Warm-Up

1. Find the probability of rolling a number greater than 2 and then rolling a multiple of 3 when a number cube is rolled twice.

$\frac{4}{6} \cdot \frac{2}{6} = \frac{8}{36} = \frac{2}{9}$

2. A drawer contains 8 blue socks, 8 black socks, and 4 white socks. Socks are picked at random. Explain why the events picking a blue sock and then another blue sock are dependent. Then find the probability.

20 total

$\frac{\cancel{8}^{2}}{\cancel{20}_{5}} \cdot \frac{7}{19} = \frac{14}{95}$

3. Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the indicated probability.

a. selecting two face cards when the first card is replaced

$\frac{\cancel{12}^{3}}{\cancel{52}_{13}} \cdot \frac{\cancel{12}^{3}}{\cancel{52}_{13}} = \frac{9}{169}$

b. selecting two face cards when the first card is not replaced

$\frac{\cancel{12}^{3}}{\cancel{52}_{13}} \cdot \frac{11}{\cancel{51}_{17}} = \frac{11}{221}$

7-4 Two-Way Tables
Going Deeper

Essential question: *How do you calculate a conditional probability?*

COMMON CORE Standards for Mathematical Content

CC.9-12.S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$ . . . .*

CC.9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table . . . to approximate conditional probabilities.*

CC.9-12.S.CP.5 Recognize and explain the concept of conditional probability . . . in everyday language and everyday situations.*

CC.9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*

A *two-way table* is a useful way to organize data that can be categorized by two variables. Suppose you asked 20 children and adults whether they liked broccoli. The table shows one way to arrange the data.

	Yes	No
Children	3	8
Adults	7	2

The **joint relative frequencies** are the values in each category divided by the total number of values, shown by the shaded cells in the table. Each value is divided by 20 the total number of individuals.

	Yes	No
Children	3/20	8/20
Adults	7/20	2/20

	Yes	No
Children	0.15	0.4
Adults	0.35	0.1

The **marginal relative frequencies** are found by adding the joint relative frequencies in each row and column.

	Yes	No	Total
Children	0.15	0.4	0.55
Adults	0.35	0.1	0.45
Total	0.5	0.5	1

To find a **conditional relative frequency**, divide the joint relative frequency by the marginal relative frequency. Conditional relative frequencies can be used to find conditional probabilities.

$\frac{.15}{.55}$

The table shows the results of randomly selected car insurance quotes for 125 cars made by an insurance company in one week. Make a table of the joint and marginal relative frequencies.

	Teen	Adult
0 accidents	15	53
1 accident	4	32
2+ accidents	9	12

Divide each value by the total of 125 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

	Teen	Adult	Total
0 acc.	0.12	0.424	0.544
1 acc.	0.032	0.256	0.288
2 + acc.	0.072	0.096	0.168
Total	0.224	0.776	1

The table shows the number of books sold at a library sale. Make a table of the joint and marginal relative frequencies.

	Fiction	Nonfiction
Hardcover	28	52
Paperback	94	36

Divide each value by the total of 210 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

	Fiction	Nonfiction	Total
Hardcover	0.133	0.248	0.381
Paperback	0.448	0.171	0.619
Total	0.581	0.419	1

A reporter asked 150 voters if they plan to vote in favor of a new library and a new arena. The table shows the results.

		Library	
		Yes	**No**
Arena	**Yes**	21	30
	No	57	42

		Library		
		Yes	No	Total
Arena	Yes	0.14	0.2	0.34
	No	0.38	0.28	0.66
	Total	0.52	0.48	1

B. If you are given that a voter plans to vote *no* to the new library, what is the probability the voter also plans to say no to the new arena?

$$\frac{.28}{.48}$$

The classes at a dance academy include ballet and tap dancing. Enrollment in these classes is shown in the table.

		Ballet	
		Yes	No
Tap	Yes	38	52
	No	86	24

2a. Copy and complete the table of the joint relative frequencies and marginal relative frequencies.

		Ballet		
		Yes	No	Total
Tap	Yes			
	No			
	Total			1

		Ballet		
		Yes	No	Total
Tap	Yes	0.19	0.26	0.45
	No	0.43	0.12	0.55
	Total	0.62	0.38	1

2b. If you are given that a student is taking ballet, what is the probability that the student is not taking tap?

$$\frac{.43}{.62}$$

Use conditional probabilities to determine for which method a customer is most likely to pay with a gift card.

