong \overline{CD}$. Be ready to share your method with the class.

c. Now construct a new line segment, labeled $\overline{EF}$, that is twice as long as $\overline{AB}$. How can you be sure that $\overline{EF}$ is twice as long as $\overline{AB}$?

9-56. In problem 9-55, you learned how to use a compass and a straightedge to copy a line segment. But how can you use these tools to copy an angle? On your Lesson 9.2.1 Resource Page, find $\measuredangle X$. With your team, discuss how you can construct a new angle ($\measuredangle Y$) that is congruent to $\measuredangle X$. If you need help, use parts (a) through (c) below to guide you.



a. On the resource page, draw a ray with endpoint Y.

b. With your compass point at X, draw an arc that intersects both sides of $\measuredangle X$. Now draw an arc with the same radius and with center Y.

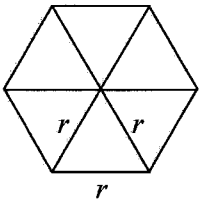
Problem continues on next page →

9-56. *Problem continued from previous page.*

c. How can you use your compass to measure the "width" of $\measuredangle X$? Discuss this with your teammates and then determine how to complete $\measuredangle Y$. Be ready to share your method with the class.

9-57. REGULAR HEXAGON

As Shui was completing her homework, she noticed that a regular hexagon has a special quality: when dissected into congruent triangles, the hexagon contains triangles that are all equilateral! "I bet I can use this fact to help me construct a regular hexagon," she told her team.

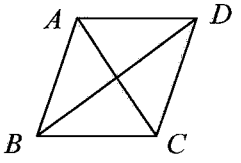


a. On the Lesson 9.2.1 Resource Page, construct a circle with radius r and center H.

b. Mark one point on the circle to be a starting vertex. Since each side of the hexagon has length r, the radius of the circle, carefully use the compass to mark off the other vertices of the hexagon on the circle. Then connect the vertices to create the regular hexagon.

c. When all vertices of a polygon lie on the same circle, the polygon is called **inscribed**. For example, the hexagon you constructed in part (b) is inscribed in $\odot H$. After consulting with your teammates, construct an equilateral triangle that is also inscribed in $\odot H$. You may want to use colored markers or pencils to help distinguish between the hexagon and the triangle.

9-58. **Examine** the diagram of $ABCD$ at right.



a. If opposite sides of the quadrilateral are parallel and all sides are congruent, what type of quadrilateral is $ABCD$?

b. List what you know about the diagonals of $ABCD$.

c. Find the area of $ABCD$ if $BC = 8$ and $m\measuredangle ABC = 60°$.

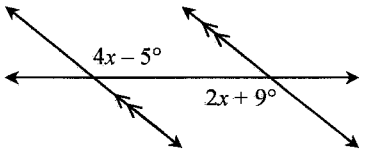
9-59. For a rectangular prism with base dimensions 3 cm and 5 cm and height 8 cm:

a. Sketch the prism on your paper.

b. Find the volume of the prism.

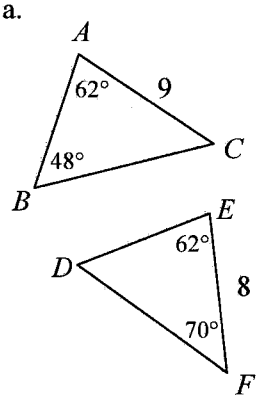
c. Find the surface area of the prism.

9-60. Use the relationships given in the diagram at right to write and solve an equation for x. Show all work.

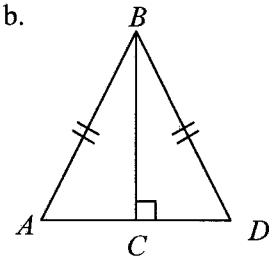


9-61. For each pair of triangles below, determine if the triangles are congruent. If they are congruent, state the congruence property that assures their congruence and write a congruence statement (such as $\triangle ABC \cong \triangle$_____).

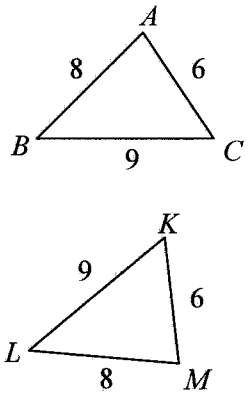
a.



b.



c.

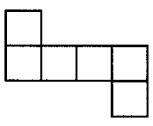


9-62. Write the equation represented by the table below.

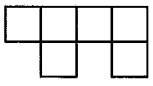
IN (x)	–4	–3	–2	–1	0	1	2	3	4
OUT (y)	–26	–20	–14	–8	–2	4	10	16	22

9-63. **Multiple Choice:** Which net below will not produce a closed cube?

