33. $-1 < 4x - 1 \leq 11$

(a) $x = 0$ **(b)** $x = 2$ **(c)** $x = 3$

34. $-3 \leq 1 - 2x \leq 3$

(a) $x = -1$ **(b)** $x = 0$ **(c)** $x = 2$

In Exercises 35–42, solve the inequality, and draw a number line graph of the solution set.

35. $x - 4 < 2$

36. $x + 3 > 5$

37. $2x - 1 \leq 4x + 3$

38. $3x - 1 \geq 6x + 8$

39. $2 \leq x + 6 < 9$

40. $-1 \leq 3x - 2 < 7$

41. $2(5 - 3x) + 3(2x - 1) \leq 2x + 1$

42. $4(1 - x) + 5(1 + x) > 3x - 1$

In Exercises 43–54, solve the inequality.

43. $\frac{5x + 7}{4} \leq -3$

44. $\frac{3x - 2}{5} > -1$

45. $4 \geq \frac{2y - 5}{3} \geq -2$

46. $1 > \frac{3y - 1}{4} > -1$

47. $0 \leq 2z + 5 < 8$

48. $-6 < 5t - 1 < 0$

49. $\frac{x - 5}{4} + \frac{3 - 2x}{3} < -2$

50. $\frac{3 - x}{2} + \frac{5x - 2}{3} < -1$

51. $\frac{2y - 3}{2} + \frac{3y - 1}{5} < y - 1$

52. $\frac{3 - 4y}{6} - \frac{2y - 3}{8} \geq 2 - y$

53. $\frac{1}{2}(x - 4) - 2x \leq 5(3 - x)$

54. $\frac{1}{2}(x + 3) + 2(x - 4) < \frac{1}{3}(x - 3)$

In Exercises 55–58, find the solutions of the equation or inequality displayed in Figure P.20.

55. $x^2 - 2x < 0$

56. $x^2 - 2x = 0$

57. $x^2 - 2x > 0$

58. $x^2 - 2x \leq 0$

X	Y1	
0	0	
1	-1	
2	0	
3	3	
4	8	
5	15	
6	24	
Y1 = X²–2X		

FIGURE P.20 The second column gives values of $y_1 = x^2 - 2x$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.

59. Writing to Learn Explain how the second equation was obtained from the first.

$$x - 3 = 2x + 3, \quad 2x - 6 = 4x + 6$$

60. Writing to Learn Explain how the second equation was obtained from the first.

$$2x - 1 = 2x - 4, \quad x - \frac{1}{2} = x - 2$$

61. Group Activity Determine whether the two equations are equivalent.

(a) $3x = 6x + 9, \quad x = 2x + 9$

(b) $6x + 2 = 4x + 10, \quad 3x + 1 = 2x + 5$

62. Group Activity Determine whether the two equations are equivalent.

(a) $3x + 2 = 5x - 7, \quad -2x + 2 = -7$

(b) $2x + 5 = x - 7, \quad 2x = x - 7$

Standardized Test Questions

63. True or False $-6 > -2$. Justify your answer.

64. True or False $2 \leq \frac{6}{3}$. Justify your answer.

In Exercises 65–68, you may use a graphing calculator to solve these problems.

65. Multiple Choice Which of the following equations is equivalent to the equation $3x + 5 = 2x + 1$?

(a) $3x = 2x$ **(b)** $3x = 2x + 4$

(c) $\frac{3}{2}x + \frac{5}{2} = x + 1$ **(d)** $3x + 6 = 2x$

(e) $3x = 2x - 4$

66. Multiple Choice Which of the following inequalities is equivalent to the inequality $-3x < 6$?

(a) $3x < -6$ **(b)** $x < 10$

(c) $x > -2$ **(d)** $x > 2$

(e) $x > 3$

67. Multiple Choice Which of the following is the solution to the equation $x(x + 1) = 0$?

(a) $x = 0$ or $x = -1$ **(b)** $x = 0$ or $x = 1$

(c) only $x = -1$ **(d)** only $x = 0$

(e) only $x = 1$

68. Multiple Choice Which of the following represents an equation equivalent to the equation

$$\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{1}{3}$$

that is cleared of fractions?

(a) $2x + 1 = x - 1$ **(b)** $8x + 6 = 3x - 4$

(c) $4x + 3 = \frac{3}{2}x - 2$ **(d)** $4x + 3 = 3x - 4$

(e) $4x + 6 = 3x - 4$

Explorations

69. Testing Inequalities on a Calculator

(a) The calculator we use indicates that the statement $2 < 3$ is true by returning the value 1 (for true) when $2 < 3$ is entered. Try it with your calculator.

(b) The calculator we use indicates that the statement $2 < 1$ is false by returning the value 0 (for false) when $2 < 1$ is entered. Try it with your calculator.

(c) Use your calculator to test which of these two numbers is larger: 799/800, 800/801.

(d) Use your calculator to test which of these two numbers is larger: −102/101, −103/102.

(e) If your calculator returns 0 when you enter $2x + 1 < 4$, what can you conclude about the value stored in x?

Extending the Ideas

70. Perimeter of a rectangle The formula for the perimeter P of a rectangle is

$$P = 2(L + W).$$

Solve this equation for W.

71. Area of a Trapezoid The formula for the area A of a trapezoid is

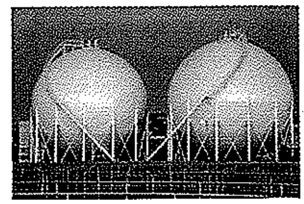
$$A = \frac{1}{2}h(b_1 + b_2).$$

Solve this equation for b_1.

72. Volume of a Sphere The formula for the volume V of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

Solve this equation for r.



73. Celsius and Fahrenheit The formula for Celsius temperature in terms of Fahrenheit temperature is

$$C = \frac{5}{9}(F - 32)$$

Solve the equation for F.

P.4 LINES IN THE PLANE

What you'll learn about

- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables

. . . and why

Linear equations are used extensively in applications involving business and behavioral science.

Slope of a Line

The slope of a nonvertical line is the ratio of the amount of vertical change to the amount of horizontal change between two points. For the points (x_1, y_1) and (x_2, y_2), the vertical change is $\Delta y = y_2 - y_1$ and the horizontal change is $\Delta x = x_2 - x_1$. Δy is read "delta" y. See Figure P.21.

