#110

P 837

) $f(y) = +y^2 - y - 4 = 0$ → leading term $x^{\textcircled{2}}$ → 2 roots

1

POS : 1 sign changes so 1 pos root

$f(-y) = (-y)^2 - (-y) - 4$

$y^2 + y - 4$

NEG : 1 sign change so 1 neg root

TOTAL	+	−	imaginary
2	1	1.	0

3 $f(+y) = +y^3 - y^2 + 2y + 3$ → leading term y^3 → 3 total roots

pos : 2 sign changes so 2 or 0 pos rt

$f(-y) = (-y)^3 - (-y)^2 + 2(-y) + 3$

$= -y^3 - y^2 - 2y + 3$

neg : 1 sign change → 1 neg Root

total	+	−	i
3	2	1	0
	0	1	2

5 $f(x) = +2x^4 - 3x^3 - x^2 + x + 1$ → leading term $x^{\textcircled{4}}$ → 4 total roots

pos : 2 sign changes → 2 or 0 pos Root

$f(-x) = 2(-x)^4 - 3(-x)^3 - (-x)^2 + (-x) + 1$

$+2x^4 + 3x^3 - x^2 - x + 1$

neg : 2 sign changes → 2 or 0 neg Root

total	+	−	imag.
4	2	0	2
	2	2	0
	0	2	2
	0	0	4

110/2

7 $f(y) = +5y^{④} - y^3 + y^2 + 2y - 4 = 0 \rightarrow$ total $= 4$ roots

POS : 3 sign changes → 3 or 1 pos roots

$f(-y) = 5(-y)^4 - (-y)^3 + (-y)^2 + 2(-y) - 4$

$5y^4 + y^3 + y^2 - 2y - 4$

NEG : 1 sign change → 1 neg root

total	+	—	i
4	3	1	0
	1	1	2

9. $x^3 - 2x^2 + 3x + 5 = 0$

$$\begin{array}{r|rrrr} & 1 & -2 & 3 & 5 \\ 1 & & 1 & -1 & 2 \\ \hline & 1 & \ominus 1 & 2 & 7 \end{array}$$

not all +

$$\begin{array}{r|rrrr} & 1 & -2 & 3 & 5 \\ 2 & & 2 & 0 & 6 \\ \hline & +1 & +0 & +3 & +11 \end{array}$$

↑ 0 can be + or −

since all + → 2 is upperbound

$$\begin{array}{r|rrrr} -1 & 1 & -2 & 3 & 5 \\ & & -1 & 3 & -6 \\ \hline & +1 & -3 & +6 & -1 \end{array}$$

alternating signs

→ −1 is lower bound

$-1 < x < 2$

11 $2x^3 - 3x^2 - 3x + 3 = 0$

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 3 \\ 1 & & 2 & -1 & -4 \\ \hline & 2 & \ominus 1 & \ominus 4 & \ominus 1 \end{array}$$

not all +

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 3 \\ 2 & & 4 & 2 & -2 \\ \hline & 2 & 1 & \ominus 1 & 1 \end{array}$$

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 3 \\ 3 & & 6 & 9 & 18 \\ \hline & +2 & +3 & +6 & +21 \end{array}$$

→ 3 = UB (all pos)

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 3 \\ -1 & & -2 & 5 & -2 \\ \hline & 2 & -5 & 2 & 1 \end{array}$$

not alternating

$$\begin{array}{r|rrrr} & 2 & -3 & -3 & 3 \\ -2 & & -4 & 14 & -22 \\ \hline & +2 & -7 & +11 & -19 \end{array}$$

alternating signs

→ −2 = Lower Bound

$-2 < x < 3$

110/3

p77

①
$$\begin{array}{r|rrrr} & 1x^3 & 0x^2 & 1 & -5 \\ -1 & & -1 & 1 & -2 \\ \hline & 1 & -1 & 2 & -7 \end{array}$$

-1 is lower bound. You don't need to find any more negative roots.

$$\begin{array}{r|rrrr} & 1 & 0 & 1 & -5 \\ 1 & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & \boxed{-3} \end{array}$$ not UB

$$\begin{array}{r|rrrr} & 1 & 0 & 1 & -5 \\ 2 & & 2 & 4 & 10 \\ \hline & 1 & 2 & 5 & 5 \end{array}$$

2 is UB so no need to go further.