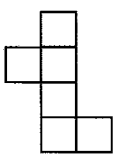
a.



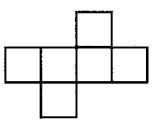
b.



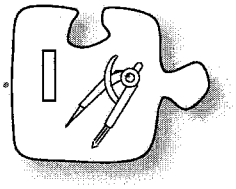
c.



d.



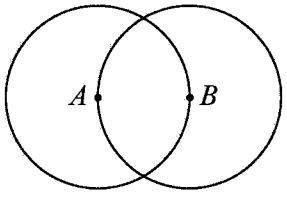
9.2.2 How can I construct it?



Constructing Bisectors

During Lesson 9.2.1, you studied how to construct geometric relationships such as congruent line segments using tools that include a compass and tracing paper. But what other geometric relationships and shapes can we construct using these tools? Today, as you **investigate** new ways to construct familiar geometric figures, look for connections to previous course material.

9-64. INTERSECTING CIRCLES



As Ventura was doodling with his compass, he drew the diagram at right.

a. Explain why $\odot A$ and $\odot B$ must have the same radius.

b. On the Lesson 9.2.2 Resource Page provided by your teacher, construct two intersecting circles so that each passes through the other's center. Label the centers A and B.

c. On your construction, locate the two points where the circles intersect each other. Label these points C and D. Then construct quadrilateral $ACBD$. What type of quadrilateral is $ACBD$? **Justify** your answer.

d. Use what you know about the diagonals of $ACBD$ to describe the relationship of $\overline{AB}$ and $\overline{CD}$. Make as many statements as you can.

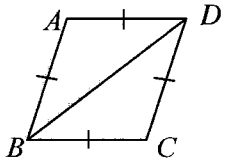
9-65. In problem 9-64, you constructed a rhombus and a perpendicular bisector.

a. In your own words, describe how this process works. That is, given any line segment, how can you find its midpoint? How can you find a line perpendicular to it? Be sure to **justify** your statements.

b. Test that your directions in part (a) work for line $\overline{KM}$ on the Lesson 9.2.2 Resource Page. In other words, construct a perpendicular bisector of $\overline{KM}$. Label the midpoint of $\overline{KM}$ point N.

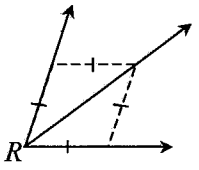
c. Return to your work from part (b) and use it to construct a 45°- 45°- 90° triangle. Prove that your triangle must be isosceles.

9-66. In problem 9-64, you used the fact that the diagonals of a rhombus are perpendicular bisectors of each other to develop a construction. In fact, most constructions are rooted in the properties of many of the geometric shapes you have studied so far. A rhombus can help us with another important construction.



a. **Examine** the rhombus *ABCD* at right. What is the relationship between $\measuredangle ABC$ and $\overline{BD}$?

b. Since the diagonals of a rhombus bisect the angles, use this relationship to construct an angle bisector of $\measuredangle R$ on the resource page. That is, construct a rhombus so that *R* is one of its vertices. Use only a compass, a straightedge, and a pencil.



9-67. CONSTRUCTION CHALLENGE

On the Lesson 9.2.2 Resource Page, locate $\overline{PQ}$, $\overline{ST}$, and $\measuredangle V$. In the space provided, use the construction **strategies** you have developed so far to construct a triangle with legs congruent to $\overline{PQ}$ and $\overline{ST}$, with an angle congruent to $\measuredangle V$ in between. Be sure you know how to do this two ways: with a compass and a straightedge and with tracing paper.

METHODS AND MEANINGS

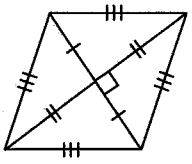
Rhombus Facts

MATH NOTES

Review what you have previously learned about a rhombus below.

A **rhombus** is a quadrilateral with four equal sides. All rhombi (the plural of rhombus) are parallelograms.

The diagonals of a rhombus are perpendicular bisectors of each other. That is, they intersect each other at their midpoints and form right angles at that point. Examine these relationships in the diagram at right.



Review & Preview

9-68. Unlike a straightedge, a ruler has measurement markings. With a ruler, it is fairly simple to construct a line segment of length 6 cm or a line segment with length 3 inches. But how can we construct a line segment of $\sqrt{2} \approx 1.414213562...$ centimeters? Consider this as you answer the questions below.

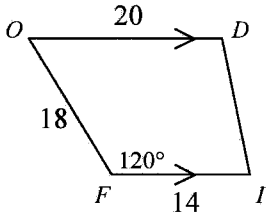
a. With a ruler, construct a line segment of 1 cm.