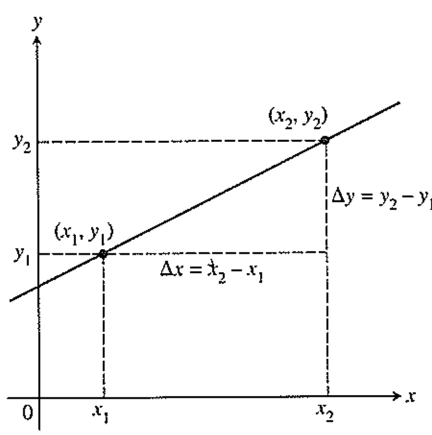


FIGURE P.21 The slope of a nonvertical line can be found from the coordinates of any two points of the line.

Definition Slope of a Line

The **slope** of the nonvertical line through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

If the line is vertical, then $x_1 = x_2$ and the slope is undefined.

EXAMPLE 1 Finding the slope of a line

Find the slope of the line through the two points. Sketch a graph of the line.

(a) $(-1, 2)$ and $(4, -2)$ **(b)** $(1, 1)$, and $(3, 4)$

SOLUTION

(a) The two points are $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (4, -2)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 2}{4 - (-1)} = -\frac{4}{5}.$$

(b) The two points are $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (3, 4)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 1} = \frac{3}{2}.$$

The graphs of these two lines are shown in Figure P.22.

Now try Exercise 3.

SLOPE FORMULA

The slope does not depend on the order of the points. We could use $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 2)$ in Example 1a. Check it out.

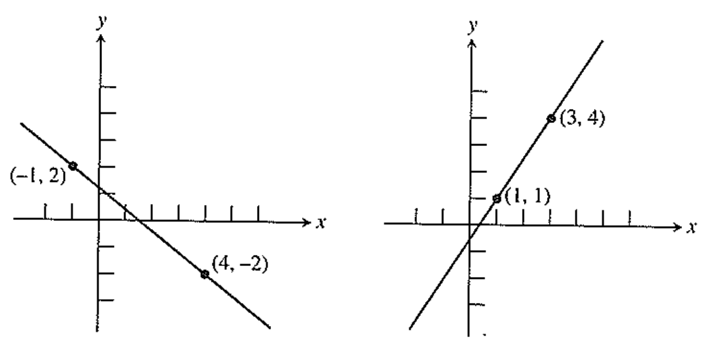


FIGURE P.22 The graphs of the two lines in Example 1.

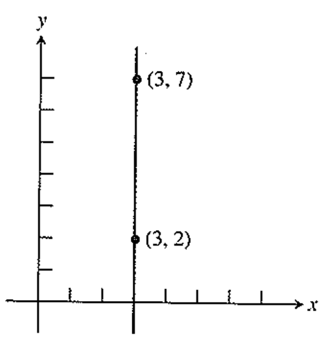


FIGURE P.23 Applying the slope formula to this vertical line gives $m = 5/0$, which is not defined. Thus, the slope of a vertical line is undefined.

Figure P.23 shows a vertical line through the points (3, 2) and (3, 7). If we try to calculate its slope using the slope formula $(y_2 - y_1)/(x_2 - x_1)$, we get zero in the denominator. So, it makes sense to say that a vertical line does not have a slope, or that its slope is undefined.

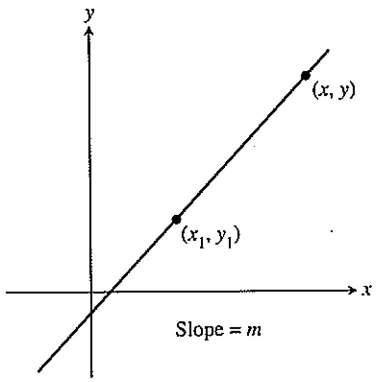


FIGURE P.24 The line through (x_1, y_1) with slope m.

Point-Slope Form Equation of a Line

If we know the coordinates of one point on a line and the slope of the line, then we can find an equation for that line. For example, the line in Figure P.24 passes through the point (x_1, y_1) and has slope m. If (x, y) is any other point on this line, the definition of the slope yields the equation

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1).$$

An equation written this way is in the *point-slope form*.

Definition Point-Slope Form of an Equation of a Line

The **point-slope form** of an equation of a line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 Using the point-slope form

Use the point-slope form to find an equation of the line that passes through the point $(-3, -4)$ and has slope 2.

SOLUTION Substitute $x_1 = -3$, $y_1 = -4$, and $m = 2$ into the point-slope form, and simplify the resulting equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-4) = 2(x - (-3))$	$x_1 = -3, y_1 = -4, m = 2$
$y + 4 = 2x - 2(-3)$	Distributive property
$y + 4 = 2x + 6$	
$y = 2x + 2$	A common simplified form

Now try Exercise 11.

y-INTERCEPT

The b in $y = mx + b$ is often referred to as "the y-intercept" instead of "the y-coordinate of the y-intercept."

Slope-Intercept Form Equation of a Line

The **y-intercept** of a nonvertical line is the point where the line intersects the y-axis. If we know the y-intercept and the slope of the line, we can apply the point-slope form to find an equation of the line.

Figure P.25 shows a line with slope m and y-intercept $(0, b)$. A point-slope form equation for this line is $y - b = m(x - 0)$. By rewriting this equation we obtain the form known as the *slope-intercept form*.

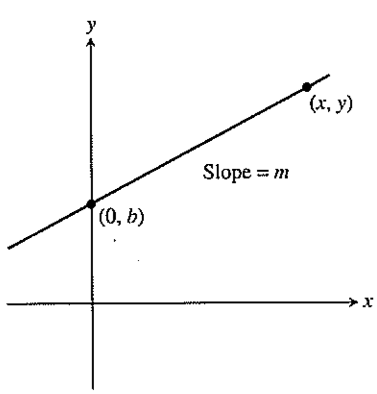


FIGURE P.25 The line with slope m and y-intercept $(0, b)$.

