CHAPTER 11

Think & Discuss *(p. 659)*

1. 6
2. The base angles of an equilateral triangle measure 60° each. $2(60°) = 120°$, $120° \times 6 = 720°$

Skill Review (p. 660)

1. $A = \frac{1}{2}bh$

$= \frac{1}{2}(12 \text{ in.})(8 \text{ in.})$

$= 48 \text{ in.}^2$

2. $m\angle A + m\angle B + m\angle C = 180°$

$57° + m\angle B + 79° = 180°$

$m\angle B + 136° = 180°$

$m\angle B = 44°$

Exterior angle to A is $180° - 57° = 123°$. Exterior angle to B is $180° - 44° = 136°$. Exterior angle to C is $180° - 79° = 101°$.

3. a. $\frac{XY}{DE} = \frac{12}{8} = \frac{3}{2}$

b. $\frac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle XYZ} = \frac{8}{12} = \frac{2}{3}$

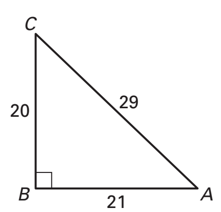
4. *Sample answer:*

$m\angle A = \sin^{-1}\left(\frac{20}{29}\right)$

$\approx 43.6°$

$m\angle C = \sin^{-1}\left(\frac{21}{29}\right)$

$\approx 46.4°$



Developing Concepts Activity (p. 661)

Polygon	*Number of sides*	*Number of triangles*	*Sum of measures of interior angles*
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	4	2	$2 \cdot 180° = 360°$
Pentagon	5	3	$3 \cdot 180° = 540°$
Hexagon	6	4	$4 \cdot 180° = 720°$
⋮	⋮	⋮	⋮
n-gon	n	$n - 2$	$(n - 2) \cdot 180°$

The sum of the measures of the interior angles of any convex n-gon is $(n - 2)(180°)$.

Lesson 11.1

11.1 Guided Practice (p. 665)

1. Interior angles are $\angle A$, $\angle B$, $\angle D$, $\angle BCD$, and $\angle AED$. Exterior angles are $\angle AEF$, $\angle BCG$, and $\angle DCH$.
2. There are $2n$ exterior angles. No; the Polygon Exterior Angles Theorem specifies only 1 angle at each vertex.
3. The sum of the measures of the interior angles of a pentagon is $(5 - 2)(180°) = 540°$.

$540° - (105° + 115° + 120° + 105°) = x$

$540° - 445° = x$

$95° = x$

4. Sum of measures of interior angles $= (6 - 2)(180°)$

$= 720° = \frac{720°}{6} = 120°$

5. Measure of an exterior angle $= \frac{360°}{n} = \frac{360°}{8} = 45°$

11.1 Practice and Applications (p. 665–668)

6. $(n - 2)(180°)$

$(10 - 2)(180°)$

$1440°$

7. $(n - 2)(180°)$

$(12 - 2)(180°)$

$1800°$

8. $(n - 2)(180°)$

$(15 - 2)(180°)$

$2340°$

9. $(n - 2)(180°)$

$(18 - 2)(180°)$

$2880°$

10. $(n - 2)(180°)$

$(20 - 2)(180°)$

$3240°$

11. $(n - 2)(180°)$

$(30 - 2)(180°)$

$5040°$

12. $(n - 2)(180°)$

$(40 - 2)(180°)$

$6840°$

13. $(n - 2)(180°)$

$(100 - 2)(180°)$

$17{,}640°$

14. $x + 113° + 80° + 130° + 90° = (5 - 2)(180°)$

$x + 413° = 540°$

$x = 127°$

15. $x + 125° + 147° + 106° + 98° + 143° = (6 - 2)(180°)$

$x + 619° = 720°$

$x = 101°$

16. $x + 102° + 146° + 120° + 124° + 170° + 158°$

$= (7 - 2)(180°)$

$x + 820° = 900°$

$x = 80°$

17. $x = \dfrac{(n-2)\cdot 180°}{n}$

$= \dfrac{(5-2)\cdot 180°}{5}$

$= 108°$

18. $x = \dfrac{(n-2)\cdot 180°}{n}$

$= \dfrac{(7-2)(180°)}{7}$

$\approx 128.57°$

19. $x = \dfrac{(n-2)\cdot 180°}{n}$

$= \dfrac{(8-2)\cdot 180°}{8}$

$= 135°$

20. $x + 80° + 110° + 80° = (4-2)(180°)$

$x + 270° = 360°$

$x = 90°$

21. $x + 60° + 80° + 120° + 140° = (5-2)(180°)$

$x + 400° = 540°$

$x = 140°$

22. $144° = \dfrac{(n-2)(180°)}{n}$

$144° \cdot n = (n-2)(180°)$

$144° \cdot n = 180° \cdot n - 360°$

$-36° \cdot n = -360°$

$n = 10$

23. $120° = \dfrac{(n-2)(180°)}{n}$

$120° \cdot n = (n-2)(180°)$

$120° \cdot n = 180° \cdot n - 360°$

$-60° \cdot n = -360°$

$n = 6$

24. $140° = \dfrac{(n-2)(180°)}{n}$

$140° \cdot n = (n-2)(180°)$

$140° \cdot n = 180° \cdot n - 360°$

$-40° \cdot n = -360°$

$n = 9$

25. $157.5° = \dfrac{(n-2)(180°)}{n}$

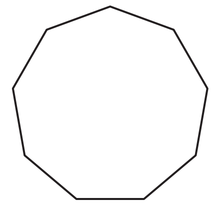
$157.5° \cdot n = (n-2)(180°)$

$157.5° \cdot n = 180° \cdot n - 360°$

$-22.5° \cdot n = -360°$

$n = 16$

26.–28.



29. exterior angle $= \dfrac{360°}{n}$

$= \dfrac{360°}{12}$

$= 30°$

30. exterior angle $= \dfrac{360°}{n}$

$= \dfrac{360°}{11}$

$\approx 32.73°$

31. exterior angle $= \dfrac{360°}{n}$

$= \dfrac{360°}{21}$

$\approx 17.14°$

32. exterior angle $= \dfrac{360°}{n}$

$= \dfrac{360°}{15}$

$= 24°$

33. $n = \dfrac{360°}{60°}$

$= 6$

34. $n = \dfrac{360°}{20°}$

$= 18$

35. $n = \dfrac{360°}{72°}$

$= 5$

36. $n = \dfrac{360°}{10°}$

$= 36$

37. $x + 48° + 52° + 55° + 62° + 68° = 360°$

$x + 285° = 360°$

$x = 75°$

38. exterior angle $= \dfrac{360°}{10}$

$= 36°$

39. The interior angles of the regular hexagons are 120°. Two of the interior angles of the red triangles form a linear pair with the interior angles of the regular polygons. Therefore, each interior angle of the triangles is 60°. The interior angles of the yellow hexagons are 120°. Two of the interior angles of the yellow pentagons form linear pair with the red triangles, so, they are 120°. A third angle of the yellow pentagons is $540° - 120° - 120° - 90° - 90° = 120°$.

40. Regular pentagons have interior angles measuring 108°. The interior angles of the red quadrilaterals which form linear pairs with the interior angles of the pentagons measure 72°. The angle in the red quadrilateral formed by the joining of two pentagons is 144°. The 4th angle of the red quadrilateral is $360° - 72° - 72° - 144° = 72°$. For the yellow pentagons, two of the angles form linear pairs with a 72° angle, so they are 108°. The fifth angle is $540° - 108° - 108° - 90° - 90° = 144°$. Each angle of the yellow decagon is 144°.

41. $m\angle 9$ and $m\angle 10$ are 70° because they form linear pairs with $\angle 4$ and $m\angle 5$, respectively. $m\angle 3$ is 140° because it forms a linear pair with $\angle 8$. $m\angle 7$ is 80° because it forms a linear pair with $\angle 2$. $m\angle 1$ is 80° because the sum of the measures of the interior angles of a pentagon must equal 540°. $m\angle 6$ is 100° because forms a linear pair with $\angle 1$.

42. Any n-gon can be divided into $n - 2$ triangles. The sum of the measures of the interior angles will be equal to $(n - 2)(180°)$. The n-gons do not need to be regular or similar because the sum of the angles of any triangle will always be 180°. What matters is that n is the same.

43. Given pentagon $ABCDE$, where $n = 5$ draw two diagonals from one vertex to divide the pentagon into three triangles. The sum of the measures of the angles of a triangle is 180°. Therefore, the sum of the measures of the angles of the three triangles must be $3(180°)$ or $(n - 2)(180°)$.

44. $ABCDE$ is a pentagon. Draw diagonals from A to C and A to D, dividing $ABCDE$ into three triangles. The sum of the measures of the interior triangles is $3(180°)$ or $(n - 2)180°$. The interior angles of a regular polygon are congruent to each other. Therefore, you need to divide the sum of the measures of the interior angles by the number of the interior angles (which equals the number of sides), to get the measure of each angle. So, each angle has the measure $(n - 2)(180°)/n$.

45. In a convex n-gon, extend each side to make an exterior angle with the n-gon. The interior angle and its adjacent exterior angle at any vertex form a linear pair, whose sum is 180°. Since the n-gon has n sides, there will be n linear pairs composed of an interior and exterior angle, so multiply n by 180°. Then subtract the sum of the measures of the interior angles, which is $(n - 2) \cdot 180°$ to get the total of the exterior angles. So, the sum of the measures of the exterior angles of a convex n-gon is $180°n - (n - 2) \cdot 180° = 180°n - 180°n - (-360°) = 360°$.

46. The measure of each interior angle of an n-gon is

$$\frac{(n - 2)(180°)}{n} = 180° - \frac{360°}{n}.$$

Each exterior angle of a regular n-gon is 180° − measure of interior angle. Therefore, the measure of an exterior angle by substitution is

$$180° - \left[\frac{(n - 2) \cdot 180°}{n}\right] = 180° - \left(\frac{180°n - 360°}{n}\right)$$
$$= \frac{360°}{n}.$$

47. *Sample answer:* If a regular hexagon were constructed, each exterior angle would measure 60° and the sum of the exterior angles would be 360°. Answers will vary.

48. The exterior angles will vary inversely with the change in the interior angle. The sum of the measures of the exterior angles will still be 360°.

49. $3x + 90° + 90° = (5 - 2)(180°)$

$3x + 180° = 540°$

$3x = 360°$

$x = 120°$

So, $m\angle A = m\angle E = 90°$, $m\angle B = m\angle C = m\angle D = 120°$.

50. $x + x + 2x + 160° + 160° + 150° + 150°$

$= (7 - 2)(180°)$

$4x + 620° = 900°$

$4x = 280°$

$x = 70°$

So, $m\angle P = m\angle V =$ °, and $m\angle S =$ °.

