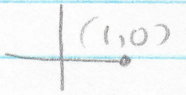
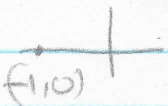
Ch 11 Review B

1. $(3 \text{ cis } \frac{\pi}{2})(1 \text{ cis } -\frac{\pi}{2}) = 3 \text{ cis } (\frac{\pi}{2} + -\frac{\pi}{2}) = 3 \text{ cis } 0°$



$3(\cos 0° + i \sin 0°) = 3(1) + i(3)(0) = 3 + 0i$

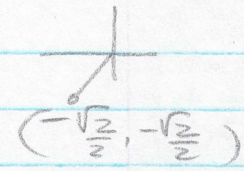
1b. $\dfrac{3 \text{ cis } \frac{\pi}{2}}{1 \text{ cis } -\frac{\pi}{2}} = 3 \text{ cis } (\frac{\pi}{2} - -\frac{\pi}{2}) = 3 \text{ cis } \pi$



$= 3(\cos \pi + i \sin \pi) = 3(-1) + i(3)(0) = -3 + 0i$

2. $(2 \text{ cis } 135°)(\frac{2}{3} \text{ cis } 90°) =$

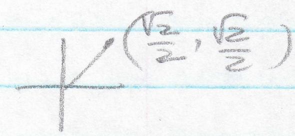
$2(\frac{2}{3}) \text{ cis } (135° + 90°) = \frac{4}{3} \text{ cis } 225°$



$= \frac{4}{3}(\cos 225° + i \sin 225°)$

$= \frac{4}{3}(-\frac{\sqrt{2}}{2}) + i(\frac{4}{3})(-\frac{\sqrt{2}}{2}) = \frac{-2\sqrt{2}}{3} - \frac{2\sqrt{2}}{3} i$

2b. $\dfrac{2 \text{ cis } 135°}{\frac{2}{3} \text{ cis } 90°} = (2 \cdot \frac{3}{2}) \text{ cis } (135° - 90°)$

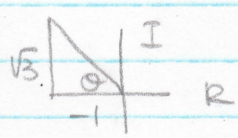


$= 3 \text{ cis } 45°$

$= 3(\cos 45° + i \sin 45°) = 3(\frac{\sqrt{2}}{2}) + i(3)(\frac{\sqrt{2}}{2})$

$= \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} i$

3. $(-1 + \sqrt{3} i)^{-5}$



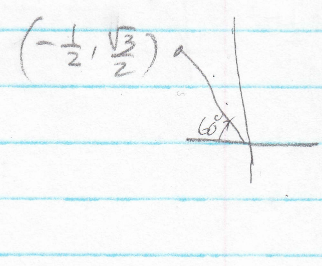
$r = \sqrt{3+1} = 2$ $\rightarrow (-1 + \sqrt{3} i) = 2 \text{ cis } 12$

$\theta = 180° - \tan^{-1}(\frac{\sqrt{3}}{1}) = 180° - 60° = 120°$

$(2 \text{ cis } 120°)^{-5} = 2^{-5} \text{ cis } -5(120°) = \frac{1}{32} \text{ cis } -600°$

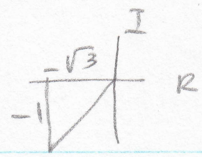
$= \frac{1}{32} \text{ cis } (-600° + 2 \cdot 360°) = \frac{1}{32} \text{ cis } 120°$

$= \frac{1}{32} \cos(120° + i \sin 120°)$



$= \frac{1}{32}(-\frac{1}{2}) + i(\frac{1}{32})(\frac{\sqrt{3}}{2})$

$= \frac{-1}{64} + \frac{\sqrt{3}}{64} i$



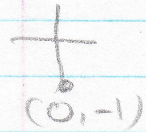
4 $(-\sqrt{3}-i)^3$

$r = \sqrt{3+1} = 2$

$\theta = 180° + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 180° + 30° = 210°$

$(2 \text{ cis } 210°)^3 = 2^3 \text{ cis } 3(210°) = 8 \text{ cis } 630°$

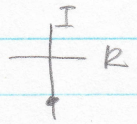
$= 8 \text{ cis } (630° - 360°) = 8 \text{ cis } 270°$



$= 8(\cos 270° + i \sin 270°)$

$= 8(0) + i(8)(-1) = 0 - 8i$

5 $(-i)^{-5}$



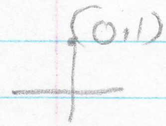
$r = 1$

$\theta = 270°$

$(1 \text{ cis } 270°)^{-5} = 1^{-5} \text{ cis } (-5 \cdot 270°)$

$= 1 \text{ cis}(-1350°) = 1 \text{ cis } (-1350° + 4 \cdot 360°$

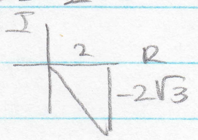
$= \text{cis } (-1350° + 1440°) = \text{cis } 90°$