Definition Slope-Intercept Form of an Equation of a Line

The slope-intercept form of an equation of a line with slope m and y-intercept $(0, b)$ is

$$y = mx + b.$$

EXAMPLE 3 Using the slope-intercept form

Write an equation of the line with slope 3 that passes through the point $(-1, 6)$ using the slope-intercept form.

SOLUTION

$y = mx + b$ — Slope-intercept form

$y = 3x + b$ — $m = 3$

$6 = 3(-1) + b$ — $y = 6$ when $x = -1$

$b = 9$

The slope-intercept form of the equation is $y = 3x + 9$.

Now try Exercise 21.

We cannot use the phrase "*the* equation of a line" because each line has many different equations. Every line has an equation that can be written in the form $Ax + By + C = 0$ where A and B are not both zero. This form is the **general form** for an equation of a line.

If $B \neq 0$, the general form can be changed to the slope-intercept form as follows:

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$y = \underbrace{-\frac{A}{B}}_{\text{slope}}x + \underbrace{\left(-\frac{C}{B}\right)}_{y\text{-intercept}}$$

Forms of Equations of Lines	
General form:	$Ax + By + C = 0$, A and B not both zero
Slope-intercept form:	$y = mx + b$
Point-slope form:	$y - y_1 = m(x - x_1)$
Vertical line:	$x = a$
Horizontal line:	$y = b$

Graphing Linear Equations in Two Variables

A **linear equation in x and y** is one that can be written in the form

$$Ax + By = C,$$

where A and B are not both zero. Rewriting this equation in the form $Ax + By - C = 0$ we see that it is the general form of an equation of a line. If $B = 0$, the line is vertical, and if $A = 0$, the line is horizontal.

The **graph** of an equation in x and y consists of all pairs (x, y) that are solutions of the equation. For example, (1, 2) is a **solution** of the equation $2x + 3y = 8$ because substituting $x = 1$ and $y = 2$ into the equation leads to the true statement $8 = 8$. The pairs $(-2, 4)$ and $(2, 4/3)$ are also solutions.

Because the graph of a linear equation in x and y is a straight line, we need only to find two solutions and then connect them with a straight line to draw its graph. If a line is neither horizontal nor vertical, then two easy points to find are its x-intercept and the y-intercept. The **x-intercept** is the point $(x', 0)$ where the graph intersects the x-axis. Set $y = 0$ and solve for x to find the x-intercept. The coordinates of the y-intercept are $(0, y')$. Set $x = 0$ and solve for y to find the y-intercept.

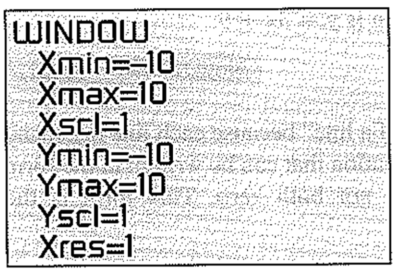


FIGURE P.26 The window dimensions for the *standard window*. The notation "[−10, 10] by [−10, 10]" is used to represent window dimensions like these.

Graphing with a Graphing Utility

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form $y =$ (an expression in x).
2. Enter the equation into the grapher.
3. Select an appropriate **viewing window** (see Figure P.26).
4. Press the "graph" key.

A graphing utility, often referred to as a *grapher*, computes y-values for a select set of x-values between Xmin and Xmax and plots the corresponding (x, y) points.

EXAMPLE 4 Use a graphing utility

Draw the graph of $2x + 3y = 6$.

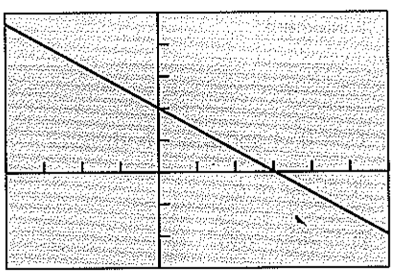
SOLUTION First we solve for y.

$$2x + 3y = 6$$

$$3y = -2x + 6 \quad \text{Solve for } y.$$

$$y = -\frac{2}{3}x + 2 \quad \text{Divide by 3.}$$

Figure P.27 shows the graph of $y = -(2/3)x + 2$, or equivalently, the graph of the linear equation $2x + 3y = 6$ in the $[-4, 6]$ by $[-3, 5]$ viewing window. Now try Exercise 27.



[−4, 6] by [−3, 5]

FIGURE P.27 The graph of $2x + 3y = 6$. The points (0, 2) (y-intercept) and (3, 0) (x-intercept) appear to lie on the graph and, as pairs, are solutions of the equation, providing visual support that the graph is correct. (Example 4)

VIEWING WINDOW

The viewing window [−4, 6] by [−3, 5] in Figure P.27 means $-4 \le x \le 6$ and $-3 \le y \le 5$.

Parallel and Perpendicular Lines

SQUARE VIEWING WINDOW

A **square viewing window** on a grapher is one in which angles appear to be true. For example, the line $y = x$ will appear to make a 45° angle with the positive x-axis. Furthermore, a distance of 1 on the x- and y-axes will appear to be the same. That is, if Xscl = Yscl, the distance between consecutive tick marks on the x- and y-axes will appear to be the same.

EXPLORATION 1 Investigating Graphs of Linear Equations

1. What do the graphs of $y = mx + b$ and $y = mx + c$, $b \neq c$, have in common? How are they different?
2. Graph $y = 2x$ and $y = -(1/2)x$ in a *square viewing window* (see margin note). On the calculator we use, the "decimal window" $[-4.7, 4.7]$ by $[-3.1, 3.1]$ is square. Estimate the angle between the two lines.
3. Repeat part 2 for $y = mx$ and $y = -(1/m)x$ with $m = 1, 3, 4$, and 5.

Parallel lines and perpendicular lines were involved in Exploration 1. Using a grapher to decide when lines are parallel or perpendicular is risky. Here is an algebraic test to determine when two lines are parallel or perpendicular.

Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if their slopes are equal.
2. Two nonvertical lines are perpendicular if and only if their slopes m_1 and m_2 are opposite reciprocals. That is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 5 Finding an equation of a parallel line

Find an equation of the line through $P(1, -2)$ that is parallel to the line L with equation $3x - 2y = 1$.