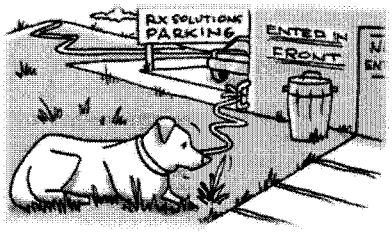
b. What about 1.4 cm? Adjust your line segment from part (a) so that its length is 1.4 cm. Did your line get longer or shorter?

c. Now change the line segment so that its length is 1.41 cm. How did it change?

d. Karen wants to continue this process until her line segment is exactly $\sqrt{2} \approx 1.414213562...$ centimeters long. What do you think will happen?

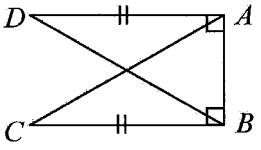
9-69. The floor plan of Marina's local drug store is shown at right. While shopping one day, Marina tied her dog, Mutt, to the building at point *F*. If Mutt's leash is 4 meters long and all measurements in the diagram are in meters, what is the area that Mutt can roam? Draw a diagram and show all work.





9-70. Find the area of the Marina's drugstore (*FIDO*) in problem 9-69. Show all work.

9-71. Given the information in the diagram at right, prove that $\measuredangle C \cong \measuredangle D$. Write your proof using any format studied so far.

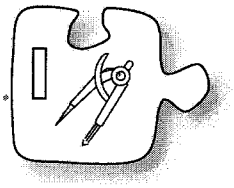


9-72. Which has greater measure: an exterior angle of an equilateral triangle or an interior angle of a regular heptagon (7-gon)? Show all work.

9-73. **Multiple Choice:** A solid with a volume of 26 in^3 was enlarged to create a similar solid with a volume of 702 in^3. What is the linear scale factor between the two solids?

a. 1 b. 2 c. 3 d. 4

9.2.3 How do I construct it?



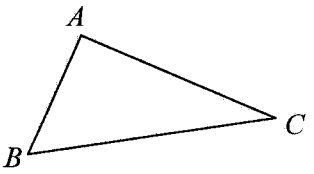
More Exploration with Constructions

So far, several geometric relationships and properties have helped you develop constructions using a compass and a straightedge. For example, constructing a rhombus helped you construct an angle bisector. Constructing intersecting circles helped you construct a perpendicular bisector. What other relationships can help you develop constructions?

Today you will review some of your triangle knowledge to **investigate** how to construct congruent triangles and special triangles, such as 30°- 60°- 90° triangles. You'll also find a way to construct a line segment with a seemingly impossible length!

In Lesson 9.2.2, you developed a method to construct a rhombus within a given angle. This not only allowed you to construct an angle bisector, but it also helped you construct parallel lines, since the opposite sides of a rhombus are parallel. Today you will explore how to use your parallel line conjectures to construct a line parallel to a given line through a point not on the line.

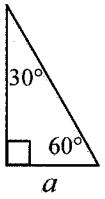
9-74. Find $\triangle ABC$ on the Lesson 9.2.3 Resource Page provided by your teacher.



a. Using a compass and a straightedge, construct $\triangle DEF$ so that $\triangle DEF \cong \triangle ABC$. Share construction ideas with your teammates.

b. Are there other ways to copy a triangle with a compass and a straightedge? Brainstorm as many ways as you can with your team. Be ready to share your ideas with the class.

9-75. Consider what you know about all 30°- 60°- 90° triangles.



a. Using the information in the triangle at right, how long is the hypotenuse? Explain how you know.

b. Negin (pronounced "Nay-GEEN") wants to use this relationship to construct a 30°- 60°- 90° triangle. On the Lesson 9.2.3 Resource Page, locate her work so far. She has constructed perpendicular lines and has constructed one side ($\overline{MN}$). Complete her construction so that her triangle has angles 30°, 60°, and 90°.

9-76. CONSTRUCTING AN IRRATIONAL LENGTH

Revisit your work from problem 9-68 from homework.

a. Explain why you cannot construct a line segment of exactly length $\sqrt{2} \approx 1.414213562...$ with a ruler.

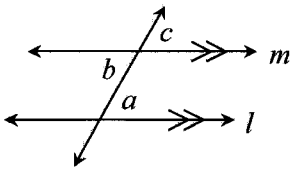
b. The number $\sqrt{2}$ is known as an **irrational number** because it cannot be represented as a fraction of two integers. One way to spot an irrational number is by looking at its decimal form. Irrational numbers have decimal numbers that never repeat and "go on forever" (meaning they never terminate, like $\frac{1}{2} = 0.5$ or $\frac{3}{8} = 0.375$ do).