$= \cos 90° + i \sin 90°$

$= 0 + i(1) = 0 + i$

6. $(2 - 2\sqrt{3}\, i)^2 =$



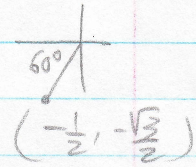
$r = \sqrt{4+12} = 4$

$\theta = 360° - \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 360° - \tan^{-1}(\sqrt{3})$

$= 360° - 60° = 300°$

$(4 \text{ cis } 300°)^2 = 4^2 \text{ cis } 2(300°) = 16 \text{ cis } 600° = 16 \text{ cis } (600° - 3$

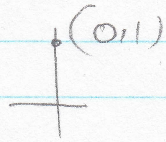
$= 16 \text{ cis } 240°$



$= 16(\cos 240° + i \sin 240°)$

$= 16\left(-\frac{1}{2}\right) + i\left(16\left(-\frac{\sqrt{3}}{2}\right)\right)$

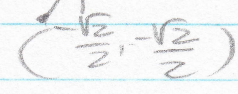
$= -8 - 8\sqrt{3}\, i$



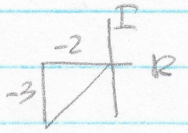
7. $2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) =$

$2(0) + i(2)(1) = 0 + 2i$

8 $3 \text{ cis } \frac{5\pi}{4} = 3\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$



$= 3\left(-\frac{\sqrt{2}}{2}\right) + i(3)\left(-\frac{\sqrt{2}}{2}\right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} i$

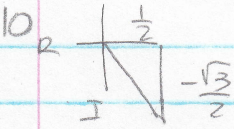
9. 

$r = \sqrt{4+9} = \sqrt{13}$

$\theta = 180° + \tan^{-1}\left(\frac{3}{2}\right)$

$= 180° + 56.3° = 236.3°$

$-2 - \sqrt{3}\, i = \sqrt{13}\ \text{cis}\ 236.3°$

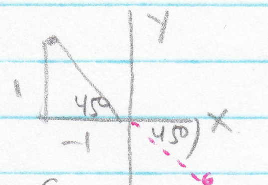
10. 

$r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = 360° - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = 360° - \tan^{-1}\left(\frac{\sqrt{3}}{2} \cdot \frac{2}{1}\right)$

$= 360° - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 360° - 60° = 300°$

$\frac{1}{2} - \frac{\sqrt{3}}{2} i = 1\ \text{cis}\ 300°$

11. 

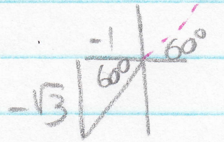
$r = \sqrt{1+1} = \sqrt{2}$

$\theta = 180° - \tan^{-1}\left(\frac{1}{1}\right) = 180° - 45° = 135°$

$(\sqrt{2}, 135°) = (\sqrt{2}, 135° - 360°) = (\sqrt{2}, -225°)$ no i no cis

$(-\sqrt{2}, 315°) = (-\sqrt{2}, 315° - 360°) = (-\sqrt{2}, -45°)$

12. $(-1, -\sqrt{3})$



$r = \sqrt{1+3} = 2$

$\theta = 180° + \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 180° + 60° = 240°$

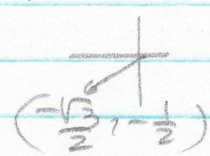
$(2, 240°) = (2, 240° - 360°) = (2, -120°)$

$(-2, 60°) = (-2, 60° - 360°) = (-2, -300°)$

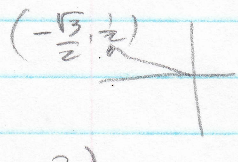
13 $\left(1, \frac{7\pi}{6}\right)$

$x = 1 \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$y = 1 \sin \frac{7\pi}{6} = -\frac{1}{2}$



$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ no cis, no i



14 $\left(-3, \frac{5\pi}{6}\right)$

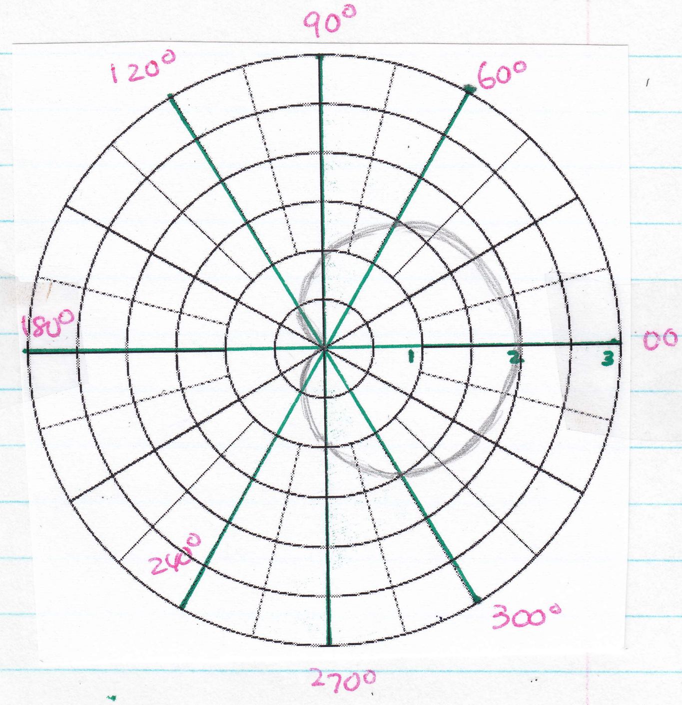
$x = -3 \cos \frac{5\pi}{6} = -3\left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$