SOLUTION We find the slope of L by writing its equation in slope-intercept form.

$$3x - 2y = 1 \qquad \text{Equation for } L$$
$$-2y = -3x + 1 \qquad \text{Subtract } 3x.$$
$$y = \frac{3}{2}x - \frac{1}{2} \qquad \text{Divide by } -2.$$

The slope of L is 3/2.

The line whose equation we seek has slope 3/2 and contains the point $(x_1, y_1) = (1, -2)$. Thus, the point-slope form equation for the line we seek is

$$y + 2 = \frac{3}{2}(x - 1)$$
$$y + 2 = \frac{3}{2}x - \frac{3}{2} \qquad \text{Distributive property}$$
$$y = \frac{3}{2}x - \frac{7}{2}$$

Now try Exercise 41(a).

EXAMPLE 6 Finding an equation of a perpendicular line

Find an equation of the line through $P(2, -3)$ that is perpendicular to the line L with equation $4x + y = 3$. Support the result with a grapher.

SOLUTION We find the slope of L by writing its equation in slope-intercept form.

$$4x + y = 3 \quad \text{Equation for } L$$

$$y = -4x + 3 \quad \text{Subtract } 4x.$$

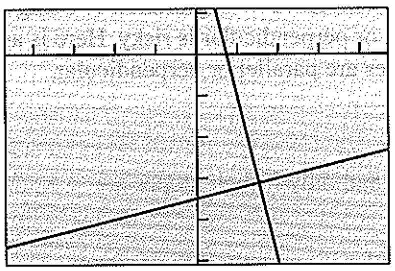
The slope of L is -4.

The line whose equation we seek has slope $-1/(-4) = 1/4$ and passes through the point $(x_1, y_1) = (2, -3)$. Thus, the point-slope form equation for the line we seek is

$$y - (-3) = \frac{1}{4}(x - 2)$$

$$y + 3 = \frac{1}{4}x - \frac{2}{4} \quad \text{Distributive property}$$

$$y = \frac{1}{4}x - \frac{7}{2}$$



[−4.7, 4.7] by [−5.1, 1.1]

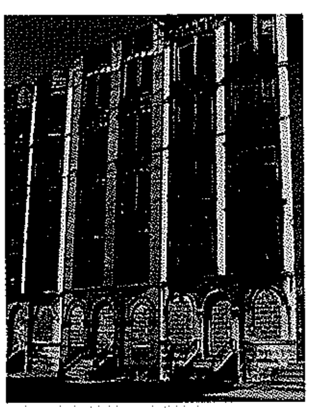
FIGURE P.28 The graphs of $y = -4x + 3$ and $y = (1/4)x - 7/2$ in this square viewing window appear to intersect at a right angle. (Example 6)

Figure P.28 shows the graphs of the two equations in a square viewing window and suggests that the graphs are perpendicular.

Now try Exercise 43(b).

Applying Linear Equations in Two Variables

Linear equations and their graphs occur frequently in applications. Algebraic solutions to these application problems often require finding an equation of a line and solving a linear equation in one variable. Grapher techniques complement algebraic ones.



EXAMPLE 7 Finding the depreciation of real estate

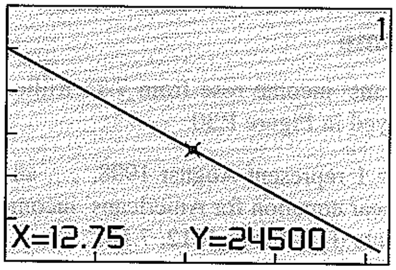
Camelot Apartments purchased a \$50,000 building and depreciates it \$2000 per year over a 25-year period.

(a) Write a linear equation giving the value y of the building in terms of the years x after the purchase.

(b) In how many years will the value of the building be \$24,500?

SOLUTION

(a) We need to determine the value of m and b so that $y = mx + b$, where $0 \le x \le 25$. We know that $y = 50{,}000$ when $x = 0$, so the line has y-intercept $(0, 50{,}000)$ and $b = 50{,}000$. One year after purchase ($x = 1$), the value of the building is $50{,}000 - 2{,}000 = 48{,}000$. So when $x = 1$, $y = 48{,}000$.



[0, 23.5] by [0, 60000]

(a)

X	Y_1	
12	26000	
12.25	25500	
12.5	25000	
12.75	24500	
13	24000	
13.25	23500	
13.5	23000	

Y_1 = −2000X+50000

(b)

FIGURE P.29 A (a) graph and (b) table of values for $y = -2000x + 50{,}000$. (Example 7)

Using algebra, we find

$$y = mx + b$$
$$48{,}000 = m \cdot 1 + 50{,}000 \qquad y = 48{,}000 \text{ when } x = 1$$
$$-2000 = m$$

The value y of the building x years after its purchase is

$$y = -2000x + 50{,}000.$$

(b) We need to find the value of x when $y = 24{,}500$.

$$y = -2000x + 50{,}000$$

Again, using algebra we find

$$24{,}500 = -2000x + 50{,}000 \qquad \text{Set } y = 24{,}500.$$
$$-25{,}500 = -2000x \qquad \text{Subtract } 50{,}000.$$
$$12.75 = x$$

The depreciated value of the building will be \$24,500 exactly 12.75 years, or 12 years 9 months, after purchase by Camelot Apartments. We can support our algebraic work both graphically and numerically. The trace coordinates in Figure P.29a show graphically that (12.75, 24,500) is a solution of $y = -2000x + 50{,}000$. This means that $y = 24{,}500$ when $x = 12.75$.

Figure P.29b is a table of values for $y = -2000x + 50{,}000$ for a few values of x. The fourth line of the table shows numerically that $y = 24{,}500$ when $x = 12.75$.

Now try Exercise 45.

Figure P.30 shows Americans' income from August 1998 through July 1999 in trillions of dollars, adjusted for inflation and other factors. In Example 8 we model the data in Figure P.30 with a linear equation.