Negin thinks that a right triangle may be able to help her construct a line segment of length $\sqrt{2}$ units. First find two lengths of the legs of a right triangle that will have a hypotenuse of $\sqrt{2}$ units. Then, on your Lesson 9.2.3 Resource Page, construct a right triangle with these dimensions.

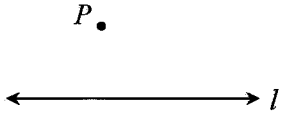
9-77. CONSTRUCTING PARALLEL LINES

So far, you have used geometric concepts such as triangle congruence and the special properties of a rhombus to create constructions. How can angle relationships formed by parallel lines help with construction? Consider this as you answer the questions below.

a. **Examine** the diagram at right. If $l \parallel m$, what do you know about $\measuredangle a$ and $\measuredangle b$? $\measuredangle a$ and $\measuredangle c$? **Justify** your answer.

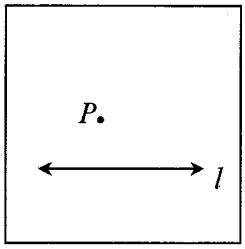


b. Negin thinks that angle relationships can help her construct a line parallel to another line through a given point not on the line. On the Lesson 9.2.3 Resource Page, find line l and point P. Help Negin construct a line parallel to l through point P by first constructing a transversal through point P that intersects line l.



c. If you have not already done so, complete Negin's construction by copying an angle formed by the transversal and line l. Explain how you used alternate interior angles or corresponding angles.

9-78. Negin started to construct a parallel line with tracing paper but got off-task. She started by tracing line l and point P on her tracing paper, as shown in the diagram at right. While experimenting, she folded the tracing paper so that line l passed through point P. After creating a crease, she unfolded the tracing paper and then folded it again at a different place so that line l still passed through point P. She continued this process until she had over 20 creases on her tracing paper!



a. With your team, predict what shape the creases created.

b. With your own tracing paper, recreate Negin's experiment. First draw a line l and a point P that is not on the line, as shown above. Then fold the tracing paper as described above so that each fold causes line l to pass through point P at a different point on line l. What shape emerged?

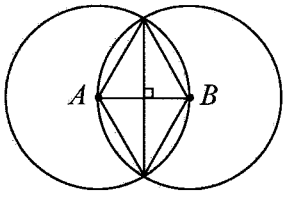
MATH NOTES — METHODS AND MEANINGS

Constructing a Perpendicular Bisector

A perpendicular bisector of a given segment can be constructed using tracing paper or using a compass and a straightedge.

With tracing paper: To construct a perpendicular bisector with tracing paper, first copy the line segment onto the tracing paper. Then fold the tracing paper so that the endpoints coincide (so that they lie on top of each other). When the paper is unfolded, the resulting crease is the perpendicular bisector of the line segment.

With a compass and a straightedge: One way to construct a perpendicular bisector with a compass and a straightedge is to construct a circle at each endpoint of the line segment with a radius equal to the length of the line segment. Then use the straightedge to draw a line through the two points where the circles intersect. This line will be the perpendicular bisector of the line segment.



Review & Preview

9-79. **Examine** the mat plan of a three-dimensional solid at right.

6	0	0	
3	0	1	RIGHT
2	6	6	

FRONT

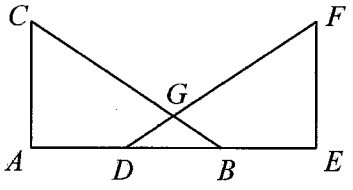
Mat Plan

a. On your paper, draw the front, right, and top views of this solid.

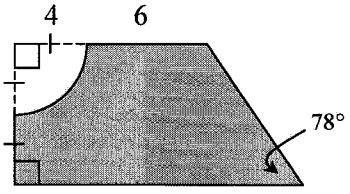
b. Find the volume of the solid.

c. If the length of each edge of the solid is divided by 2, what will the new volume be? Show how you got your answer.

9-80. **Examine** the diagram at right. Given that $\triangle ABC \cong \triangle EDF$, prove that $\triangle DBG$ is isosceles. Use any format of proof that you prefer.



9-81. The Portland Zoo is building a new children's petting zoo. One of the designs being considered is shown at right (the shaded portion). If the measurements are in meters, find:



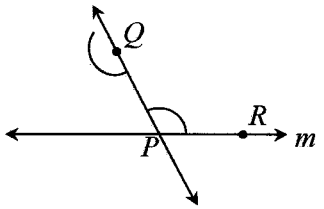
a. The area of the petting zoo.

b. The length of fence needed to enclose the petting zoo area.

9-82. Sylvia has 14 coins, all nickels and quarters. If the value of the coins is \$2.90, how many of each type of coin does she have? Explain your method.