51. $\dfrac{(n - 2)(180°)}{n} = 150°$

$180°n - 360° = 150°n$

$30°n = 360°$

$n = 12$

Yes. The polygon would be a dodecagon.

52. $\dfrac{(n - 2)(180°)}{n} = 90°$

$180°n - 360° = 90°n$

$90°n = 360°$

$n = 4$

Yes. The polygon would be a square.

53. $\dfrac{(n - 2)(180°)}{n} = 72°$

$180°n - 360° = 72°n$

$108°n = 360°$

$n = 3\frac{1}{3}$

No. Because n must be an integer, a regular poygon can't have angles of 72°.

54. $\dfrac{(n - 2)(180°)}{n} = 18°$

$180°n - 360° = 18°n$

$162°n = 360°$

$n = 2.22$

No. Because n must be an integer, a regular poygon can't have angles of 18°.

55. The function is showing the relationship between the number of sides, n, and the measure of each interior angle for a regular n-gon. As n gets larger and larger, $f(n)$ approaches 180°.

56. $f(n)$ represents the measure of the exterior angle of a regular polygon. As n gets larger and larger, $f(n)$ decreases.

57. The exterior angles measure 55° and 17°. $360° \div 72° = 5$, so $n = 10$.

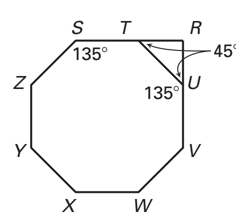
58. B. Column A = 1440°. Column B = 2340°.

59. C. Both columns equal = 360°.

60. A. Column A = 74°. Column B = 73°.

61. D. If these were regular polygons, the quantities could be determined.

62. Each interior angle has a measure of $(8 - 2)(180)/8 = 135°$. Each exterior angle is $45° = 180° - 135°$. Since the sum of the angles of a $\triangle$ is 180°, $\angle R$ must be $180° - 2(45°) = 90°$.



11.1 Mixed Review (p. 668)

63. $A = \frac{1}{2}bh$

$= \frac{1}{2}(11)(5)$

$= 27.5 \text{ in.}^2$

64. $A = \frac{1}{2}bh$

$= \frac{1}{2}(43)(11)$

$= 236.5 \text{ m}^2$

65. base = 5

height = 15

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(15)$

$= 37.5$ square units

66. base = 6

height = 8

$A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(8)$

$= 24$ square units

67. $9^2 + 13^2 \stackrel{?}{=} 16^2$

$81 + 169 \stackrel{?}{=} 256$

$250 \neq 256$

Not a right $\triangle$.

68. $21^2 + 72^2 \stackrel{?}{=} 75^2$

$441 + 5184 \stackrel{?}{=} 5625$

$5625 = 5625$

Is right $\triangle$.

69. $7^2 + 5^2 \stackrel{?}{=} \left(2\sqrt{17}\right)^2$

$49 + 25 \stackrel{?}{=} 68$

$74 \neq 68$

Not a right $\triangle$.

70. $m\widehat{DH} = 80°$

71. $m\widehat{ED} = m\widehat{FH} - m\widehat{FE} - m\widehat{DH}$

$= 180° - 35° - 80°$

$= 65°$

72. $m\widehat{EH} = m\widehat{FH} - m\widehat{FE}$

$= 180° + 35°$

$= 145°$

73. $m\widehat{EHG} = m\widehat{GD} + m\widehat{DE}$

$= m\widehat{GD} + \left(m\widehat{FH} - m\widehat{FE} - m\widehat{DH}\right)$

$= 180° + 180° - 35° - 80°$

$= 245°$

Lesson 11.2

Activity in a Lesson (p. 670)

1. Six equilateral triangles
2. To find the area of one triangle, one must draw an altitude from one side of the hexagon to the center forming two 30°-60°-90° right triangles. If the side of the hexagon is s, the height of the triangle would be $\frac{\sqrt{3}}{2}s$.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right)$

$= \frac{1}{4}\sqrt{3}\, s^2$

Area hexagon $= 6 \cdot$ (Area one triangle)

$= 6\left(\frac{1}{4}\sqrt{3}\, s^2\right)$

$= \frac{3}{2}\sqrt{3}\, s^2$

11.2 Guided Practice (p. 672)

1. J **2.** $\overline{JB}$ or $\overline{JC}$

3. *Sample answers:* $\angle BJA$, $\angle AJE$, $\angle BJC$, $\angle EJD$, or $\angle DJC$

4. $\overline{KJ}$ **5.** Divide 360° by the number of sides.

6. $A = \frac{1}{4}\sqrt{3}\, s^2$

$= \frac{1}{4}\sqrt{3}\,(3)^2$

$= \frac{9\sqrt{3}}{4}$

$\approx 3.9 \text{ in.}^2$

7. $\frac{360°}{8} = 45°$

8. Each side would be 80 inches ÷ 8 or 10 inches. The apothem would be half the height of the sign or 12 inches. The apothem creates a 45°-45°-90° triangle, so the radius of the octagon would be equal to the hypotenuse which is 13 inches.

$A = \frac{1}{2}aP$

$= \frac{1}{2}(12)(80)$

$= 480 \text{ in.}^2$

11.2 Practice and Applications (pp. 672–675)

9. $A = \frac{1}{4}\sqrt{3}\, s^2$

$= \frac{1}{4}\sqrt{3}\,(5)^2$

$= \frac{25\sqrt{3}}{4}$

$\approx 10.8 \text{ units}^2$

10. $A = \frac{1}{4}\sqrt{3}\, s^2$

$= \frac{1}{4}\sqrt{3}\,(11)^2$

$= \frac{121\sqrt{3}}{4}$

$\approx 52.4 \text{ units}^2$

11. $A = \frac{1}{4}\sqrt{3}\, s^2$

$= \frac{1}{4}\sqrt{3}\left(7\sqrt{5}\right)^2$

$= \frac{254\sqrt{3}}{4}$

$\approx 106.1 \text{ units}^2$

12. $\frac{360°}{9} = 40°$

13. $\dfrac{360°}{12} = 30°$ **14.** $\dfrac{360°}{15} = 24°$ **15.** $\dfrac{360°}{180} = 2°$

16. $s = 2\sqrt{8^2 - (4\sqrt{2})^2}$
$= 2\sqrt{64 - 32}$
$= 8\sqrt{2}$

$A = \dfrac{1}{2}a \cdot ns$
$= \dfrac{1}{2}(4\sqrt{2})(4)(8\sqrt{2})$
$= 128 \text{ units}^2$

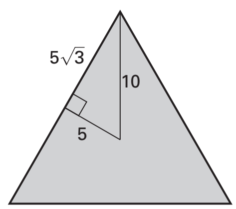
17. $s = 2\sqrt{12^2 - 6^2}$
$= 2\sqrt{144 - 36}$
$= 12\sqrt{3}$

$A = \dfrac{1}{2}a \cdot ns$
$= \dfrac{1}{2}(6)(3)(12\sqrt{3})$
$= 108\sqrt{3} \approx 187.1 \text{ units}^2$

18. $s = 2\sqrt{20^2 - (10\sqrt{3})^2}$
$= 2\sqrt{400 - 300}$
$= 20$

$A = \dfrac{1}{2}a \cdot ns$
$= \dfrac{1}{2}(10\sqrt{3}) \cdot (6)(20)$
$= 600\sqrt{3} \approx 1039.2 \text{ units}^2$

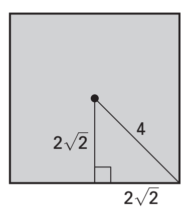
19. One side $= 10\sqrt{3}$



$P = ns$
$= 3(10\sqrt{3})$
$= 30\sqrt{3} \approx 52.0 \text{ units}$

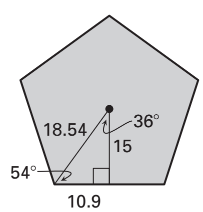
$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(5)(30\sqrt{3})$
$= 75\sqrt{3} \approx 129.9 \text{ units}^2$

20. $P = ns$
$= 4(4\sqrt{2})$
$= 16\sqrt{2} \text{ units}$



$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(2\sqrt{2})(16\sqrt{2})$
$= 32 \text{ units}^2$

21. $\dfrac{360°}{5} = 72°$



The apothem bisects the central angle. If a radius is drawn, a 36°-54°-90° triangle will be formed. Use trigonometry ratios to find the length of the radius ($r = 15/(\cos 36°)$) and a side. The side of the pentagon would be equal to: $2(15 \tan 36°) = 30 \tan 36°$

$P = ns$
$= (5)(30 \tan 36°)$
$= 150 \tan 36°$
$\approx 109.0 \text{ units}$

$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(15)(150 \tan 36°)$
$= 1125 \tan 36°$
$\approx 817.36 \text{ units}^2$

22. $\dfrac{360°}{6} = 60°$

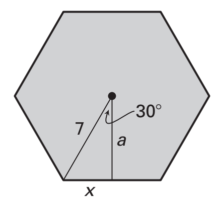
The apothem bisects the central angle. So, a 30°-60°-90° triangle is formed by the apothem and the radius of the hexagon.

$\dfrac{a}{7} = \cos 30°$
$a = 7 \cos 30°$
$a = \dfrac{7\sqrt{3}}{2} \text{ units}$

$\dfrac{x}{7} = \sin 30°$
$x = 7 \sin 30°$
$x \approx 3.5 \text{ units}$

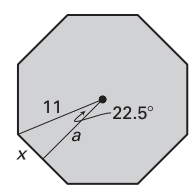
$s = 2x = 2(3.5) = 7 \text{ units}$

$P = ns$
$= (6)(7)$
$= 42 \text{ units}$



$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}\left(\dfrac{7\sqrt{3}}{2}\right)(42)$
$= \dfrac{147\sqrt{3}}{2}$
$= 127.31 \text{ units}^2$

23. $\dfrac{360°}{8} = 45°$



The apothem bisects the central angle. So, $\dfrac{x}{11} = \sin 22.5°$.

$x = 11(\sin 22.5°)$

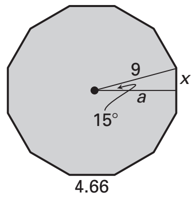
One side of the octagon $= 2x = 22 \sin 22.5°$.

$\dfrac{a}{11} = \cos 22.5°$
$a = 11(\cos 22.5°)$

$P = ns$
$P = 8(22 \sin 22.5°)$
$= 176 \sin 22.5$
$\approx 67.35 \text{ units}$

$A = \dfrac{1}{2}aP$
$= \dfrac{1}{2}(11 \cos 22.5°)(176 \sin 22.5°)$
$\approx 342.24 \text{ units}^2$

24. $\frac{360°}{12} = 30°$



The central angle of a dodecagon is 30°. The central angle is bisected by the apothem forming a 15°-75°-90° triangle.