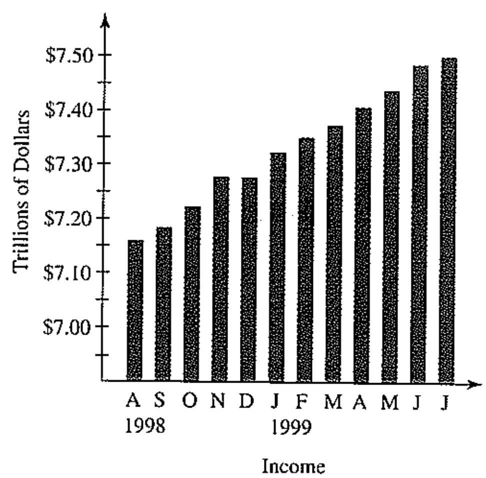


FIGURE P.30 Americans' income in July 1998 was 7.13 trillion dollars and in July 1999 was 7.5 trillion dollars.
Source: AP, Commerce Department as reported in *The Columbus Dispatch* on August 28, 1999. (Example 8)

EXAMPLE 8 Finding a linear model for Americans' personal income

From July 1998 to July 1999, Americans' income rose from 7.13 trillion dollars to 7.50 trillion dollars as displayed in Figure P.30.

(a) Let $x = 0$ represent July 1998, $x = 1$ represent August 1998, ..., and $x = 12$ represent July 1999. Write a linear equation for Americans' income y in terms of the month x using the points $(0, 7.13)$ and $(12, 7.50)$.

(b) Use the equation in (a) to estimate Americans' income in December 1998.

(c) Use the equation in (a) to predict Americans' income in July 2002.

SOLUTION

(a) Let $y = mx + b$. The value of 7.13 trillion dollars in July 1998 gives $y = 7.13$ when $x = 0$. So, $b = 7.13$. The value of 7.50 trillion in July 1999 gives $y = 7.5$ when $x = 12$.

$$y = mx + b$$
$$y = mx + 7.13 \qquad y = 7.13 \text{ when } x = 0$$
$$7.5 = m \cdot 12 + 7.13 \qquad y = 7.50 \text{ when } x = 12$$
$$0.37 = 12m$$
$$m = 0.0308\overline{3}$$

The linear equation we seek is $y = 0.0308\overline{3}x + 7.13$.

(b) December 1998 is represented by $x = 5$.

$$y = 0.0308\overline{3}x + 7.13$$
$$y = 0.0308\overline{3} \cdot 5 + 7.13 \qquad \text{Set } x = 5.$$
$$y \approx 7.28 \qquad \text{Simplify.}$$

Using the linear model found in (a) we estimate American's income in December 1998 to be 7.28 trillion dollars.

(c) July 2002 is represented by $x = 48$.

$$y = 0.0308\overline{3}x + 7.13$$
$$y = 0.0308\overline{3} \cdot 48 + 7.13 \qquad \text{Set } x = 48.$$
$$y = 8.61 \qquad \text{Simplify.}$$

Using the linear model found in (a) we predict Americans' income in July 2002 to have been 8.61 trillion dollars. Now try Exercise 51.

PROBLEM: Assume that the speed of light is approximately 186,000 miles per second. (It took a long time to arrive at this number. See the note below about the speed of light.)

(a) If the distance from the Moon to the Earth is approximately 237,000 miles, find the length of time required for light to travel from the Earth to the Moon.

(b) If light travels from the Earth to the Sun in 8.32 minutes, approximate the distance from the Earth to the Sun.

(c) If it takes 5 hours and 29 seconds for light to travel from the Sun to Pluto, approximate the distance from the Sun to Pluto.

SOLUTION: We use the linear equation $d = r \times t$ (distance = rate × time) to make the calculations with $r = 186{,}000$ miles/second.

(a) Here $d = 237{,}000$ miles, so

$$t = \frac{d}{r} = \frac{237{,}000 \text{ miles}}{186{,}000 \text{ miles/second}} \approx 1.27 \text{ seconds.}$$

The length of time required for light to travel from the Earth to the Moon is about 1.27 seconds.

(b) Here $t = 8.32$ minutes $= 499.2$ seconds, so

$$d = r \times t = 186{,}000 \, \frac{\text{miles}}{\text{second}} \times 499.2 \text{ seconds} = 92{,}851{,}200 \text{ miles.}$$

The distance from the Earth to the Sun is about 93 million miles.

(c) Here $t = 5$ hours and 29 minutes $= 329$ minutes $= 19{,}740$ seconds, so

$$d = r \times t = 186{,}000 \, \frac{\text{miles}}{\text{second}} \times 19{,}740 \text{ seconds}$$
$$= 3{,}671{,}640{,}000 \text{ miles.}$$

The distance from the Sun to Pluto is about 3.7×10^9 miles.

THE SPEED OF LIGHT

Many scientists have tried to measure the speed of light. For example, Galileo Galilei (1564–1642) attempted to measure the speed of light without much success. Visit the following web site for some interesting information about this topic: *http://www.what-is-the-speed-of-light.com/*

QUICK REVIEW P.4

In Exercises 1–4, solve for x.

1. $-75x + 25 = 200$
2. $400 - 50x = 150$
3. $3(1 - 2x) + 4(2x - 5) = 7$
4. $2(7x + 1) = 5(1 - 3x)$

In Exercises 5–8, solve for y.

5. $2x - 5y = 21$
6. $\frac{1}{3}x + \frac{1}{4}y = 2$
7. $2x + y = 17 + 2(x - 2y)$
8. $x^2 + y = 3x - 2y$

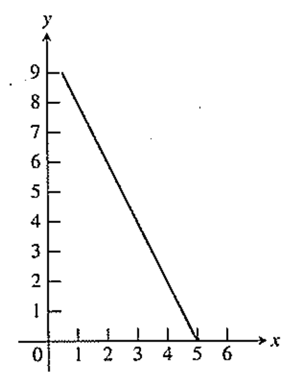
In Exercises 9 and 10, simplify the fraction.

9. $\dfrac{9 - 5}{-2 - (-8)}$
10. $\dfrac{-4 - 6}{-14 - (-2)}$

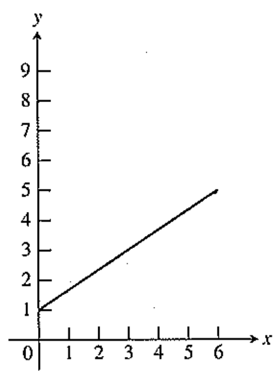
SECTION P.4 EXERCISES

In Exercises 1 and 2, estimate the slope of the line.