9-83. West High School has a math building in the shape of a regular polygon. When Mrs. Woods measured an interior angle of the polygon (which was in her classroom), she got 135°. How many sides does the math building have? Show how you got your answer.

9-84. **Multiple Choice:** Jamila has started to construct a line parallel to line m through point Q at right. Which of the possible **strategies** below make the most sense to help her find the line parallel to m through point Q?



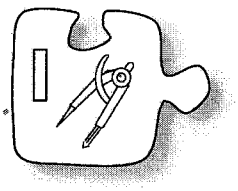
a. Measure $\measuredangle QPR$ with a protractor.

b. Use the compass to measure the arc centered at P, then place the point of the compass where the arc centered at Q meets $\overline{QP}$, and mark that measure off on the arc.

c. Construct $\overline{QR}$.

d. Measure PR with a ruler.

9.2.4 What more can I construct?



Finding a Centroid

So far in this section, you have developed a basic library of constructions that can help create many of the geometric shapes and relationships you have studied in Chapters 1 through 8. For example, you can construct a rhombus, an isosceles triangle, a right triangle, a regular hexagon, and an equilateral triangle.

As you continue your **investigation** of geometric constructions today, keep in mind the following focus questions:

What geometric principles or properties can I use?

Why does it work?

Is there another way?

9-85. TEAM CHALLENGE

Albert has a neat trick. Given any triangle, he can place it on the tip of his pencil and it balances on his first try! The whole class wonders, "How does he do it?"

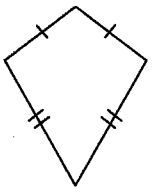
Your Task: Construct a triangle and find its point of balance. This point, called a **centroid**, is special not only because it is the center of balance, but also because it is where the **medians** of the triangle meet. Read more about medians of a triangle in the Math Notes box for this lesson and then follow the directions below.

a. After reading about medians and centroids in the Math Notes box for this lesson, draw a large triangle on a piece of unlined paper provided by your teacher. (Note: Your team will work together on one triangle.)

b. Working together, carefully construct the three medians and locate the centroid of the triangle.

c. Once your team is convinced that your centroid is accurate, glue the paper to a piece of cardstock or cardboard provided by your teacher. Carefully cut out the triangle and demonstrate that your centroid is, in fact, the center of balance of your triangle! Good luck!

9-86. CONSTRUCTING OTHER GEOMETRIC SHAPES



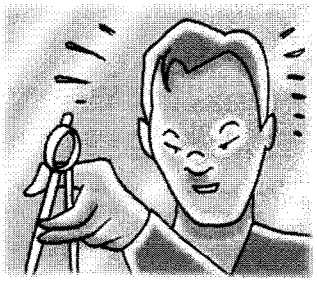
On a plain, unlined piece of paper, use a compass and a straightedge to construct a kite. Remember that a kite is defined as a quadrilateral with two pairs of adjacent, congruent sides. Be prepared to explain to the class how you constructed your kite.

9-87. In Lesson 9.2.3, you figured out how to construct a triangle congruent to a given triangle. However, what if you are not given the triangle and are instead given only its side lengths?

a. On a plain, unlined piece of paper, use a compass and a straightedge to construct a triangle that has side lengths 3, 4, and 5 units. Note that the length of 1 unit is provided above.

b. What kind of triangle did you construct? **Justify** your conclusion.

9-88. Albert wants to construct a triangle with side lengths 2, 3, and 6 units. On a plain, unlined piece of paper, use a compass and a straightedge to construct Albert's triangle. Use the unit length provided below. Explain to him what happened.

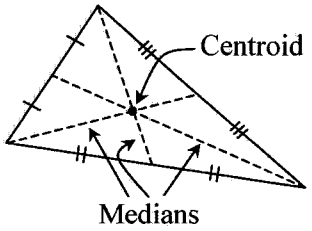


MATH NOTES

METHODS AND MEANINGS

Centroid and Medians of a Triangle

A line segment connecting a vertex of a triangle to the midpoint of the side opposite the vertex is called a **median**.



Since a triangle has three vertices, it has three medians. An example of a triangle with its three medians is provided at right.

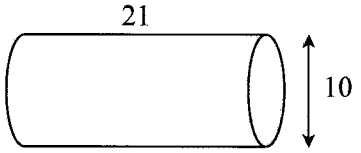
The point at which the three medians intersect is called a **centroid**. The centroid is also the center of balance of a triangle.

Since the three medians intersect at a single point, this point is called a **point of concurrency.** You will learn about other points of concurrency in a later chapter.