$\frac{a}{9} = \cos 15°$ $\qquad$ $\frac{x}{9} = \sin 15°$

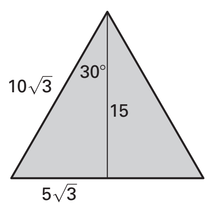
$a = 9 \cos 15°$ $\qquad$ $x = 9 \sin 15°$

$s = 2x = 2(9 \sin 15°) = 18 \sin 15°$

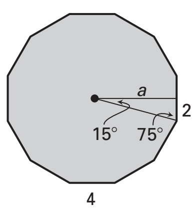
$P = ns$
$= (12)(18 \sin 15°)$
$= 216 \sin 15°$
$= 55.9$ units

$A = \frac{1}{2}aP$
$= \frac{1}{2}(9 \cos 15°)(216 \sin 15°)$
$= 972(\sin 15°)(\cos 15°)$
≈ 243 units^2

25. $A = \frac{1}{2}bh$
$= \frac{1}{2}\left(10\sqrt{3}\right)(15)$
$= 75\sqrt{3}$
$= 129.9$ in.^{2}



26. $\frac{360°}{12} = 30°$



The apothem bisects the central angle forming a 15°-75°-90° triangle.

$\frac{a}{2} = \tan 75°,\ a = 2 \tan 75°$

$P = ns$
$= (12)(4)$
$= 48$ in.

$A = \frac{1}{2}aP$
$= \frac{1}{2}(2 \tan 75°)(48)$
$= 48 \tan 75°$
$= 179.14$ in.^{2}

27. True. Let θ be the central angle and r the radius. The side length s would be

$2r\left(\sin \frac{\theta}{2}\right)$ and the apothem would be

$r\left(\cos \frac{\theta}{2}\right)$. Both increase as r increases.

Then $\frac{1}{2}aP$ increases as r increases.

28. True. The apothem and radius are the leg and hypotenuse of a right △ respectively. The hypotenuse is always longer than the legs so the apothem will always be less than the radius.

29. False. It depends on the polygon. They are the same for a regular hexagon.

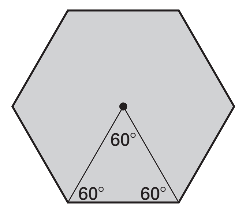
30. $A = \left(16\sqrt{3}\right)(6)$
$= 96\sqrt{3}$
≈ 166.3 units^2

31. $A = (4 \tan 67.5°)(8)$
$= 32 \tan 67.5°$
≈ 77.3 units^2

32. $A = (\tan 54°)(5)$
≈ 6.9 units^2

33. $\frac{360°}{6} = 60° =$ central angle

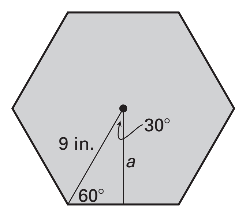
$\frac{(6 - 2)(180°)}{6} = 120° =$ interior angle



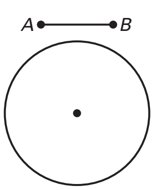
The radius bisects each interior angle, and with the apothem forms an equilateral △. The area of an equilateral △ is $A = \frac{1}{4}\sqrt{3}\,s^2$. Since a regular hexagon can be divided into six equilateral △s, the area of a hexagon is $A = 6\left(\frac{1}{4}\sqrt{3}\,s^2\right)$.

34. $\frac{a}{9} = \sin 60°$

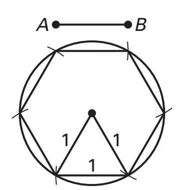
$a = 9 \sin 60°$
$= \frac{9\sqrt{3}}{2}$
≈ 7.8 in.



35.



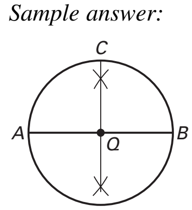
36.–37.



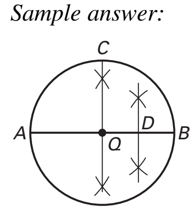
38. $A = 6 \cdot \left(\frac{1}{4}\sqrt{3}\,s^2\right)$
$= 6 \cdot \left(\frac{1}{4}\sqrt{3}(1)^2\right)$
≈ 2.6 in.^{2}

39. Do the same construction but connect every other compass mark.

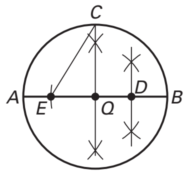
40. *Sample answer:*



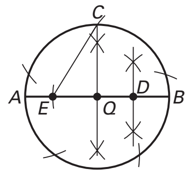
41. *Sample answer:*



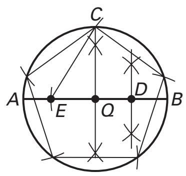
42. *Sample answer:*



43. *Sample answer:*

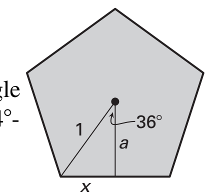


44. *Sample answer:*



Sample answer:

$\frac{360°}{5} = 72°$ central angle.



The apothem bisects the central angle so with the radius, it forms a 36°-54°-90° triangle. The apothem can be found by using trigonometry $r = 1$ in.

$\frac{a}{1} = \cos 36°$ $\qquad$ $\frac{x}{1} = \sin 36°$

$a \approx 0.81$ in. $\qquad$ $x \approx 0.59$ in.

$s = 2x = 1.18$ in.

$P = ns$
$= 5(1.18)$
$= 5.9$ in.

$A = \frac{1}{2}aP$
$= \frac{1}{2}(0.81 \text{ in.})(5.9 \text{ in.})$
$\approx 2.4 \text{ in.}^2$

45. The radius $=$ side $= 0.5$ m. The apothem bisects a side.

So, the base of the right triangle $= \frac{1}{2}(0.5) = 0.25$ m.
The triangle formed by the apothem and radius is a 30°-60°-90° triangle with the 0.25 side being opposite the 30° angle. The apothem $= \sqrt{3}\,(0.25) = 0.43$ m.

46. $P = ns$
$= 6(0.5)$
$= 3$ m

$A = \frac{1}{2}aP$
$= \frac{1}{2}(0.43)(3)$
$= 0.65 \text{ m}^2$

Note: See problem 45 for explanation.

47. 3 colors

48. The apothem will be $3\sqrt{3}$ in. or 5.2 in. long based on a 30°-60°-90° triangle. $P = ns = 6(6) = 36$ in.

$A = \frac{1}{2}aP = \frac{1}{2}(5.2)(36) = 93.6 \text{ in.}^2$

49. 6 ft $=$ 72 in., 8 ft $=$ 96 in.
$A = (72)(96) = 6912 \text{ in.}^2$

number tiles $= \frac{6912}{93.6} \approx 74$ tiles for the floor total

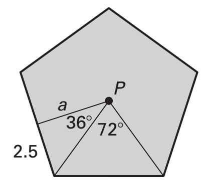
$\frac{74}{3} \approx 25$ tiles of each color

50. B. Column $A = 45°$. Column $B \approx 51.43$.

51. A. Column $A \approx 0.92$. Column $B \approx 0.90$.

52. A. Column $A = 6.12$. Column $B = 6.07$.

53. $A = \frac{1}{2}aP$
$= \frac{1}{2}\left(\frac{2.5}{\tan 36°}\right)(5)(5)$
$\approx 43 \text{ units}^2$



Area triangle $= \frac{1}{2}(5 \sin 54°)(5 \cos 54°)(2)$
$= 11.89 \text{ units}^2$

Area trapezoid $= \frac{1}{2}(b_1 + b_2)(h)$
$= \frac{1}{2}(5 + 2 \cdot 5 \sin 54°)(5 \cos 18°)$
$= \frac{1}{2}(13.09)(4.76)$
$= 31.12$

Area total $= 11.89 + 31.12$
≈ 43 units

Answers should be the same. Differences may arise due to rounding.

11.2 Mixed Review (p. 675)

54. $\frac{x}{6} = \frac{11}{12}$
$12x = 66$
$x = \frac{11}{2}$

55. $\frac{20}{4} = \frac{15}{x}$
$20x = 60$
$x = 3$

56. $\frac{12}{x + 7} = \frac{13}{x}$
$12x = 13(x + 7)$
$12x = 13x + 91$
$-x = 91$
$x = -91$

57. $\frac{x + 6}{9} = \frac{x}{11}$
$11(x + 6) = 9x$
$11x + 66 = 9x$
$66 = -2x$
$-33 = x$

58. True. Corresponding sides of similar triangles are proportional.

59. True. The perimeters of similar triangles are proportional.

60. True. The corresponding angles of similar triangles are congruent.

61. False. Corresponding sides of similar triangles are proportional, not necessarily congruent.

62. $14 \cdot x = 7 \cdot 12$
$14x = 84$
$x = \frac{84}{14}$
$x = 6$

63. $9(9 + x) = 8(8 + 10)$
$81 + 9x = 144$
$9x = 63$
$x = 7$

64. $8^2 = 4(4 + x)$
$64 = 16 + 4x$
$48 = 4x$
$12 = x$

Exploring the Concept Activity *(p. 676)*

For example on p. 676-*Sample Answer:*

Original Polygon	*Area*	*Similar Polygon*	*Area*
Rectangle 1	5 sq units	Rectangle 1	20 sq units
Rectangle 2	8 sq units	Rectangle 2	32 sq units
Rectangle 3	6 sq units	Rectangle 3	24 sq units
⋮	⋮	⋮	⋮
Total	11 sq units	Total	44 sq units

Note: The areas for similar polygons should be 4 times the area of the original polygons.

Drawing Conclusions

1. The ratio of the area of the similar polygons to the area of the original polygons.

2. The ratio of the areas of two similar polygons is the square of the scale factor.

Lesson 11.3

11.3 Guided Practice (p. 679)

1. $a: b,\ a^2: b^2$

2. False. In order to be similar, then the lengths of the corresponding sides must be proportional and all corresponding angles must be congruent.

3. False. The scale factor is 1:2, so the ratio of the areas is 1:4 and the area is quadrupled.

4. $P_{red} = 3(6) = 18$ $\quad P_{blue} = 9(6) = 54$ $\quad \frac{P_{red}}{P_{blue}} = \frac{18}{54} = \frac{1}{3}$

The ratios of the areas is equal to the square of the ratio of the perimeters so $\frac{1^2}{3^2} = \frac{1}{9}$.

5. $A_{red} = 6 \cdot 5 = 30$ $\quad A_{blue} = 4 \cdot 3\frac{1}{3} = 13\frac{1}{3}$

$\frac{A_{red}}{A_{blue}} = \frac{30}{\frac{40}{3}} = \frac{90}{40} = \frac{9}{4}$

$\frac{P_{red}}{P_{blue}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

6. The ratio of the lengths of the pieces of paper is 2:1, so the ratio of the areas of the piece of paper is 4:1. The cost of the smaller piece should be $\frac{1}{4}$ as much as the larger sheet or $0.11.