1.



2.



In Exercises 3–6, find the slope of the line through the pair of points.

3. $(-3, 5)$ and $(4, 9)$
4. $(-2, 1)$ and $(5, -3)$
5. $(-2, -5)$ and $(-1, 3)$
6. $(5, -3)$ and $(-4, 12)$

In Exercises 7–10, find the value of x or y so that the line through the pair of points has the given slope.

	Points	Slope
7.	$(x, 3)$ and $(5, 9)$	$m = 2$
8.	$(-2, 3)$ and $(4, y)$	$m = -3$
9.	$(-3, -5)$ and $(4, y)$	$m = 3$
10.	$(-8, -2)$ and $(x, 2)$	$m = 1/2$

In Exercises 11–14, find a *point-slope form* equation for the line through the point with given slope.

	Point	Slope		Point	Slope
11.	$(1, 4)$	$m = 2$	12.	$(-4, 3)$	$m = -2/3$
13.	$(5, -4)$	$m = -2$	14.	$(-3, 4)$	$m = 3$

In Exercises 15–20, find a *general form equation* for the line through the pair of points.

15. $(-7, -2)$ and $(1, 6)$
16. $(-3, -8)$ and $(4, -1)$
17. $(1, -3)$ and $(5, -3)$
18. $(-1, -5)$ and $(-4, -2)$
19. $(-1, 2)$ and $(2, 5)$
20. $(4, -1)$ and $(4, 5)$

In Exercises 21–26, find a slope-intercept form equation for the line.

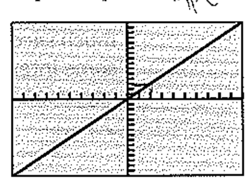
21. The line through $(0, 5)$ with slope $m = -3$
22. The line through $(1, 2)$ with slope $m = 1/2$
23. The line through the points $(-4, 5)$ and $(4, 3)$
24. The line through the points $(4, 2)$ and $(-3, 1)$
25. The line $2x + 5y = 12$
26. The line $7x - 12y = 96$

In Exercises 27–30, graph the linear equation on a grapher. Choose a viewing window that shows the line intersecting both the x- and y-axes.

27. $8x + y = 49$
28. $2x + y = 35$
29. $123x + 7y = 429$
30. $2100x + 12y = 3540$

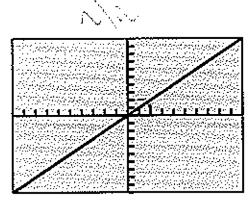
In Exercises 31 and 32, the line contains the origin and the point in the upper right corner of the grapher screen.

31. **Writing to Learn** Which line shown here has the greater slope? Explain.



[–10, 10] by [–15, 15]

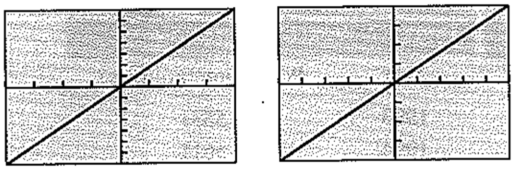
(a)



[–10, 10] by [–10, 10]

(b)

32. Writing to Learn Which line shown here has the greater slope? Explain.



[−20, 20] by [−35, 35]
(a)

[−5, 5] by [−20, 20]
(b)

In Exercises 33–36, find the value of x and the value of y for which $(x, 14)$ and $(18, y)$ are points on the graph.

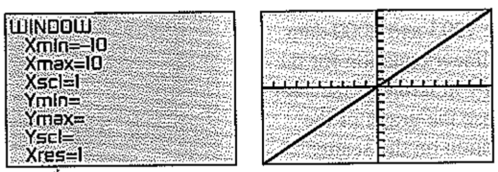
33. $y = 0.5x + 12$

34. $y = -2x + 18$

35. $3x + 4y = 26$

36. $3x - 2y = 14$

In Exercises 37–40, find the values for Ymin, Ymax, and Yscl that will make the graph of the line appear in the viewing window as shown here.



37. $y = 3x$

38. $y = 5x$

39. $y = \frac{2}{3}x$

40. $y = \frac{5}{4}x$

In Exercises 41–44, (a) find an equation for the line passing through the point and parallel to the given line, and (b) find an equation for the line passing through the point and perpendicular to the given line. Support your work graphically.

	Point	Line
41.	(1, 2)	$y = 3x - 2$
42.	(−2, 3)	$y = -2x + 4$
43.	(3, 1)	$2x + 3y = 12$
44.	(6, 1)	$3x - 5y = 15$

45. Real Estate Appreciation Bob Michaels purchased a house 8 years ago for \$42,000. This year it was appraised at \$67,500.

(a) A linear equation $V = mt + b$, $0 \le t \le 15$, represents the value V of the house for 15 years after it was purchased. Determine m and b.

(b) Graph the equation and trace to estimate in how many years after purchase this house will be worth \$72,500.

(c) Write and solve an equation algebraically to determine how many years after purchase this house will be worth \$74,000.

(d) Determine how many years after purchase this house will be worth \$80,250.

46. Investment Planning Mary Ellen plans to invest \$18,000, putting part of the money x into a savings that pays 5% annually and the rest into an account that pays 8% annually.

(a) What are the possible values of x in this situation?

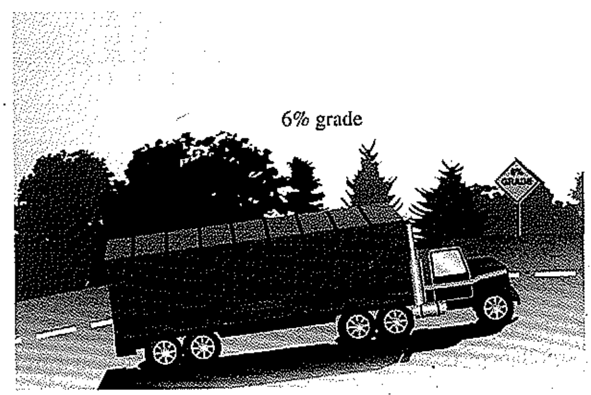
(b) If Mary Ellen invests x dollars at 5%, write an equation that describes the total interest I received from both accounts at the end of one year.