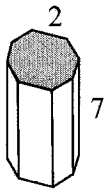
Review & Preview

9-89. Find the volumes of the solids below.

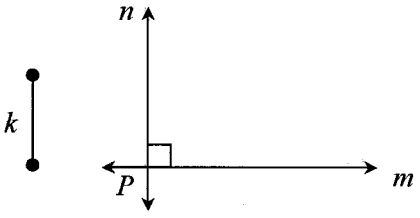
a. cylinder



b. regular octagonal prism

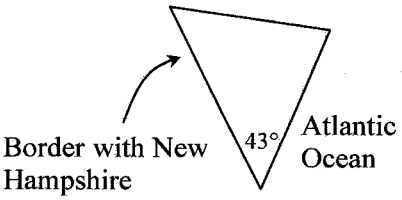


9-90. Jillian is trying to construct a square. She has started by constructing two perpendicular lines, as shown at right. If she wants each side of the square to have length k, as defined at right, describe how she should finish her construction.



9-91. Without using a calculator, find the sum of the interior angles of a 1,002-gon. Show all work.

9-92. York County, Maine, is roughly triangular in shape. To help calculate its area, Sergio has decided to use a triangle, as shown at right. According to his map, the border with New Hampshire is 165 miles long, while the coastline along the Atlantic Ocean is approximately 100 miles long. If the angle at the tip of Maine is 43°, as shown in the diagram, what is the area of York County?

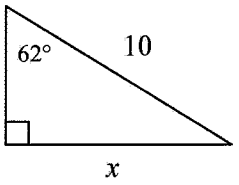


9-93. Copy the following words and their lines of reflection onto your paper. Then use your **visualization** skills to help draw the reflected images.

a. REFLECT

b. PRISM

9-94. **Multiple Choice:** Solve this problem without a calculator:



Examine the triangle at right. Find the approximate value of x. Use the values in the trigonometric table below as needed.

a. 4.69 b. 5.32

c. 8.83 d. 18.81

e. None of these

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
28°	0.883	0.469	0.532
62°	0.469	0.883	1.881

Chapter 9 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following three topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Problem Solving: What did you do to solve problems? What different **strategies** did you use?

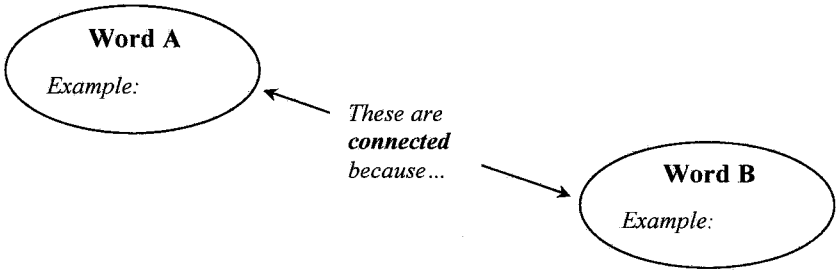
Connections: How are the topics, ideas, and words that you learned in previous courses are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

base	bisect	**centroid**
circle	**compass**	**concentric circles**
construction	**cylinder**	**inscribed**
irrational number	**lateral face**	line segment
linear scale factor	**mat plan**	**median**
net	**oblique**	perimeter
perpendicular bisector	polygon	**polyhedra**
prism	ratio	rhombus
similar	**solid**	**straightedge**
surface area	three-dimensional	**volume**

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch of an example.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a "GO" page to work on. "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

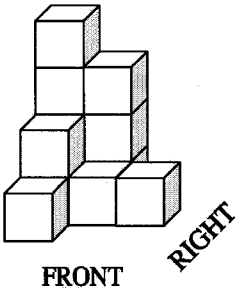
④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section will appear at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 9-95. On her paper, Kaye has a line with points A and B on it. Explain how she can use a compass to find a point C so that B is a midpoint of $\overline{AC}$. If you have access to a compass, try this yourself.

CL 9-96. Assume that the solid at right has no hidden cubes.

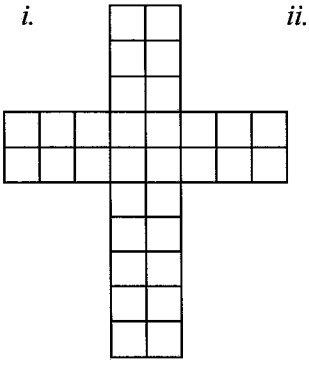


a. On graph paper, draw the front, right, and top views of this solid.

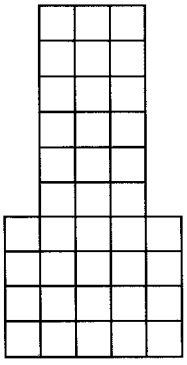
b. Find the volume and surface area of the cube.

c. Which net(s) below would have the same volume as the solid at right when it is folded to create a box?

