Lesson 7 - 4

Two-Way Tables

Going Deeper

Essential question: *How do you calculate a conditional probability?*

The probability that event *B* occurs given that event *A* has already occurred is called the **conditional probability** of *B* given *A* and is written $P(B \mid A)$.

CC.9-12.S.CP.6

1 EXAMPLE Finding Conditional Probabilities

One hundred people who frequently get migraine headaches were chosen to participate in a study of a new anti-headache medicine. Some of the particpants were given the medicine; others were not. After one week, the participants were asked if they got a headache during the week. The two-way table summarizes the results.

A **To the nearest percent, what is the probability that a participant who took the medicine did not get a headache?**

	Took Medicine	No Medicine	TOTAL
Headache	12	15	27
No Headache	48	25	73
TOTAL	60	40	100

Let event *A* be the event that a participant took the medicine. Let event *B* be the event that a participant did not get a headache.

To find the probability that a participant who took the medicine did not get a headache, you must find $P(B \mid A)$. You are only concerned with participants who took the medicine, so look at the data in the "Took Medicine" column.

There were 60 participants who took the medicine.

Of these participants, 48 participants did not get a headache.

So, $P(B \mid A) = \frac{48}{60} = 80\%$

	Took Medicine A	No Medicine	TOTAL
Headache	12	15	27
No Headache B	$\boxed{48}$	25	73
TOTAL	60	40	100

B **To the nearest percent, what is the probability that a participant who did not get a headache took the medicine?**

To find the probability that a participant who did not get a headache took the medicine, you must find $P(A \mid B)$. You are only concerned with participants who did not get a headache, so look at the data in the "No headache" row.

There were 73 participants who did not get a headache.

Of these participants, 48 participants took the medicine.

So, $P(A \mid B) = \frac{\overset{P(A \cap B)}{48}}{\underset{P(B)}{73}} \approx 66\%$

	Took Medicine A	No Medicine	TOTAL
Headache	12	15	27
B **No Headache**	48	25	73
TOTAL	60	40	100

REFLECT

1a. In general, do you think $P(B \mid A) = P(A \mid B)$? Why or why not?

No, order matters & determines outcomes in sample set

1b. How can you use set notation to represent the event that a participant took the medicine and did not get a headache? Is the probability that a participant took the medicine and did not get a headache equal to either of the conditional probabilities you calculated in the example?

$P(A \cap B) = 48/100 = .48$. It is not = to $P(B|A)$ or $P(A|B)$

intersection

CC.9-12.S.CP.3

2 EXPLORE Developing a Formula for Conditional Probability

You can generalize your work from the previous example to develop a formula for finding conditional probabilities.

A Recall how you calculated $P(B \mid A)$, the probability that a participant who took the medicine did not get a headache.

		Event A: Took Medicine	No Medicine	TOTAL
	Headache	12	15	27
Event B	No Headache	48 = $n(A \cap B)$	25	73 = $n(B)$
	TOTAL	60 = $n(A)$	40	100

You found that $P(B \mid A) = \frac{48}{60}$.

Use the table shown here to help you write this quotient in terms of events A and B.

$P(B \mid A) = \frac{n(A \cap B)}{n(A)}$ $\frac{\text{\# people who take med + get no headache}}{\text{took meds}}$

B Now divide the numerator and denominator of the quotient by $n(S)$, the number of outcomes in the sample space. This converts the counts to probabilities.

48 /100 # of items in sample space

$P(B \mid A) = \frac{n(A \cap B) / n(S)}{n(A) / n(S)} = \frac{P(A \cap B)}{P(A)}$

		Event A: Took Medicine	No Medicine	TOTAL
	Headache	12	15	27
Event B	No Headache	48 = $n(A \cap B)$	25	73 = $n(B)$
	TOTAL	60 = $n(A)$	40	100

REFLECT

2a. Write a formula for $P(A \mid B)$ in terms of $n(A \cap B)$ and $n(B)$.

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2b. Write a formula for $P(A \mid B)$ in terms of $P(A \cap B)$ and $P(B)$.

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You may have discovered the following formula for conditional probability.

Conditional Probability

The conditional probability of B given A (the probability that event B occurs given that event A occurs) is given by the following formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \quad \frac{P(\text{both})}{P(A)} \quad P(A \overset{\cap}{\text{and}} B) = P(A) \cdot P(B|A)$$

3 EXAMPLE CC.9–12.S.CP.3 **Using the Conditional Probability Formula**

In a standard deck of playing cards, find the probability that a red card is a queen. P(Q, if R)

A Let event Q be the event that a card is a queen. Let event R be the event that a card is red. You are asked to find $P(Q \mid R)$. First find $P(R \cap Q)$ and $P(R)$.

$R \cap Q$ represents cards that are both red and a queen; that is, red queens.

There are __2__ red queens in the deck of 52 cards, so $P(R \cap Q) =$ __2/52__.

There are __26__ red cards in the deck, so $P(R) =$ __$\frac{26}{52}$__.

B Use the formula for conditional probability.

$$P(Q \mid R) = \frac{\overset{\text{red Q}}{P(Q \cap R)}}{\underset{\text{given}}{P(R)}} = \frac{2/52 \cdot 52}{26/52 \cdot 52} \qquad \text{Substitute probabilities from above.}$$

$$= \frac{2}{26} \qquad \text{Multiply numerator and denominator by 52.}$$

$$= \frac{1}{13} \qquad \text{Simplify.}$$

$$\frac{\text{Red Q}}{\text{red cards}} = \frac{2}{26} \quad \frac{1}{13}$$

So, the probability that a red card is a queen is __$\frac{1}{13}$__.