(c) Graph and trace to estimate how much Mary Ellen invested at 5% if she earned \$1020 in total interest at the end of the first year.

(d) Use your grapher to generate a table of values for I to find out how much Mary Ellen should invest at 5% to earn \$1185 in total interest in one year.

47. Navigation A commercial jet airplane climbs at takeoff with slope $m = 3/8$. How far in the horizontal direction will the airplane fly to reach an altitude of 12,000 ft above the takeoff point?

48. Grade of a Highway Interstate 70 west of Denver, Colorado, has a section posted as a 6% grade. This means that for a horizontal change of 100 ft there is a 6-ft vertical change.



(a) Find the slope of this section of the highway.

(b) On a highway with a 6% grade what is the horizontal distance required to climb 250 ft?

(c) A sign along the highway says 6% grade for the next 7 mi. Estimate how many feet of vertical change there are along those next 7 mi. (There are 5280 ft in 1 mile.)

49. Writing to Learn Building Specifications Asphalt shingles do not meet code specifications on a roof that has less than a 4-12 pitch. A 4-12 pitch means there are 4 ft of vertical change in 12 ft of horizontal change. A certain roof has slope $m = 3/8$. Could asphalt shingles be used on that roof? Explain.

50. Revisiting Example 8 Use the linear equation found in Example 8 to estimate Americans' income in the months displayed in Figure P.28.

51. Americans' Spending From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars as illustrated in Figure P.31.

(a) Let $x = 0$ represent July 1998, $x = 1$ represent August 1998, ..., and $x = 12$ represent July 1999. Write a linear equation for Americans' spending in terms of the month x using the pairs (0, 5.82) and (12, 6.20).

(b) Use the equation in (a) to estimate Americans' spending in the months displayed in Figure P.31.

(c) Use the equation in (a) to predict Americans' spending in July 2002.

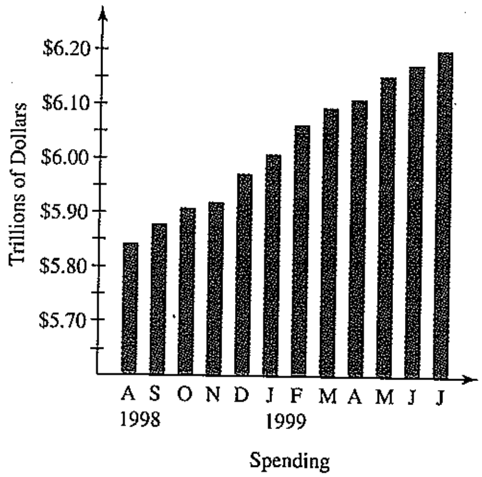


FIGURE P.31 Americans' spending in July 1998 was 5.82 trillion dollars and in July 1999 was 6.20 trillion dollars. *Source*: AP, Commerce Department as reported in *The Columbus Dispatch* on August 28, 1999.

52. U.S. Imports from Mexico The total y in billions of dollars of U.S. imports from Mexico for each year x from 1991 to 1998 is given in the table. *(Source: Bureau of the Census, Foreign Trade Division, FINAL 1991–1998 and 2001.)*

x	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
y	31.1	35.2	39.9	49.5	61.7	74.3	85.9	94.6	109.7	135.9

(a) Use the pairs (1992, 35.2) and (1996, 74.3) to write a linear equation for x and y.

(b) Superimpose the graph of the linear equation in (a) on a scatter plot of the data.

(c) Use the equation in (a) to predict the total U.S. Imports from Mexico in 2004.

53. World Population The midyear world population for the years 1993 to 2000 (in millions) is shown in Table P.7.

TABLE P.7 WORLD POPULATION

Year	Population (millions)
1993	5531
1994	5611
1995	5691
1996	5769
1997	5847
1998	5924
1999	6002
2000	6080

Source: U.S. Bureau of the Census, International Data Base, Data updated 2000.

(a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Draw a scatter plot of the data.

(b) Use the 1993 and 2000 data to write a linear equation for the population y in terms of the year x. Superimpose the graph of the linear equation on the scatter plot in (a).

(c) Use the equation in (b) to predict the midyear world population in 2004. Compare it with the Census Bureau estimate of 6386.

54. U.S. Exports to Japan The total in billions of dollars of U.S. Exports to Japan from 1993 to 2000 is given in Table P.8.

TABLE P.8 U.S. EXPORTS TO JAPAN

Year	U.S. Exports (billions of dollars)
1993	47.9
1994	53.4
1995	64.3
1996	67.6
1997	65.5
1998	57.8
1999	57.5
2000	65.3

Source: Bureau of the Census, Foreign Trade Division, Final 1993–2001.

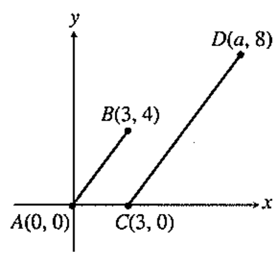
(a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Draw a scatter plot of the data.

(b) Use the 1993 and 2000 data to write a linear equation for the U.S. Exports to Japan y in terms of the year x. Superimpose the graph of the linear equation on the scatter plot in (a).

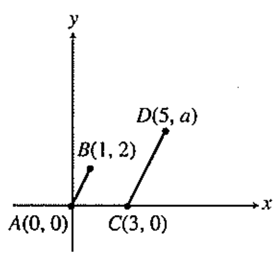
(c) Use the equation in (b) to predict the the U.S. Exports to Japan in 2004.

In Exercises 55 and 56, determine a so that the line segments AB and CD are parallel.

55.

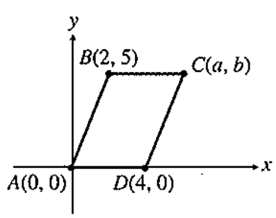


56.

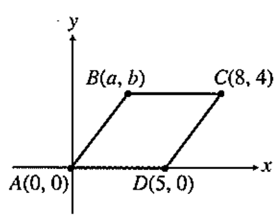


In Exercises 57 and 58, determine a and b so that figure $ABCD$ is a parallelogram.

57.



58.