11.3 Practice and Applications (p. 679–681)

7. The ratio of the perimeters is 2:1.
The ratio of the areas is 4:1.

8. The ratio of the perimeter is 5:7.
The ratio of the areas is 25:49.

9. The ratio of the perimeter is 5:6.
The ratio of the areas is 25:36.

10. The ratio of the perimeter is 5:3.
The ratio of the areas is 25:9.

11. sometimes **12.** sometimes **13.** always **14.** 4:25

15. $\sqrt{\frac{49}{100}} = \frac{7}{10}$

16. The scale factor of the hypotenuses is 8:20 or 2:5. The ratio of the areas is the square of the scale factor, or 4:25.

$\frac{4}{25} = \frac{13.9}{x}$
$4x = 347.5$
$x = 86.875$ inches

17. *Sample answer:* $\overline{AB} \parallel \overline{DC}$, so $\angle AEB \cong \angle DEC$ because vertical angles are $\cong$. $\angle BAE \cong \angle ECD$ because alternate interior angles are $\cong$. $\triangle CDE \sim \triangle ABE$ by the AA Similarity Postulate.

Area of $\triangle ABE = \frac{1}{2}(12)(3) =$ sq 18 units

$\frac{7}{3} = \frac{DC}{12}$
$28 = DC$
$\frac{7(12)}{3} = DC$

Area of $\triangle CDE = \frac{1}{2}(28)(7) = 98$ sq units.

18. $\overline{DC} \parallel \overline{AB}$ and $\overline{LK} \parallel \overline{AB}$, so $\overline{DC} \parallel \overline{LK}$ because 2 lines $\parallel$ to the same line are parallel. Then $\angle k \cong \angle C$ and $\angle A \cong \angle J$, and all corresponding $\angle s$ are $\cong$. Ratio of sides is 3:1, so the ratio of the areas is $3^2 = 1^2$. The area of $\square ABCD$ is 9 times the area of $\square JBKL = 9(15.3) = 137.7$ sq in.

19. Perimeter of $ABCDE = 30\sqrt{5}$. Perimeter of $QRSTU = 40$. Ratio of perimeters $= 30\sqrt{5}$: 40 or $3\sqrt{5}$:4.

20. Perimeter of small square = 16.

Ratio of perimeters = 36:16 = 9:4.

Ratio of areas $= 9^2:4^2 = 81:16$.

21. The ratio of areas is 90:25. The ratio of perimeters is $\sqrt{90}:\sqrt{25} = 3\sqrt{10}:5$.

22. *Sample answer:* Given two rectangles which are similar with the lengths of corresponding sides in the ratio $a:b$. If the length of rectangle 1 is L and the width of the rectangle 2 is W then the length of rectangle 1 $= \frac{a}{b}L$ and the width would be $\frac{a}{b}W$. The area of rectangle 2 $= LW$ and the area of rectangle 1 is $\frac{a^2}{b^2}LW$. Therefore, (Area rectangle 1/Area rectangle 2) $= \frac{a^2}{b^2}$.

23. Area small rug = 29 in. × 47 in. = 1363 sq in.

Area large rug = 58 in. × 94 in. = 5452 sq in.

$\frac{\text{Area small}}{\text{Area large}} = \frac{1363}{5452}$ or 1:4

24. Small rug: $\frac{\$79}{1363 \text{ sq in.}} = \$.06$ sq in.

Large rug: $\frac{\$299}{5452 \text{ sq in.}} = \$.05$ sq in.

The large rug is a good buy because it costs slightly less per square inch than a small rug.

25. $A = \frac{1}{2}bh$

$= \frac{1}{2}(40)(41)$

$= 820 \text{ ft}^2$

26. $AB = 1.3 \times 40$ ft

$= 52$ ft

27. Area $\triangle ABC = (1.3)^2(820 \text{ ft}^2)$

$= 1385.8 \text{ ft}^2$

Area walkway = Area of $\triangle ABC$ − Area of $\triangle DEF$

$= 1385.8 - 820$

$= 565.8 \text{ ft}^2$

28. $\frac{\text{Area outer}}{\text{Area inner}} = \frac{466{,}170}{446{,}400} \approx \frac{1.04}{1}$

Ratio of perimeters $= \frac{\sqrt{1.04}}{\sqrt{1}} \approx \frac{1.02}{1}$

$\frac{1.02}{1} = \frac{477}{x}$

$1.02x = 477$

$x = \frac{4.77}{1.02}$

$= 467$ ft

29. **a.** $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{15^2}{2^2} = \frac{225}{4}$

b. $\frac{25x}{x-5} = \frac{225}{4}$

$100x = 225(x - 5)$

$100x = 225x - 1125$

$-125x = -1125$

$x = 9$

c. 15:2

d. $\frac{8 + y}{3y - 19} = \frac{15}{2}$

$2(8 + y) = 15(3y - 19)$

$16 + 2y = 45y - 285$

$301 = 43y$

$7 = y$

e. Because $\overline{AB}$ and $\overline{DE}$ are corresponding sides of $\sim\triangle$, the ratio of their lengths is the sclae factor. You can solve the proportion $\frac{22.5}{13z - 10} = \frac{15}{z}$ to find z.

30. *Sample answer:*

$\triangle PVQ \sim \triangle RVT$

$\triangle RVQ \sim \triangle PVU$

$\triangle TQR \sim \triangle TUS$

31. $\frac{PV}{RV} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\frac{3}{5} = \frac{PV}{10}$

$PV = 6$

32. $\frac{3}{5} = \frac{VQ}{VT}$

$\frac{3}{5} = \frac{VQ}{15}$

$45 = 5VQ$

$9 = VQ$

$\frac{3}{5} = \frac{VQ}{VT}$

33. *Sample answer:*

$\frac{\text{Area } \triangle PVQ}{\text{Area } \triangle RVT} = \frac{9}{25}$

$\frac{\text{Area } \triangle RVQ}{\text{Area } \triangle PVU} = \frac{45}{16.2} = \frac{25}{9}$

$\frac{\text{Area } \triangle TQR}{\text{Area } \triangle TUS} = \frac{120}{19.2} = \frac{25}{4}$

—CONTINUED—

32. —CONTINUED—

$\frac{VU}{VQ} = \frac{3}{5}$

$\frac{VU}{9} = \frac{3}{5}$

$5VU = 27$

$VU = \frac{27}{5}$

$= 5\frac{2}{5}$

$UT = 15 - 5\frac{2}{5} = 9\frac{3}{5}$

11.3 Mixed Review (p. 681)

34. 80° **35.** 145° **36.** 145° **37.** 215°

38. $10x + 8x = 180°$

$18x = 180°$

$x = 10°$

$17y + 19y = 180°$

$36y = 180°$

$y = 5°$

$\angle R = 100°, \angle S = 85°, \angle T = 80°, \angle U = 95°$

39. $m\angle 1 = \frac{1}{2}(160°)$

$= 80°$

40. $m\angle 1 = \frac{1}{2}(110° + 50°)$

$= 80°$

41. $m\angle 1 = \frac{1}{2}(126° - 40°)$

$= \frac{1}{2}(86°)$

$= 43°$

Quiz 1 (p. 682)

1. $(20 - 2)(180°) = 3240°$ **2.** $\frac{360°}{25} = 14.4°$

3. $A = \frac{1}{4}\sqrt{3}\, s^2$

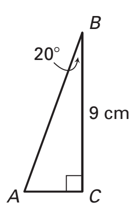
$= \frac{1}{4}\sqrt{3}\,(17)^2$

$\approx 125.1 \text{ in.}^2$

4. The central angle of a nonagon is 40°. The apothem bisects the central angle.

$\frac{9 \text{ cm}}{AB} = \cos 20°$ $\qquad \frac{AC}{9} = \tan 20°$

$AB \approx 9.58 \text{ cm}$ $\qquad AC \approx 3.28$



$A = \frac{1}{2}AP$

$= \frac{1}{2}(9)(9 \tan 20°)(2)(9)$

$= 265.33 \text{ cm}^2$

5. $\frac{P_{red}}{P_{blue}} = \frac{8}{6} = \frac{4}{3}$

$\frac{A_{red}}{A_{blue}} = \frac{16}{9}$

6. $\frac{P_{red}}{P_{blue}} = \frac{3.25}{5} = \frac{13}{20}$

$\frac{A_{red}}{A_{blue}} = \frac{169}{400}$

7. The scale factor is 3:7; therefore the ratios of the area is $\frac{9}{49}$.

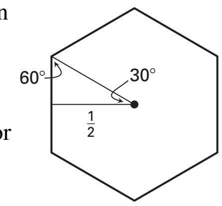
$\frac{9}{49} = \frac{480}{x}$

$9x = 23{,}520$

$x = \$2613$

11.3 Math & History (p. 682)

1. Each side of the inscribed hexagon will be equal to $\frac{1}{2}$ (diameter). So, the perimeter of the inscribed hexagon is equal to 3 · diameter or 3 units. If the apothem is $\frac{1}{2}$ unit, then the length of one side of the hexagon is $2 \cdot \frac{1}{2} \tan 30° = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{3}$ or $\frac{\sqrt{3}}{3}$. The perimeter of the circumscribed hexagon is $6 \cdot \left(\sqrt{3}/3\right) = 2\sqrt{3}$ so $3 < \pi < 3.46$.



Lesson 11.4

11.4 Guided Practice (p. 686)

1. Arc measure is the number of degrees of the angle whose endpoints are the ends of the arc. Arc length the measure of a portion of the circumference.

2. Arc length $\widehat{AB} = \frac{m\angle 1}{360°} \cdot 2\pi r = \frac{m\angle 2}{360°} \cdot 2\pi r =$ arc length $\widehat{CD}$.

3. F **4.** D **5.** C **6.** B **7.** A **8.** E

9. False. Arc length depends on the radius of the circle. So if the radii are different, the arc length will be different.

10. False. The circumference is proportional to the radius. If the radius doubles, the circumference doubles.

11. False. Arc length is determined by the measure of the arc and the radius of the circle. If you solve for the measure of the arc, then it is dependent upon both the arc length and the radius of the circle.