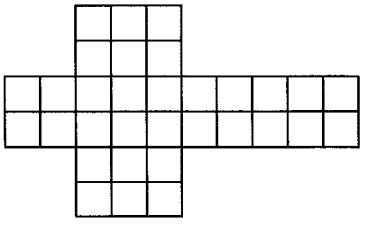
i.



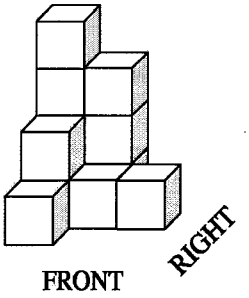
ii.



iii.



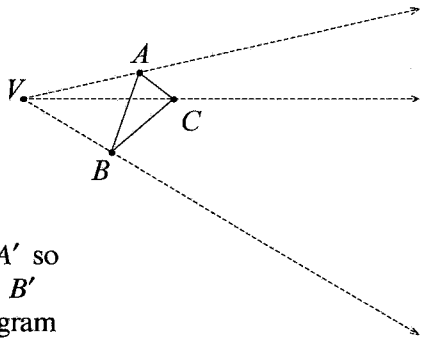
CL 9-97. The solid from problem CL 9-96 is redrawn at right.



a. If this solid were enlarged by a linear scale factor of 4, what would the volume and surface area of the new solid be?

b. Enrique enlarged the solid at right so that its volume was 1500 cubic units. What was his linear scale factor? **Justify** your answer.

CL 9-98. After constructing a $\triangle ABC$, Pricilla decided to try a little experiment. She chose a point V outside of $\triangle ABC$ and then constructed rays $\overrightarrow{VA}$, $\overrightarrow{VB}$, and $\overrightarrow{VC}$. Her result is shown at right. Copy this diagram onto your paper.

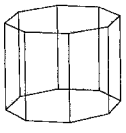


a. Pricilla then used a compass to mark point A' so that $VA = AA'$. She also constructed points B' and C' using the same method. For the diagram on your paper, locate A', B', and C'.

b. Now connect $\triangle A'B'C'$. What do you notice? What appears to be the relationship between $\triangle ABC$ and $\triangle A'B'C'$? Explain what happened.

c. If the area of $\triangle ABC$ is 19 cm^2 and its perimeter is 15 cm, find the area and perimeter of $\triangle A'B'C'$.

CL 9-99. Find the volume and surface area of the prism at right if the base is a regular octagon with side length 14 mm and the height of the prism is 28 mm.



CL 9-100. Answer the questions about the angles of polygons below, if possible. If it is not possible, explain how you know it is not possible.

a. Find the sum of the interior angles of a 28-gon.

b. If the exterior angle of a regular polygon is 42°, how many sides does the polygon have?

c. Find the measure of each interior angle of a pentagon.

d. Find the measure of each interior angle of a regular decagon.

CL 9-101. Fill in the blanks in each statement below with one of the quadrilaterals listed at right so that the statement is true. Use each quadrilateral name only once.

List:
Kite
Rectangle
Rhombus
Trapezoid

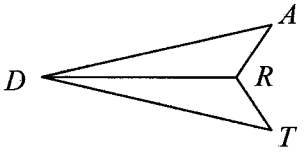
a. If a shape is a square, then it must also be a _____.

b. The diagonals of a _____ must be perpendicular to each other.

c. If the quadrilateral has only one line of symmetry, then it could be a _____.

d. If a quadrilateral has only two sides that are congruent, then the shape could be a _____.

CL 9-102. Copy quadrilateral *DART*, shown at right, onto your paper. If $\overline{DR}$ bisects $\measuredangle ADT$ and if $\measuredangle A \cong \measuredangle T$, prove that $\overline{DA} \cong \overline{DT}$.

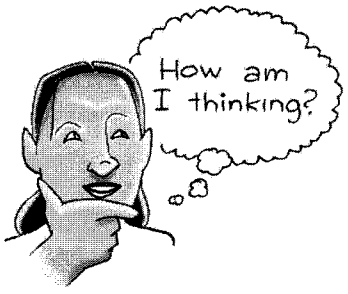


CL 9-103. After Myong's cylindrical birthday cake was sliced, she received the slice at right. If her birthday cake originally had a diameter of 14 inches and a height of 6 inches, find the volume of her slice of cake.

CL 9-104. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

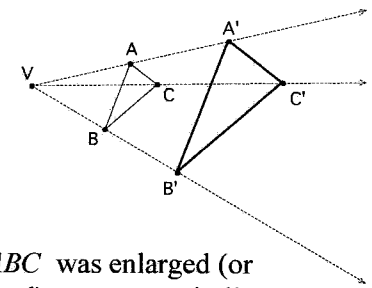
⑤ HOW AM I THINKING?