59. Writing to Learn Perpendicular Lines

(a) Is it possible for two lines with positive slopes to be perpendicular? Explain.

(b) Is it possible for two lines with negative slopes to be perpendicular? Explain.

60. Group Activity Parallel and Perpendicular Lines

(a) Assume that $c \neq d$ and a and b are not both zero. Show that $ax + by = c$ and $ax + by = d$ are parallel lines. Explain why the restrictions on a, b, c, and d are necessary.

(b) Assume that a and b are not both zero. Show that $ax + by = c$ and $bx - ay = d$ are perpendicular lines. Explain why the restrictions on a and b are necessary.

Standardized Test Questions

61. True or False The slope of a vertical line is zero. Justify your answer.

62. True or False The graph of any equation of the form $ax + by = c$, where a and b are not both zero, is always a line. Justify your answer.

In Exercises 63–66, you may use a graphing calculator to solve these problems.

63. Multiple Choice Which of the following is an equation of the line through the point $(-2, 3)$ with slope 4?

(a) $y - 3 = 4(x + 2)$ **(b)** $y + 3 = 4(x - 2)$

(c) $x - 3 = 4(y + 2)$ **(d)** $x + 3 = 4(y - 2)$

(e) $y + 2 = 4(x - 3)$

64. Multiple Choice Which of the following is an equation of the line with slope 3 and y-intercept -2?

(a) $y = 3x + 2$ **(b)** $y = 3x - 2$

(c) $y = -2x + 3$ **(d)** $x = 3y - 2$

(e) $x = 3y + 2$

65. Multiple Choice Which of the following lines is perpendicular to the line $y = -2x + 5$?

(a) $y = 2x + 1$ **(b)** $y = -2x - \frac{1}{5}$

(c) $y = -\frac{1}{2}x + \frac{1}{3}$ **(d)** $y = -\frac{1}{2}x + 3$

(e) $y = \frac{1}{2}x - 3$

66. Multiple Choice Which of the following is the slope of the line through the two points $(-2, 1)$ and $(1, -4)$?

(a) $-\frac{3}{5}$ **(b)** $\frac{3}{5}$

(c) $-\frac{5}{3}$ **(d)** $\frac{5}{3}$

(e) -3

Explorations

67. Exploring the Graph of $\frac{x}{a} + \frac{y}{b} = c, a \neq, 0, b \neq 0$

Let $c = 1$.

(a) Draw the graph for $a = 3$, $b = -2$.

(b) Draw the graph for $a = -2$, $b = -3$.

(c) Draw the graph for $a = 5$, $b = 3$.

(d) Use your graphs in (a), (b), (c) to conjecture what a and b represent when $c = 1$. Prove your conjecture.

(e) Repeat (a)–(d) for $c = 2$.

(f) If $c = -1$, what do a and b represent?

68. Investigating Graphs of Linear Equations

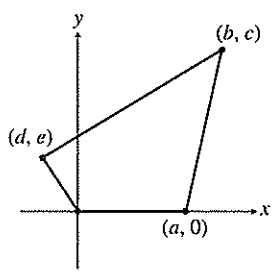
(a) Graph $y = mx$ for $m = -3, -2, -1, 1, 2, 3$ in the window $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?

(b) If $m > 0$, what do the graphs of $y = mx$ and $y = -mx$ have in common? How are they different?

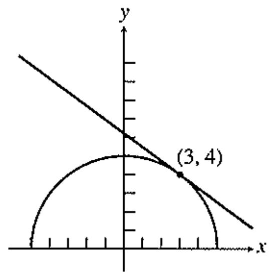
(c) Graph $y = 0.3x + b$ for $b = -3, -2, -1, 0, 1, 2, 3$ in $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?

Extending the Ideas

69. Connecting Algebra and Geometry Show that if the midpoints of consecutive sides of any quadrilateral (see figure) are connected, the result is a parallelogram.



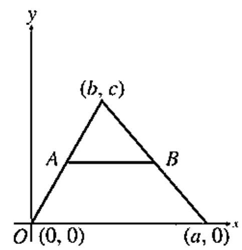
Art for Exercise 69



Art for Exercise 70

70. Connecting Algebra and Geometry Consider the semicircle of radius 5 centered at (0, 0) as shown in the figure. Find an equation of the line tangent to the semicircle at the point (3, 4). (*Hint:* A line tangent to a circle is perpendicular to the radius at the point of tangency.)

71. Connecting Algebra and Geometry Show that in any triangle (see figure), the line segment joining the midpoints of two sides is parallel to the third side and half as long.



P.5 SOLVING EQUATIONS GRAPHICALLY, NUMERICALLY, AND ALGEBRAICALLY

What you'll learn about

- Solving Equations Graphically
- Solving Quadratic Equations
- Approximating Solutions of Equations Graphically
- Approximating Solutions of Equations Numerically with Tables
- Solving Equations by Finding Intersections

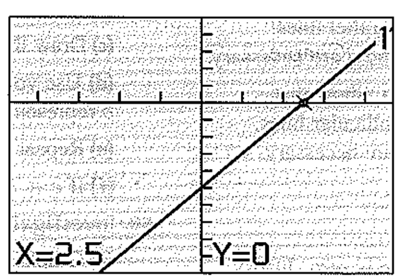
. . . and why

These basic techniques are involved in using a graphing utility to solve equations in this textbook.

Solving Equations Graphically

The graph of the equation $y = 2x - 5$ (in x and y) can be used to solve the equation $2x - 5 = 0$ (in x). Using the techniques of Section P.3, we can show algebraically that $x = 5/2$ is a solution of $2x - 5 = 0$. Therefore, the ordered pair $(5/2, 0)$ is a solution of $y = 2x - 5$. Figure P.32 suggests that the x-intercept of the graph of the line $y = 2x - 5$ is the point $(5/2, 0)$ as it should be.

One way to solve an equation graphically is to find all its x-intercepts. There are many graphical techniques that can be used to find x-intercepts.



[–4.7, 4.7] by [–10, 5]

FIGURE P.32 Using the Trace feature of a grapher, we see that (2.5, 0) is an x-intercept of the graph of $y = 2x - 5$ and, therefore, $x = 2.5$ is a solution of the equation $2x - 5 = 0$.