12. Length $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$= \frac{140°}{360°} \cdot 2\pi \cdot 29.5$

$\approx 22.9\pi$ cm or 72.1 cm

13. $\text{Length } \overset{\frown}{CD} = \frac{m\overset{\frown}{CD}}{360°} \cdot 2\pi r$

$= \frac{160°}{360°} \cdot 2 \cdot \pi \cdot 29$

$\approx 25.8\,\pi \text{ cm or } 81.0 \text{ cm}$

14. $\text{Length } \overset{\frown}{EF} = \frac{m\overset{\frown}{EF}}{360°} \cdot 2\pi r$

$67.6 \text{ cm} = \frac{m\overset{\frown}{EF}}{360°} \cdot 2\pi(25)$

$\frac{(360°)(67.6 \text{ cm})}{50\pi} = m\overset{\frown}{EF}$

$155° \approx m\overset{\frown}{EF}$

11.4 Practice and Applications (pp. 686-689)

15. $C = 2\pi r$

$= 2\pi(5)$

$= 10\pi$

$\approx 31.4 \text{ in.}$

16. $C = 2\pi r$

$44 = 2\pi r$

$\frac{44}{2\pi} = r$

$7.0 \text{ ft} \approx r$

17. $C = \pi d$

$= \pi(8)$

$\approx 25.1 \text{ m}$

18. $C = 2\pi r$

$= 2\pi(15)$

$= 30\pi \text{ in.}$

$\approx 94.2 \text{ in.}$

19. $C = 2\pi r$

$32 = 2\pi r$

$\frac{32}{2\pi} \approx r$

$5.1 \text{ yd} \approx r$

20. $\text{Length } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$= \frac{45°}{360°} \cdot 2\pi(3)$

$\approx 2.36 \text{ cm}$

21. $\text{Length } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$= \frac{60°}{360°} \cdot 2\pi(7)$

$\approx 7.33 \text{ in.}$

22. $\text{Length } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$= \frac{120°}{360°} \cdot 2\pi(10)$

$= 20.9 \text{ ft}$

23.

Radius	12	3	0.6	3.5	5.1	$3\sqrt{3}$
$m\overset{\frown}{AB}$	45°	30°	120°	192°	90°	107°
Length of $m\overset{\frown}{AB}$	3π	0.5π	0.4π	3.73π	2.55π	3.09π

24. $\text{Length of } \overset{\frown}{XY} = \frac{m\overset{\frown}{XY}}{360°} \cdot 2\pi r$

$= \frac{30°}{360°} \cdot 2\pi(8)$

$\approx 4.2 \text{ units}$

25. $\text{Length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$5.5 = \frac{55°}{360°} \cdot 2\pi r$

$5.5\left(\frac{360°}{55°}\right) = 2\pi r$

$36 \text{ units} = 2\pi r$

26. $\text{Length of } \overset{\frown}{CD} = \frac{m\overset{\frown}{CD}}{360°} \cdot 2\pi r$

$20 = \frac{160°}{360°} \cdot 2\pi r$

$20 = 2.79r$

$7.17 \text{ units} \approx r$

27. $\text{Length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$

$= \frac{118°}{360°} \cdot 2(10.14)$

$\approx 20.88 \text{ units}$

28. $\text{Length of } \overset{\frown}{ST} = \frac{m\overset{\frown}{ST}}{360°} \cdot 2\pi r$

$12.4 = \frac{84°}{360°} \cdot 2\pi r$

$53.14 \text{ units} = 2\pi r$

29. $\text{Length of } \overset{\frown}{LM} = \frac{m\overset{\frown}{LM}}{360°} \cdot 2\pi r$

$42.56 = \frac{240°}{360°} \cdot 2\pi r$

$42.56 = 4.19r$

$10.16 \text{ units} \approx r$

30. $P = 2(l) + 2\pi r$
$= 2(12) + 2\pi(2.5)$
≈ 45.99 units

31. $P = 5 + 5 + 5 + 2\pi(2)$
$= 15 + 15.71$
$= 30.71$ units

32. $P = 4(2) + 2\pi(2)$
$= 8 + 12.57$
$= 20.57$ units

33. $2x + 15° = 135°$
$2x = 120°$
$x = 60°$

$$\text{Length of arc} = \frac{m \text{ arc}}{360°} \cdot 2\pi r$$

$$(y - 3)\pi = \frac{135°}{360°} \cdot 2\pi(8)$$

$y - 3 = 6$
$y = 9$

34. $15y - 30° = 270°$
$15y = 300°$
$y = 20$

$$\text{Length of arc} = \frac{m \text{ arc}}{360°} \cdot 2\pi r$$

$$(13x + 2)\pi = \frac{270°}{360°} \cdot 2\pi(10)$$

$13x + 2 = 15$
$13x = 13$
$x = 1$

35. $18x = 45°$
$x = 2.5$

$$\text{Length of arc} = \frac{m \text{ arc}}{360°} \cdot 2\pi r$$

$$(14y - 3)\pi = \frac{45°}{360°} \cdot 2\pi(7)$$

$14y - 3 = 1.75$
$14y = 4.75$
$y \approx 0.34$ units

36. $r = 3$
$C = 2\pi r$
$C = 6\pi$

37. $r = 2\sqrt{7}$
$C = 2\pi r$
$C = 4\pi\sqrt{7}$

38. $r = 2$
$C = 2\pi r$
$C = 4\pi$

39. Tire A = 15 in. + 4.6 in. + 4.6 in.
= 24.2 in.
Tire B = 16 in. + 4.43 in. + 4.47 in.
= 24.86 in.
Tire A = 17 in. + 4.33 in. + 4.33 in.
= 25.66 in.

40. 500 ft = 6000 in.

Tire A: $\dfrac{6000 \text{ in.}}{\pi(24.2 \text{ in.})} \approx 79$ rev

Tire B: $\dfrac{6000 \text{ in.}}{\pi(24.86 \text{ in.})} \approx 77$ rev

Tire C: $\dfrac{6000 \text{ in.}}{\pi(25.66 \text{ in.})} \approx 74$ rev

41. The sidewall width must be added twice to the rim diameter to get the tire diameter.

42. L = straightaways + curves

$$L = 30 + 8 + 12 + 12 + 30 + 20 + 6\left(\frac{1}{4}\right)(2\pi)(3) + \frac{1}{2}(2\pi)(3) + 2\left(\frac{1}{2}\right)(2\pi)(2.25)$$

$= 100 + 2\pi(8.25)$
$= 163.84$ m

43. $\dfrac{1609 \text{ m}}{163 \text{ m/lap}} = 9.8$ laps

44. Arc = 4.2°, radius = 4000 mi

$$\text{Length of arc} = \frac{m \text{ Arc}}{360°} \cdot 2\pi r$$

$$= \frac{4.2°}{360°} \cdot 2\pi(4000)$$

≈ 293.22 miles

45. $\dfrac{\text{Length of arc}}{\text{rear sprocket}} = \dfrac{164°}{360°} \cdot 8$

≈ 3.64 in.

$\dfrac{\text{Length of arc}}{\text{front sprocket}} = \dfrac{196°}{360°} \cdot 22$

≈ 11.98 in.

Length of chain = 3.64 + 16 + 11.98 + 16
= 47.62 in.

46. number of teeth = 47.62 in. ÷ (0.5 in./tooth)
≈ 95 teeth

47. $P = \dfrac{3}{4}(2\pi r)$

$= \dfrac{3}{4}(2\pi)(8)$

≈ 37.7 ft

48. B. Length of $\widehat{XZ} = \dfrac{m\widehat{XZ}}{360°} \cdot 2\pi r$

$= \dfrac{60°}{360°} \cdot 2\pi(4)$

$= \dfrac{4}{3}\pi$ units

49. B. $x = \dfrac{2(360°)}{2\pi r}$ $\quad y = \dfrac{360°}{2\pi r}$

$\dfrac{x}{y} = \dfrac{\frac{2(360°)}{2\pi r}}{\frac{360°}{2\pi r}}$

$= \dfrac{2}{1}$

50. The four semicircles form 2 complete circles so that the length would be $2(2\pi r) = 4\pi r$.

51. 8 segments $= 8\pi r$

16 segments $= 16\pi r$

n segements $= n\pi r$

No. The number of segments doesn't matter.

11.4 Mixed Review (p. 689)

52. $A = \pi r^2$

$= \pi(9)^2$

≈ 254.47 ft^2

53. $A = \pi r^2$

$= \pi(3.3)^2$

≈ 34.21 in.^{2}

54. $A = \pi r^2$

$= \pi\left(\dfrac{27}{5}\right)^2$

$= 91.61$ cm^2

55. $A = \pi\left(4\sqrt{11}\right)^2$

$= 552.92$ m^2

56. $y = mx + b$

$-2 = \dfrac{2}{3}(9) + b$

$-2 = 6 + b$

$-8 = b$

$y = \dfrac{2}{3}x - 8$

57. $\dfrac{x}{3.5} = \dfrac{5}{6}$

$6x = 17.5$

$x = 2.92$

58. $\dfrac{30}{15} = \dfrac{y}{18}$

$540 = 15y$

$36 = y$

59. 96° **60.** 176° **61.** 258° **62.** 31°

Technology Activity 11.4 (p. 690)

1. The perimeter increases as n increases, getting closer and closer to 2π.

2. $n = 6;\ p = 2(6)\sin\left(\dfrac{180°}{6}\right)$

$= 12 \sin 30°$

$= 6$

3. 12-gon: $P = 6.21$ units

15-gon: $P = 6.24$ units

18-gon: $P = 6.25$ units

24-gon: $P = 6.27$ units

Lesson 11.5

11.5 Guided Practice (p. 695)

1. A sector of a circle is bounded by two radii of the circle and their intercepted arc.

2. The radius is one half of the diameter or $\dfrac{1}{2} \cdot 4$. The area is $\pi r^2 = \pi\left(\dfrac{1}{2} \cdot 4\right)^2$.