$y = -3 \sin \frac{5\pi}{6} = -3\left(\frac{1}{2}\right) = -\frac{3}{2}$

$\left(\frac{3\sqrt{3}}{2}, \frac{-3}{2}\right)$

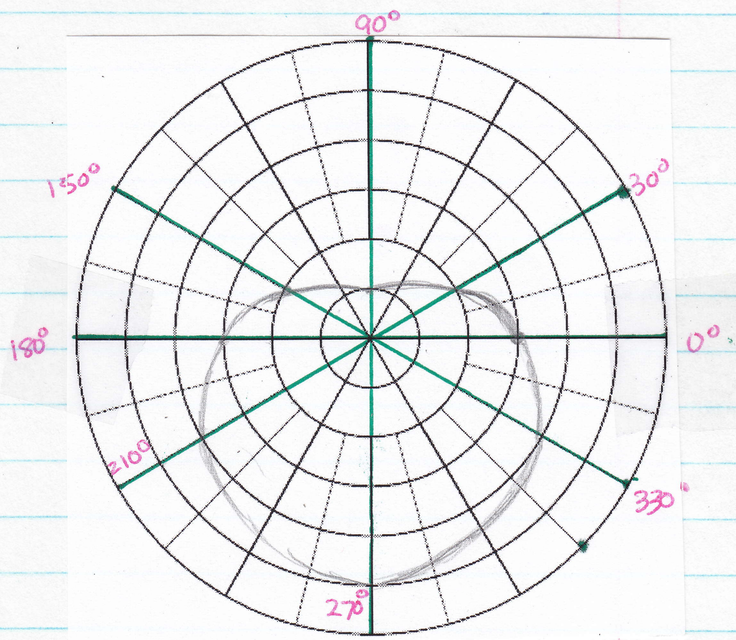
15 $r = \cos\theta - 1$

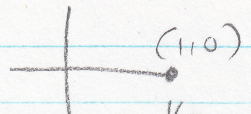
θ	$\cos\theta$	$\cos\theta - 1$
0°	1	0
60°	0.5	-0.5
90°	0	-1
120°	-0.5	-1.5
180°	-1	-2
240°	-0.5	-1.5
270°	0	-1
300°	0.5	-0.5
360°	1	0



16 $r = 3 - 2\sin\theta$

θ	$\sin\theta$	r
0	0	3
30°	0.5	2
90°	1	1
150°	0.5	2
180°	0	3
210°	-0.5	4
270°	-1	5
330°	-0.5	4
360°	0	3



17 cube roots of 8  $8 = 8 \operatorname{cis} 0° + n \cdot 360°)$

cube roots of $8 = 8^{1/3} = (8 \operatorname{cis} 0° + n \cdot 360°)^{1/3}$

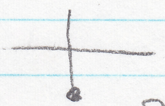
$8^{\frac{1}{3}} \operatorname{cis} 0° + n \cdot 120°$

$n=0$ $\quad 2 \operatorname{cis} 0° = 2(\cos 0° + i \sin 0°) = 2(1 + i \cdot 0) = 2 + 0i = 2$

$n=1$ $\quad 2 \operatorname{cis} 120° = 2(\cos 120° + i \sin 120°) = 2(-\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}) =$

$-1 + i\sqrt{3}$

$n=2$ $\quad 2 \operatorname{cis} 240° = 2(\cos 240° + i \sin 240°) = 2(-\frac{1}{2} + i(-\frac{\sqrt{3}}{2})) = -1 - i\sqrt{3}$



18 $-8i = 8 \operatorname{cis} 270° + n \cdot 360°$

cube roots of $-8i = (-8i)^{1/3} = (8 \operatorname{cis} 270° + n \cdot 360°)^{\frac{1}{3}}$

$= 8^{\frac{1}{3}} \operatorname{cis} 90° + n \cdot 120° = 2 \operatorname{cis} 90° + n \cdot 120°$

$n=0$ $\quad 2 \operatorname{cis} 90° = 2(\cos 90° + i \sin 90°) = 2(0 + i \cdot 1) = 0 + 2i = 2i$

$n=1$ $\quad 2 \operatorname{cis} 210° = 2(\cos 210° + i \sin 210°) = 2(-\frac{\sqrt{3}}{2} + i(-\frac{1}{2}))$

$= -\sqrt{3} - i$

$n=2$ $\quad 2 \operatorname{cis} 330° = 2(\cos 330° + i \sin 330°) =$

$2(+\frac{\sqrt{3}}{2} + i \cdot (-\frac{1}{2})) = \sqrt{3} - i$