$f(1)=-3$

$f(2)=5$

Since $f(1)$ is negative and $f(2)$ is positive $f(x)$ must cross the x axis between $x=1$ & 2 so there must be a real root between 1 & 2

3.
$$\begin{array}{r|rrrrr} & 1x^4 & -2x^3 & 0x^2 & -3 & -3 \\ -1 & & -1 & 3 & -3 & 6 \\ \hline & 1 & -3 & 3 & -6 & 3 \end{array}$$ → -1 is LB

$$\begin{array}{r|rrrrr} & 1 & -2 & 0 & -3 & -3 \\ 1 & & 1 & -1 & -1 & -4 \\ \hline & 1 & -1 & -1 & -4 & \end{array}$$ → not UB

$$\begin{array}{r|rrrrr} & 1 & -2 & 0 & -3 & -3 \\ 2 & & 2 & 0 & 0 & -6 \\ \hline & 1 & 0 & 0 & -3 & -9 \end{array}$$ not UB

$$\begin{array}{r|rrrrr} & 1 & -2 & 0 & -3 & -3 \\ 3 & & 3 & 3 & 9 & 18 \\ \hline & 1 & 1 & 3 & 6 & 15 \end{array}$$ → UB

x	y
-1	3
0	-3
1	-7
2	-9
3	15

real root between -1 & 0 and between 2 & 3

110/4

p93

1c
$$\begin{array}{r|rrrr} & 4 & 0 & -5 & -2 \\ i & & 4i & 4i^2=-4 & -9i \\ \hline & 4 & 4i & -9 & -2-9i \end{array}$$

2. $P(2)=0 \rightarrow 0=3(2)^2+k(2)-8$

$0=12+2k-8$

$0=4+2k$

$-4=2k \rightarrow k=-2$

3
$$\begin{array}{r|l} & x^2-x+5 \\ \hline x+3 & P(x) \\ & \\ \hline & \quad 2 \end{array}$$

$\rightarrow P(x)=(x^2-x+5)(x+3)\ +2$

x^3-x^2+5x

$3x^2-3x+15$

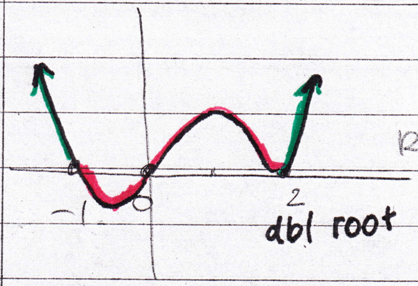
$+2$

$P(x)=x^3+2x^2+2x+17$

4.
$$\begin{array}{r|rrrrr} & 1x^4-3 & -14 & 12 & 40 \\ 2 & 2 & -2 & -32 & -40 \\ \hline & 1x^3-1 & -16 & -20 & \boxed{0} \\ 5 & 5 & 20 & 20 & \\ \hline & 1x^2+4x+4 & & \boxed{0} & \end{array}$$

$(x+2)^2=0 \rightarrow x=-2$ (dbl root)

5



$y=x\ (x+1)(x-2)^2$

$\downarrow \quad \downarrow \quad \downarrow$

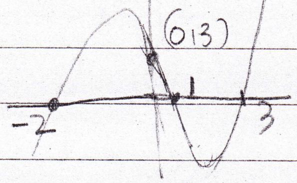
Roots 0 -1 2(dbl)

Highest exp = 4 ; coef = +

110/5

p93

6.



Roots : -2 ; 1 ; 3

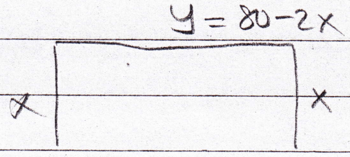
$y=a(x+2)(x-1)(x-3)$

$(0,3) \rightarrow 3=a(0+2)(0-1)(0-3)$

$3=6a \rightarrow a=\frac{1}{2}$

$y=\frac{1}{2}(x+2)(x-1)(x-3)$

7



$x+x+y=80$

$y=80-2x$

Area $= xy = x(80-2x)$

zeros = 0   40

AOS: $x=\frac{0+40}{2}=20$

$x=20 \rightarrow y=80-2(20)=40$

Area $= 40 \cdot 20 = 800\ m^2$

8a $P(x)=2x^4-3x^3-1$

$P(1)=2(1)^4-3(1)^3-1=-2$

$P(2)=2(2)^4-3(2)^3-1= \quad 32-24-1=7$

Since P(1) is neg & P(2) is pos, the location principle states that the graph has to cross the x-axis between 1 & 2 so there's a real root between x=1 & x=2