A company sells items in a store, online, and through a catalog. A manager recorded whether or not the 50 sales made one day were paid for with a gift card.

	Gift Card	Another Method
Store	𝍸 𝍷	𝍸 𝍷𝍷𝍷𝍷
Online	𝍸 𝍷𝍷𝍷𝍷	𝍸 𝍸 𝍷𝍷𝍷
Catalog	𝍸	𝍸 𝍷𝍷𝍷

	Gift Card	Another Method	TOTAL
Store	0.12	0.18	0.30
Online	0.18	0.26	0.44
Catalog	0.10	0.16	0.26
TOTAL	0.40	0.60	1

P(gift card if in store) = 0.4
P(gift card if online) ≈ 0.41
P(gift card if by catalog) ≈ 0.38
so most likely if buying online.

A customer is most likely to pay with a gift card if buying online.

Francine is evaluating three driving schools. She asked 50 people who attended the schools whether they passed their driving tests on the first try. Use conditional probabilities to determine which is the best school.

	Pass	Fail
Al's Driving	𝍸 𝍸 𝍷𝍷𝍷𝍷	𝍸 𝍷𝍷𝍷
Drive Time	𝍸 𝍸 𝍷	𝍸 𝍷𝍷
Crash Course	𝍸	𝍸

	Pass	Fail	TOTAL
Al's Driving	0.28	0.16	0.44
Drive Time	0.22	0.14	0.36
Crash Course	0.10	0.10	0.20
TOTAL	0.60	0.40	1

Al's Driving has the best pass rate, about 64%, versus 61% for Drive Time and 50% for Crash Course.

Conditional Probability

The conditional probability of *B* given *A* (the probability that event *B* occurs given that event *A* occurs) is given by the following formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

given

In a standard deck of playing cards, find the probability that a red card is a queen.

$$P(Q \mid R) = \frac{P(Q \text{ and } R)}{P(R)} = \frac{2}{26} = \frac{1}{13} \quad R = 26 \quad Q \text{ and } R = 2$$

In a standard deck of playing cards, find the probability that a queen is a red card.

$$P(R \mid Q) = \frac{P(R \text{ and } Q)}{P(Q)} = \frac{2}{4} = \frac{1}{2}$$

In order to study the connection between the amount of sleep a student gets and his or her school performance, data was collected about 120 students. The two-way table shows the number of students who passed and failed an exam and the number of students who got more or less than 6 hours of sleep the night before.

	Passed Exam	Failed Exam	TOTAL
Less than 6 hours of sleep	12	$\boxed{10}$	22
More than 6 hours of sleep	90	8	98
TOTAL	102	18	120

56%

a. To the nearest percent, what is the probability that a student who failed the exam got less than 6 hours of sleep? $P(\text{less 6} \mid F) = \dfrac{P(\text{less 6 \& fail})}{P(F)} = \dfrac{10}{18}$

45%

b. To the nearest percent, what is the probability that a student who got less than 6 hours of sleep failed the exam? $P(F \mid \text{less 6}) = \dfrac{P(F \,\&\, \text{less 6})}{P(\text{less 6})} = \dfrac{10}{22}$

c. To the nearest percent, what is the probability that a student got less than 6 hours of sleep and failed the exam? $\dfrac{10}{120}$

In Exercises 4–9, consider a standard deck of playing cards and the following events: *A*: the card is an ace; *B*: the card is black; *C*: the card is a club. Find each probability as a fraction.

4 26 13

4. $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{2}{26}$ $\dfrac{1}{13}$

$P(B \mid A) = \dfrac{P(B \text{ and } A)}{P(A)} = \dfrac{2}{4}$ $\dfrac{1}{2}$

$P(A \mid C) = \dfrac{P(A \text{ and } C)}{P(C)} = \dfrac{1}{13}$

7. $P(C \mid A) = \dfrac{1}{4}$

$P(B \mid C) = \dfrac{13}{13}$

$P(C \mid B)$ $\dfrac{1}{2}$