3. $A = \pi r^2$

$= \pi(9)^2$

≈ 254.47 in.^{2}

4. $A = \pi r^2$

$= \pi(3.8)^2$

≈ 45.36 cm^2

5. $A = \pi r^2$

$= \pi\left(\dfrac{1}{2} \cdot 6\right)^2$

≈ 28.27 ft^2

6. $A = \dfrac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \dfrac{110°}{360°} \cdot \pi(6)^2$

≈ 34.56 ft^2

7. $A = \dfrac{m\widehat{AB}}{360°} \cdot \pi r^2$

$= \dfrac{70°}{360°} \cdot \pi(10)^2$

≈ 61.09 m^2

8. $A =$ Area of circle $-$ Area of sector

$= \pi(3)^2 - \left(\dfrac{60°}{360°}\right) \cdot \pi(3)^2$

$= 28.27 - 4.71$

≈ 23.56 in.^{2}

9. $A = \dfrac{45°}{360°}\pi(8)^2$

≈ 25.13 in.^{2}

11.5 Practice and Applications (p. 695–698)

10. $A = \pi(31)^2$

≈ 3019.07 ft^2

11. $A = \pi(0.4)^2$

$\approx .50$ cm^2

12. $A = \pi(4)^2$

≈ 50.27 m^2

13. $A = \pi(10)^2$

≈ 314.16 in.^{2}

14. $A = \dfrac{60°}{360°} \cdot \pi(11)^2$

≈ 63.36 ft^2

15. $A = \dfrac{80°}{360°} \cdot \pi\left(\dfrac{7}{2}\right)^2$

≈ 8.55 in.^{2}

16. $A = \dfrac{293°}{360°} \cdot \pi(10)^2$

≈ 255.69 cm^2

17. $A = \dfrac{120°}{360°} \cdot \pi(4.6)^2$

≈ 22.16 m^2

18. $A = \frac{250°}{360°} \cdot \pi(8)^2$

$\approx 139.63 \text{ in.}^2$

19. $A = \pi(10)^2$

$\approx 314.16 \text{ ft}^2$

20. $50 = \pi(r^2)$

$15.92 = r^2$

$3.99 \text{ m} \approx r$

21. $A = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

$59 = \frac{40°}{360°} \cdot \pi r^2$

$59 = 0.35r^2$

$169.02 = r^2$

$13 \text{ in.} \approx r$

22. $A = \frac{m \text{ arc}}{360°} \cdot \pi\left(\frac{1}{2}d\right)^2$

$277 = \frac{72°}{360°} \cdot \pi\left(\frac{1}{4}\right)d^2$

$277 = 0.157d^2$

$1763.44 = d^2$

$42 \text{ m} \approx d$

23. $A =$ Area large circle $-$ Area small circle

$= \pi(24)^2 - \pi(6)^2$

$\approx 1696.46 \text{ m}^2$

24. $A =$ Area large semicircle $-$ Area small semicircle

$= \frac{1}{2}\pi(38)^2 - \frac{1}{2}\pi(19)^2$

$\approx 1701.17 \text{ cm}^2$

25. $A =$ Area of circle $-$ Area of pentagon

$= \pi(4)^2 - \frac{1}{2}(4 \sin 54°)(2)(5)(4 \cos 54°)$

$\approx 12.22 \text{ ft}^2$

26. Area $= \pi(3)^2 - \pi(2)^2$

$= \pi(9 - 4)$

$= 15.7 \text{ ft}^2$

27. $A =$ Area square $-$ Area circles

$= (18)^2 - 4\pi(4.5)^2$

$= 324 - 254.47$

$= 69.53 \text{ in.}^2$

28. $A =$ Area semicircle $-$ Area triangle

$= \frac{1}{2}\pi(2)^2 - \frac{1}{4}\sqrt{3}(2)^2$

$= 6.28 - 1.73$

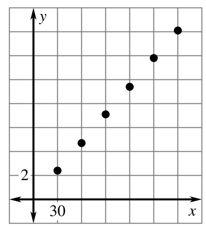
$= 4.55 \text{ cm}^2$

29.

Measure of arc, x	30°	60°	90°
Area of corresponding sector y	2.36	4.71	7.07

Measure of arc, x	120°	150°	180°
Area of corresponding sector y	9.42	11.78	14.14

30.



31. Yes it appears to be linear. The equation would be

$y = \frac{\pi}{40}x.$

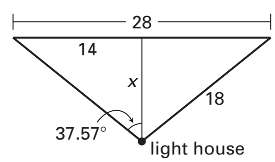
32. With a 5-inch radius, the area of the sectors would be larger. Howerver the relationship would still be linear.

$y = \frac{5\pi}{72}x.$

33. $A = \frac{245°}{360°} \cdot \pi(18)^2$

$= 692.72 \text{ mi}^2$

34.



$14^2 + x^2 = 18^2$

$196 + x^2 = 324$

$x^2 = 128$

$x = 11.31$ miles

35. $A =$ Area of sector $-$ Area of triangle

$= \frac{60}{360} \cdot \pi(6)^2 - \frac{1}{2}(2)(3)(3\sqrt{3})$

$\approx 18.85 - 15.59$

$= 3.26 \text{ cm}^2$

36. $A =$ Area of sector $-$ Area of triangle

$= \frac{90°}{360°} \cdot \pi(14)^2 - \frac{1}{2}(2)(7\sqrt{2})(7\sqrt{2})$

$\approx 153.94 - 98$

$= 55.94 \text{ m}^2$

37. $A =$ Area of sector $-$ Area of triangle

$= \frac{120°}{360°} \cdot \pi(48)^2 - \frac{1}{2}(24)(2)(24\sqrt{3})$

$\approx 2412.74 - 997.66$

$= 1415.08 \text{ cm}^2$

38. A = Total area − Area of larger sector

$= \frac{72°}{360°} \cdot \pi(3.5)^2 - \frac{72°}{360°} \cdot \pi(3)^2$

$\approx 7.70 - 5.65$

$= 2.05 \text{ ft}^2$

39. To find the area of the sector, you would need to either know the arc length or the measure of the central angle, and the radius. To find the area of the kite you would have to know the lengths of the diagonals or be given sufficient information to determine the length of the diagonals, such as a side with an angle or two sides.

40. Area $ABCD$ = Area of sector formed by $\angle APB$ − Area of sector formed by $\angle DPC$

$= \frac{45°}{360°} \cdot \pi(4)^2 - \frac{45°}{360°} \cdot \pi(2)^2$

$= 6.28 - 1.57$

$= 4.71 \text{ ft}^2$

41. If you double the radius, the area is quadrupled because it is proportional to the square of the radius. If you double the radius, the circumference doubles also because the circumference is directly proportional to the radius.

42. As n gets larger, the area of the polygon approaches π, the area of the circle.

Sample spreadsheet:

n-gon Data

	A	B
1	# of sides	Area
2	n	.5*cos(180/n)*2*n*sin(180/n)
3	3	1.299038106
4	4	2
5	5	2.377641291
6	6	2.598076211
7	7	2.736410189
8	8	2.828427125
9	9	2.892544244
10	10	2.938926261
11	11	2.973524496
12	12	3
13	13	3.020700618
14	14	3.037186174
15	15	3.050524823
16	16	3.061467459

43. C

$A = \pi(6)^2 - \pi(3)^2$

$= 36\pi - 9\pi$

$= 27\pi$

44. D

$A = \frac{108°}{360°} \cdot \pi(6)^2$

$= 10.8\pi$

45. A = Area of triangle − 3 · Area of sector

$= \frac{1}{4}\sqrt{3}(12)^2 - 3 \cdot \frac{60°}{360°} \cdot \pi(6)^2$

$\approx 62.35 - 56.55$

$= 5.81 \text{ cm}^2$

11.5 Mixed Review (p. 698)

46. $\frac{2}{5}$ **47.** $\frac{3}{16}$ **48.** $\frac{4}{21}$ **49.** $\frac{4}{11}$

50. $15\sqrt{2} \approx 21.21$ units

51. $DB = \frac{18}{\sin 68°}$

≈ 19.4 cm

52. $DC = 18$ cm

53. $m\angle DBC = 68°$

54. $BC = \frac{18}{\tan 68°}$

≈ 7.3 cm

55. $(x + 2)^2 + (y + 7)^2 = 36$

56. $x^2 + (y + 9)^2 = 100$

57. $(x + 4)^2 + (y - 5)^2 = 10.24$

58. $(x - 8)^2 + (y - 2)^2 = 11$

59. $C = 2\pi(12.5)$

≈ 78.54 in.

60. Length of $\widehat{AB} = \frac{53°}{360°} \cdot 2\pi(13)$

≈ 12.03 ft

61. $31.6 = \frac{129°}{360°} \cdot 2\pi(r)$

$31.6 = 2.25r$

$14.04 \approx r$

11.6 Guided Practice

1. A geometric probability involves geometric measures such as length and area instead of counting events or outcomes.

2. Area method should be used because you are looking for something within a region of space.

3. Length method should be used because time can be related to a line.

4. $P = \frac{AB}{AF} = \frac{2}{18} = \frac{1}{9} \approx 11\%$

5. $P = \frac{BD}{AF} = \frac{9}{18} = \frac{1}{2} = 50\%$

6. $P = \frac{BF}{AF} = \frac{16}{18} = \frac{8}{9} = 89\%$

7. $\overline{AF} = \overline{AB} + \overline{BF}$. Therefore any point not on $\overline{AB}$ must be on $\overline{BF}$. So the sum of the probabilities is

$\frac{1}{9} + \frac{8}{9} = 1.$

8. $P = \frac{\text{Area shaded}}{\text{Area total}}$

$= \frac{\frac{1}{2}(5)(4) + \frac{1}{2}(4)(4)}{\frac{1}{2}(7 + 16)(4)}$

$= \frac{18}{46}$

$\approx 39\%$

11.6 Practice and Applications (p. 702–704)

9. $P = \frac{GH}{GN} = \frac{2}{14} = \frac{1}{7} \approx 14\%$

10. $P = \frac{JL}{GN} = \frac{4}{14} = \frac{2}{7} \approx 29\%$

11. $P = \frac{JN}{GN} = \frac{8}{14} = \frac{4}{7} \approx 57\%$

12. $P = \frac{GJ}{GN} = \frac{6}{14} = \frac{3}{7} = 43\%$

13. $P = \frac{PQ}{PU} = \frac{12}{48} = \frac{1}{4} = 25\%$

14. $P = \frac{PS}{PU} = \frac{28}{48} = \frac{7}{12} \approx 58\%$

15. $P = \frac{SU}{PU} = \frac{20}{48} = \frac{5}{12} \approx 42\%$

16. $P = \frac{PU}{PU} = 1 = 100\%$

17. $P = \frac{\text{Area square} - \text{Area circle}}{\text{Area square}}$

$= \frac{144 - 113.1}{144}$

$\approx 21.5\%$

18. $P = \frac{\text{Area circle}}{\text{Area square} + \text{Area circle}}$

$= \frac{50.27}{64 + 50.27}$

$\approx 44\%$

19. $P = \frac{\text{Area triangle}}{\text{Area rectangle}}$

$= \frac{\frac{1}{2}(16 \cdot 5)}{160}$

$= \frac{40}{160}$

$= 25\%$

20. $P = \frac{\text{Area of semicircle}}{\text{Area of triangle}}$

$= \frac{\frac{1}{2}\pi(5)^2}{\frac{1}{2}(20)(10)}$

$= \frac{39.27}{100}$

$= 39.27\%$

21. $P = \frac{\text{Area of circle}}{\text{Area of hexagon}}$

$= \frac{\pi(1.5)^2}{6\left(\frac{1}{4}\right)\left(\sqrt{3}\right)(14)^2}$

$= \frac{7.07}{509.22}$

$\approx 1.39\%$

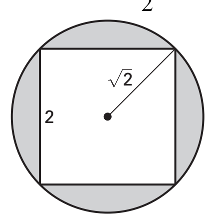
22. Since the probability is between 1 and 2%, 1 to 2 darts would probably hit the bulls-eye.