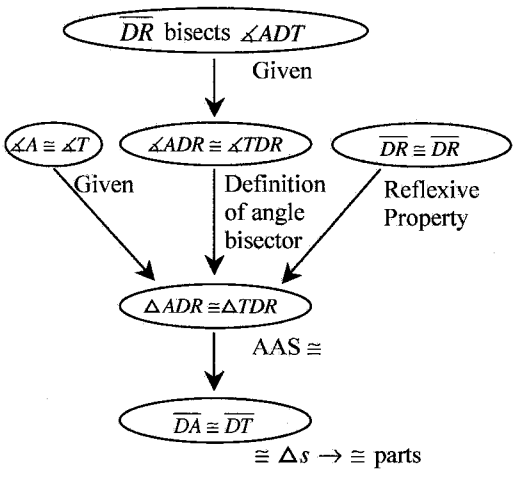
This course focuses on five different **Ways of Thinking**: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!



Choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 9-95.	She should match the length of $\overline{AB}$ with her compass. Then, with the point of the compass at B, she should mark a point on the line on the side of point B opposite point A. Then she should label that point C.	Lesson 7.3.3 Math Notes box, problem 9-55	Problems 7-108, 8-30, 9-64, 9-65, 9-67, 9-74, 9-75, 9-76, 9-86, 9-87, 9-90
CL 9-96.	a.  Front Right Top b. $V = 12$ un^3, $SA = 42$ un^2 c. All three nets will form a box with volume 12 un^3.	Lesson 9.1.3 Math Notes box, problems 9-1, 9-2, and 9-14	Problems 9-3, 9-4, 9-5, 9-7, 9-13, 9-15, 9-16, 9-20, 9-25, 9-35, 9-63, 9-79
CL 9-97.	a. $V = 12(4)^3 = 768$ un^3, $SA = 42(4)^2 = 672$ un^2 b. Linear scale factor = 5	Lesson 9.1.5 Math Notes box, problems 9-1, 9-35 and 9-36	Problems 9-37, 9-39, 9-40, 9-45, 9-46, 9-47, 9-48, 9-73, 9-79
CL 9-98.	a.  b. $\triangle ABC$ was enlarged (or dilated) to create a similar triangle with a linear scale factor of 2. c. $A = 19(2)^2 = 76$ un^2; $P = 15(2) = 30$ un	Lessons 3.1.1, 7.2.6, and 9.1.5 Math Notes boxes	Problems 7-114, 7-103, 8-63, 8-65, 8-74, 8-76, 8-114, 9-8, 9-19, 9-29
CL 9-99.	Area of base ≈ 946.37 un^2 Volume $\approx 26,498.41$ un^3 Surface Area ≈ 5028.74 un^2	Lessons 8.3.1, 9.1.2, and 9.1.3 Math Notes boxes, problem 9-15	Problems 9- 16, 9-17, 9-26, 9-27, 9-28, 9-33, 9-40, 9-59, 9-89

Problem	Solution	Need Help?	More Practice
CL 9-100.	a. 4680° b. Not possible because 42° does not divide evenly into 360°. c. Not possible because it is not stated that the pentagon is regular. d. 144°	Lessons 7.1.4, 8.1.1, and 8.1.4 Math Notes boxes, problem 8-1	Problems 8-15, 8-25, 8-29, 8-33, 8-34, 8-35, 8-40, 8-49, 8-55, 8-56, 8-87, 8-99, 8-109, 9-21, 9-50, 9-72, 9-83, 9-91
CL 9-101.	a. Rectangle b. Rhombus c. Kite d. Trapezoid	Lessons 7.2.3, 8.1.2, and 9.2.2 Math Notes boxes	Problems 7-101, 7-106, 7-116, 7-117, 7-121, 8-11, 8-56, 9-50
CL 9-102.		Lessons 3.2.4, 6.1.3, 7.1.3, and 7.2.1 Math Notes boxes, problems 7-56 and 7-79	Problems 7-61, 7-78, 7-85, 7-87, 7-96, 7-104, 7-105, 8-20, 8-28, 8-58, 8-79, 8-88, 9-12, 9-31, 9-71, 9-80
CL 9-103.	$V \approx 97.49$ cubic inches	Lessons 8.3.2, 8.3.3, 9.1.2, and 9.1.3 Math Notes boxes	Problems 8-94, 8-96, 8-103, 8-104, 8-106, 9-16, 9-17, 9-26, 9-27, 9-28, 9-33, 9-40, 9-59, 9-89