23. $P = \frac{28.28}{509.22}$

$\approx 5.55\%$

24. 

Let L be the midpoint of JM and N be the midpoint of MK. Then LN is the part of the line closer to M than to J or K, so $P = \frac{1}{2} = 50\%$.

25. 

Area of square $= 4$

Area of circle $= \pi\left(\sqrt{2}\right)^2 = 6.28$

Area shaded $= 6.28 - 4 = 2.28$ units^2

$P = \frac{\text{Area shaded}}{\text{Area of circle}} = \frac{2.28}{6.28} = 36.31\%$

26. $P = \frac{3.5}{7.5} \approx 46.7\%$

27. $P = \frac{25}{60} \approx 41.7\%$ **28.** $P = \frac{3}{15} = 20\%$

29. $A = (2000)(5000) = 10{,}000{,}000 \text{ yd}^2$

30. Area of circle $= \pi\left(\frac{500}{3}\right)^2 = 87{,}266.4 \text{ yd}^2$

$P = \frac{87{,}266.46}{10{,}000{,}000} \approx 0.87\%$

31. Area Total $= \pi(24)^2 = 1809.58$

Area of 10 point region $= \pi(2.4)^2 = 18.1$

$P = \frac{18.1}{1809.58} \approx 1\%$

32. Area yellow region $= \pi(4.8)^2 = 72.38$

$P = \frac{72.38}{1809.58} \approx 4\%$

33. Area white region $=$ Total area $-$ Area regions (3–10)

$= 1809.58 - \pi(19.2)^2$

$= 651.46$

$P = \frac{651.46}{1809.58} \approx 36\%$

34. Area regions 5–10 $= \pi(14.4)^2 = 651.44$

Area regions 6–10 $= \pi(12)^2 = 452.39$

Area regions 1–4 $= 1809.58 - 651.44$

$= 1158.14$

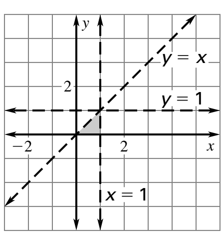
Area region 5 $= 1809.58 - 452.39 - 1158.14$

$= 199.05$

$P = \frac{199.05}{1809.58} \approx 11\%$

35. *Sample answer:* It would not hold because an expert archer is more likely to hit the bull's eye, so it is no longer true that every point on the target has an equal probability of being hit.

36.



Total Area $= 1$

$P = \frac{1}{2} = 50\%$

37. $\frac{1}{3}(360°) = 120° \div 2 = 60°$

38. $\frac{1}{4}(360°) = 90° \div 2 = 45°$

39. $\frac{1}{6}(360°) = 60° \div 2 = 30°$

40. $P = \frac{(15)^2}{(200)(250)} = \frac{225}{50{,}000} = 0.45\%$

41. $P = \frac{450}{50{,}000} = 0.9\%$

If the area is doubled, the probability is doubled.

42. $P = \frac{900}{50{,}000} = 1.8\%$

The probability is four times as much.

43. **a.** Area of jar $= \pi(12.5)^2 \approx 490.87 \text{ cm}^2$

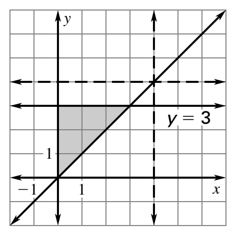
Area of dish $= \pi(2.5)^2 \approx 19.63 \text{ cm}^2$

$P = \frac{19.63}{490.87} = 4\%$

b. 25 coins **c.** $\frac{(250)(5)}{25} = 50$ prizes

d. Yes, the probability will change because now pennies just have to touch the circle, not be inside it so the target area is larger and the probability increases.

44.



$P = \frac{4.5}{16} = \frac{9}{32} \approx 28\%$

11.6 Mixed Review (p.705)

45. $\overleftrightarrow{AB}$ is not tangent to $\odot C$ because ABC is not a right $\triangle$ as follows.

$10^2 + 4^2 \neq 11^2$

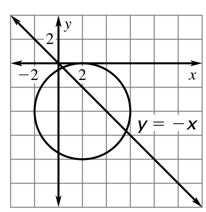
46. $\overleftrightarrow{AB}$ is tangent to $\odot C$. ABC is a right $\triangle$ as follows.

$5^2 + 12^2 = 13^2$

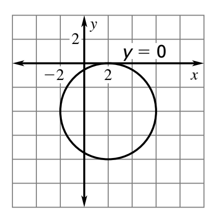
47. $\overleftrightarrow{AB}$ is tangent to $\odot C$. ABC is a right $\triangle$ as follows.

$24^2 + 7^2 = 25^2$

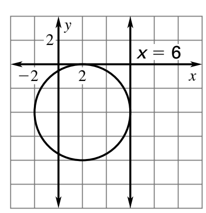
48. secant



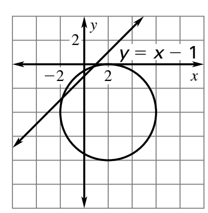
49. tangent



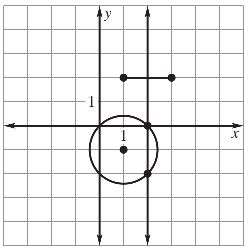
50. tangent



51. secant



52. The locus of points a distance of $\sqrt{2}$ from $(1, -1)$ is a circle. The locus of all points equidistant from $(3, 2)$ and $(1, 2)$ is the $\perp$ bisector. The $\perp$ bisector and circle intersect at 2 points, $(2, 0)$ and $(2, -2)$. The locus is all points $(2, y)$ where $-2 < y < 0$.



11.6 Quiz 2 (p.705)

1. $8.2 = \frac{68°}{360°} \cdot 2\pi r$

$43.41 \text{ m} = 2\pi r$

2. Length of $\widehat{AB} = \frac{88°}{360°} \cdot 26\pi$

≈ 19.97 in.

3. $24.6 \text{ ft} = \frac{138°}{360°} \cdot 2\pi r$

$24.6 \text{ ft}\left(\frac{360°}{138°}\right) \cdot \left(\frac{1}{2\pi}\right) = r$

$10.21 \text{ ft} \approx r$

4. $A = \pi r^2$

$= \pi(50)^2$

$\approx 7854.0 \text{ mi}^2$

5. $A = \frac{105°}{360°} \cdot \pi(7)^2$

$\approx 44.9 \text{ cm}^2$

6. $A = 2\left(\frac{145°}{360°}\right) \cdot \pi(10 \text{ ft})^2$

$\approx 253.07 \text{ ft}^2$

7. Area of triangle $= \frac{1}{4}\sqrt{3} \cdot 5^2 \approx 10.83$

Area of square $= (20)^2 = 400$

$P = \frac{10.83}{400}$

$\approx 2.7\%$

11.6 Technology Activity (p.706)

1. This is the equation for the region inside the circle centered at the origin with radius 5.

2. Answers may vary. *Sample answer:* Experimental probability is approximately 27.5% with the graphing calculator program and theoretical probability is 19.6%.

3. Answers may vary.

4. The more trials there are, the closer the theoretical and experimental probabilities will be to each other.

Chapter 11 Review ***(p. 708–710)***

1. measure of interior $\angle = \frac{(9-2)(180°)}{9} = 140°$

measure of exterior $\angle = 180° - 140° = 40°$

2. measure of interior $\angle = \frac{(13-2)(180°)}{13} \approx 152.3°$

measure of exterior $\angle = 180° - 152.3° = 27.7°$

3. measure of interior $\angle = \frac{(16-2)(180°)}{16} = 157.5°$

measure of exterior $\angle = 180° - 157.5° = 22.5°$

4. measure of interior $\angle = \frac{(24-2)(180°)}{24} = 165°$

measure of exterior $\angle = 180° - 165° = 15°$

5. $\frac{(n-2)(180°)}{n} = 172°$

$180°n - 360° = 172°n$

$8°n = 360°$

$n = 45$

6. $\frac{(n-2)(180°)}{n} = 135°$

$180°n - 360° = 135°n$

$45°n = 360°$

$n = 8$

7. $\frac{(n-2)(180°)}{n} = 150°$

$180°n - 360° = 150°n$

$30°n = 360°$

$n = 12$

8. $\frac{(n-2)(180°)}{n} = 170°$

$180°n - 360° = 170°n$

$10°n = 360°$

$n = 36$

9. $A = \frac{1}{4}\sqrt{3}s^2$

$= \frac{1}{4}\sqrt{3}(12)^2$

$\approx 62.35 \text{ cm}^2$

10. $S = \frac{6}{\tan 60°}$

$= 4\sqrt{3}$

$A = \frac{1}{4}\sqrt{3}s^2$

$= \frac{1}{4}\sqrt{3}(4\sqrt{3})^2$

$= 20.78 \text{ in.}^2$

11. $A = 6 \cdot \frac{1}{4}\sqrt{3}\ 5^2$

$= 6 \cdot \frac{1}{4}\sqrt{3}(5)^2$

$= 64.95 \text{ m}^2$

12. Measure of interior angle $= \frac{(10-2)180}{10} = 144°$

$a = 0.75 \tan 72°$

$A = \frac{1}{2}aP$

$= \frac{1}{2}(0.75 \tan 72°)(10)(1.5)$

$= 17.31 \text{ ft}^2$

13. sometimes **14.** sometimes **15.** always

16. $\frac{\text{Perimeter } CDE}{\text{Perimeter } BDF} = \frac{3}{9} = \frac{1}{3}$

$\frac{\text{Area } CDE}{\text{Area } BDF} = \frac{1^2}{3^2} = \frac{1}{9}$

17. $\dfrac{\text{Perimeter } ADG}{\text{Perimeter } BDF} = \dfrac{15}{9} = \dfrac{5}{3}$

$\dfrac{\text{Area } ADG}{\text{Area } BDF} = \dfrac{5^2}{3^2} = \dfrac{25}{9}$

18. $C = 2\pi r$

$= 2\pi(4)$

≈ 25.13 cm

Length of $\overset{\frown}{AB} = \dfrac{35°}{360°} \cdot 2\pi(4)$

≈ 2.44 cm

19. $C = 2\pi r$

$= 2\pi(7.6)$

≈ 47.75 m

Length of $\overset{\frown}{AB} = \dfrac{162°}{360°} \cdot 2\pi(7.6)$

≈ 21.49 m

20. $C = 2\pi r$

$= 2\pi(12)$

≈ 75.4 ft

Length of $\overset{\frown}{AB} = \dfrac{118°}{360°} \cdot 2\pi(12)$

≈ 24.71 ft

21. $r = \dfrac{C}{2\pi}$

$= \dfrac{12}{2\pi}$

≈ 1.91 in.

22. $d = \dfrac{C}{\pi}$

$= \dfrac{15\pi}{\pi}$

$= 15$ m

23. $A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(10)^2$

$\approx 157.08 \text{ in}^2$

24. $A = \dfrac{135°}{360°} \cdot \pi(5.5)^2$

$\approx 35.64 \text{ ft}^2$

25. $A = \pi(15)^2 - \pi(8)^2$

$\approx 505.8 \text{ cm}^2$

26. $A = \dfrac{300°}{360°} \cdot \pi(6)^2$

$\approx 94.25 \text{ cm}^2$

27. $A = \pi(14)^2$

$\approx 615.75 \text{ ft}^2$

28. $A = \pi r^2$

$\sqrt{(A/\pi)} = r$

$\sqrt{40/\pi} = r$

3.57 in. $= r$

29. $P = \dfrac{\text{Length of } \overline{LM}}{\text{Length of } \overline{JN}} = \dfrac{12}{40} = \dfrac{3}{10}$

30. $P = \dfrac{\text{Length of } \overline{JL}}{\text{Length of } \overline{JN}} = \dfrac{20}{40} = \dfrac{1}{2}$

31. $P = \dfrac{\text{Length of } \overline{KM}}{\text{Length of } \overline{JN}} = \dfrac{24}{40} = \dfrac{3}{5}$

32. Radius of a circle $= \frac{1}{2}\sqrt{8^2 + 6^2} = 5$

$P = \dfrac{\text{Area of rectangle}}{\text{Area of circle}}$

$= \dfrac{(8)(6)}{\pi(5)^2}$

$\approx \dfrac{48}{78.54}$

$= 0.611$ or 61.1%

33. $P = \dfrac{\text{Area shaded}}{\text{Area of semicircle}}$

$= \dfrac{2\pi {r_1}^2}{\frac{1}{2}\pi {r_2}^2}$

$= \dfrac{2\pi(6)^2}{\frac{1}{2}\pi(24)^2}$

$= \frac{1}{4}$

$= 25\%$

34. $P = \dfrac{\text{Area shaded}}{\text{Total area}}$

$= \dfrac{(7\sqrt{2})^2}{(14)^2}$

$= \dfrac{98}{196}$

$= \dfrac{1}{2}$

$= 50\%$

Chapter 11 Test *(p. 711)*

1. $x + 120° + 135° + 90° + 115° + 120° = (6 - 2)(180°)$

$x + 580° = 720°$

$x = 140°$

2. $40° + 60° + 45° + 90° + 65° + 60° = 360°$

3. measure of interior $\angle = \dfrac{(n - 2)(180°)}{n}$

$= \dfrac{(30 - 2)(180°)}{30}$

$= 168°$

4. exterior $\angle = \dfrac{360°}{27}$

$= 13\frac{1}{3}°$

5. $A = \dfrac{1}{4}\sqrt{3}s^2$

$= \dfrac{1}{4}\sqrt{3}(10)^2$

$\approx 43.30 \text{ ft}^2$

6. Measure of interior angle $= \dfrac{(5 - 2)(180°)}{5} = 108°$

$A = \dfrac{1}{2}a\,P$

$= \dfrac{1}{2}(8)(5)(2)\dfrac{8}{\tan 54°}$

$= 232.49 \text{ in.}^2$

7. $A = 6 \cdot \dfrac{1}{4}\sqrt{3}(9)^2$

$= 210.44 \text{ cm}^2$

8. Measure of interior angle $= \dfrac{(9 - 2)(180°)}{9} = 140°$

$A = \dfrac{1}{2}a\,P$

$= \dfrac{1}{2}(\cos 70°)(9)(2)(\sin 70°)$

$= 2.89 \text{ m}^2$

9. $\dfrac{\text{Perimeter } ABCD}{\text{Perimeter } EFGH} = \dfrac{8}{6} = \dfrac{4}{3}$

$\dfrac{\text{Area of } ABCD}{\text{Area of } EFGH} = \dfrac{4^2}{3^2} = \dfrac{16}{9}$

10. $\dfrac{56}{x} = \dfrac{16}{9}$

$16x = 504$

$x = 31.5 \text{ cm}^2$

11. $C = 2\pi r$

$= 2\pi(5)$

≈ 31.42 cm

$A = \pi r^2$

$= \pi(5)^2$

$\approx 78.54 \text{ cm}^2$

12. Length of $\widehat{AB} = \dfrac{m\,\widehat{AB}}{360°} \times 2\pi r$

$= \dfrac{105°}{360°} \cdot 2\pi(5)$

≈ 9.16 cm

13. Area sector $ARB = \dfrac{m\widehat{ARB}}{360°} \cdot \pi r^2$

$= \dfrac{105°}{360°} \cdot \pi(5)^2$

$\approx 22.91 \text{ cm}^2$

14. $A = \dfrac{1}{2}\pi r^2$

$= \dfrac{1}{2}\pi(15)^2$

$\approx 353.43 \text{ ft}^2$

15. Area of sector $= \dfrac{245°}{360°} \cdot \pi(16)^2$

$\approx 547.34 \text{ in.}^2$

16. Area of sectors $= \dfrac{120°}{360°} \cdot \pi(7)^2$

$\approx 51.31 \text{ m}^2$

17. $P = \dfrac{\text{Area of circle}}{\text{Area of square}}$

$= \dfrac{\pi(10)^2}{(20)^2}$

≈ 0.785 or 78.5%

18. $P = \dfrac{\text{Area of triangle}}{\text{Area of square}}$

$= \dfrac{\frac{1}{2}(10)(10)}{20^2}$

$= \dfrac{50}{400}$

$= \dfrac{1}{8}$ or 12.5%

19. boat $= 2\pi(80) \approx 502.65$ ft

skiier $= 2\pi(110) \approx 691.15$ ft

$691.15 - 502.65 = 188.50$ ft

20. $P = \dfrac{20 \text{ min}}{120 \text{ min}} = \dfrac{1}{6} \approx 16.67\%$

Chapter 11 Standardized Test *(p. 712–713)*

1. C $\quad \dfrac{(n - 2)(180°)}{n} = 160°$

$180°n - 360° = 160°n$

$20°n = 360°$

$n = 18$

2. A $\quad 180° - x = (7 - 2)(180°) - (130° + 128°$

$+ 133° + 139° + 110° + 136°)$

$180° - x = 900° - 776°$

$180° - x = 124°$

$56° = x$

3. A $\quad$ Polygon a: $a = 15 \cos 36° \approx 12.14$

Polygon b: $a = 14 \cos 30° \approx 12.12$

4. A Polygon a: $P = 2(5)(15 \sin 36°) \approx 88.17$ units

Polygon b: $P = 6(14)2(6)(14 \sin 30°) = 84$ units

5. A Polygon a: $A = \frac{1}{2}(12.14)(88.17) = 535.19 \text{ units}^2$

Polygon b: $A = \frac{1}{2}(12.12)(84) = 509.04 \text{ units}^2$

6. A $\dfrac{\text{Area of smaller octagon}}{\text{Area of larger octagon}} = \dfrac{12^2}{18^2} = \dfrac{144}{324} = \dfrac{4}{9}$

7. B Length of $\widehat{AB} = \dfrac{86°}{360°} \cdot 2\pi(8.3)$

≈ 12.46 ft

8. E Area shaded $= \dfrac{(360° - 86°)}{360°} \cdot \pi(8.3)^2$

$= 164.72 \text{ ft}^2$

9. C Area shaded $= \dfrac{(90° + 30°)}{360°} \cdot \pi(8)^2$

$= 67.02 \text{ cm}^2$

10. C $P = \dfrac{\text{Area } \odot P - \text{Area } \odot Q}{\text{Area } \odot P}$

$= \dfrac{\pi(6)^2 - \pi(3)^2}{\pi(6)^2}$

$\approx 75\%$

11. A $P = \dfrac{24}{60} = \dfrac{2}{5} = 0.4$

12. $C = 2\pi(16)$

≈ 100.53 m

$A = \pi(16)^2$

$\approx 804.25 \text{ m}^2$

13. $\dfrac{(8 - 2)(180°)}{8} = 135°$

14. $180° - 135° = 45°$

15. $P = 8(2)(16 \sin 22.5°)$

$\approx 97.97 \text{ m}^2$

$A = \frac{1}{2}(16 \cos 22.5°)(8)(2)(16 \sin 22.5°)$

$= 724.08 \text{ m}^2$

16. Length of $\widehat{AB} = \dfrac{45°}{360°} \cdot 2\pi(16)$

$= 12.57$ m

17. *Sample answer:* One method is to find the area of the circle and from that subtract the area of the octagon. A second method is to find the area of a sector. From that, subtract the area of a triangle. Then multiply by 8.

From problems 12–16, Area $\odot J = 804.25 \text{ m}^2$ and area octagon $= 724.08 \text{ m}^2$

Shaded area $= 804.25 - 724.08 = 80.17 \text{ m}^2$

Area sector $= \dfrac{45°}{360°} \cdot \pi(16)^2 = 100.53 \text{ m}^2$

Area triangle $= \dfrac{1}{2}(2)(16 \sin 22.5°)(16 \cos 22.5°)$

$= 90.51 \text{ m}^2$

Shaded Area $= 8(100.53 - 90.51)^2 = 80.16 \text{ m}^2$

Each method yields the same result.

18. $r = 3$ in.

$C = \pi(6) \approx 18.85$ in.

$A = \pi(3)^2 \approx 28.27 \text{ in.}^2$

19. $P = \dfrac{\text{Area of circle}}{\text{Area of dart board}} = \dfrac{\pi(3)^2}{(36)(24)} \approx 3.27\%$

20. $P = \dfrac{\text{Area of blue region}}{\text{Area of dart board}} = \dfrac{(22)(5)}{(36)(24)} = 12.73\%$

21. $P = \dfrac{\text{area green} + \text{area red} + \text{area blue}}{\text{area dart board}}$

$= \dfrac{\pi(3)^2 + 2\left(\frac{1}{2}\right)(9 + 6)(7) + (22)(5)}{(36)(24)}$

$\approx \dfrac{243.27}{864}$

$= 28.16\%$

22. $P = 71.84\%$

$= 100\% - 28.16\%$ (answer from question 21)

Chapter 11 Project (p. 714)

1. yes **2.** yes

3. No; any two great circles must intersect.

4. *Sample answer:* No. There are no parallel lines in geometry on a sphere.

5. yes

6. Answers may vary. *Sample answer:* 90°, 90°, 90°

7. Answers may vary. The sum should be larger than 180°. *Sample answer:* 270°

8. Answers may vary. *Sample answer:* 90°, 90°, 90°

9. no; yes; yes

10. Answers may vary. The sum of the angles is between 180° and 540°.