AP Stats $\boxed{10.1, 2, 3}$

$\boxed{10.1}$ a) 95% Confidence Interval 47 ± 3
(44% to 50%)

b) We don't have information about the whole population. Our sample should give us a good estimate of the parameter, but it may not be exactly correct.

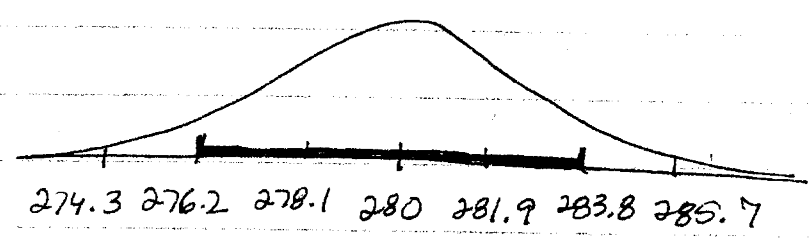
c) If we repeated our sampling procedure many times we would expect to get an estimate that is within 3% of the true value in 95% of all samples.

$\boxed{10.2}$ No. The interval 267.8 to 276.2 is a statement about the <u>mean</u> score, not about individual scores. 95% is not a probability it is a confidence level.

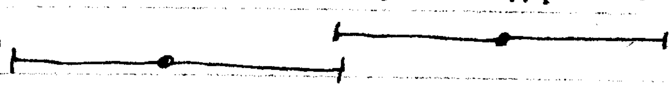
$\boxed{10.3}$ $n = 1000$
$\mu = 280$
$\sigma = 60$

a) $N(280, 60/\sqrt{1000}) = N(280, 1.897)$

b)



c) $2\sigma = 2(1.9)$
$= 3.8$



d) see sketch

e) 95% of intervals will contain $\mu = 280$ ($\bar{x} \pm m$)

AP Stats $\boxed{10.5, 6, 7, 8}$

$\boxed{10.5}$ $n = 114$ $\qquad \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

$\bar{x} = 11.78$ $\qquad 11.78 \pm 2.576\left(\frac{3.2}{\sqrt{114}}\right) = 11.78 \pm .772$

$\sigma = 3.2$

z^* for 99% $= 2.576$ $\qquad (11.008, 12.552)$

$\boxed{10.6}$ $n = 44$

1	44
1	5899
2	2
2	55667789
3	1334
3	555589
4	0011234
4	5667789
5	1224

1|4 means 14

$\boxed{\bar{x} = 35.091}$

a) The stemplot is not severly skewed. It is somewhat normal (not overwhelmingly so.)

b) $\sigma = 11$ $\quad z^*$ for 99% $= 2.576$

$35.091 \pm 2.576(11/\sqrt{44})$

35.091 ± 4.272

$(30.819, 39.363)$

c) Our confidence is based on a procedure which relies on a random sample to produce a sample that represents the population.

$\boxed{10.7}$

2239	01
2239	66788889
2240	01
2240	589
2241	2

2241|2 means 224.12

$\bar{x} = 224.002$

a) The distribution is slightly skewed right

b) z^* for 95% $\Rightarrow 1.960$

$224.002 \pm 1.96\left(\frac{.060}{\sqrt{16}}\right)$

$224.002 \pm .0294$

$(223.9726, 224.0314)$

$\boxed{10.8}$ $\sigma = .2$

$x = 3.2$

z^* for 90% = 1.645

a) $3.2 \pm 1.645\left(.2/\sqrt{1}\right)$

$3.2 \pm .329$

$(2.871, 3.529)$

b) $n = 3$

$\bar{x} = 3.2$

$3.2 \pm 1.645\left(.2/\sqrt{3}\right)$

$3.2 \pm .1899$

$(3.0101, 3.3899)$

AP Stats $\boxed{10.11 - 16, 18}$

$\boxed{10.11}$ $n = 1077$, $\bar{x} = 275$, $\sigma = 60$

95% $z^* = 1.960$

90% $z^* = 1.645$

99% $z^* = 2.576$

a) $275 \pm 1.960(60/\sqrt{1077})$ 95%

$275 \pm 3.583 \Rightarrow (271.4, 278.6)$

b) $275 \pm 1.645(60/\sqrt{1077})$ 90%

$275 \pm 3.006 \Rightarrow (271.993, 278.006)$

b) $275 \pm 2.576(60/\sqrt{1077})$ 99%

$275 \pm 4.710 \Rightarrow (270.29, 279.71)$

c) Margins of error

95% 3.583

90% 3.006

99% 4.710

Margin of error increases as confidence level increases.

$\boxed{10.12}$

a) same as 10.11a) $(271.4, 278.6)$

b) $275 \pm 1.960(60/\sqrt{250}) \Rightarrow 275 \pm 7.44 \Rightarrow (267.6, 282.4)$

c) $275 \pm 1.960(60/\sqrt{4000}) \Rightarrow 275 \pm 1.86 \Rightarrow (273.1, 276.9)$

d) $n = 250$ $m = 7.44$

$n = 1077$ $m = 3.58$

$n = 4000$ $m = 1.86$

margin of error decreases as sample size increases.

$\boxed{10.14}$ $m = \pm 1$, $\bar{x} = 11.78$, $\sigma = 3.2$, 99% $z^* = 2.576$

$m = z^*(\sigma/\sqrt{n})$

$1 = 2.576(3.2/\sqrt{n})$

$\sqrt{n} = 8.2432$

$n = 67.95$ Sample size $= \boxed{68}$

$\boxed{10.15}$ $\bar{x} = 224.002$, 95% $z^* = 1.960$, $m = \pm .020$, $\sigma = .060$

$.020 = 1.960(.060/\sqrt{n})$

$\sqrt{n} = \dfrac{1.960(.060)}{.020}$

$\sqrt{n} = 5.88$

$n = 34.5744$ Sample size $= \boxed{35}$

$\boxed{10.16}$ $\bar{x} = 8740$

$s = 1125$

$n = 958$

z^* for 95% = 1.960

a) $8740 \pm 1.960\left(\frac{1125}{\sqrt{958}}\right)$

$8740 \pm 71.24 \Rightarrow (8668.8, 8811.2)$

The calculations are correct

b) Since the calculations are based on a voluntary response sample the conclusions are not valid for the whole population.

$\boxed{10.18}$ a) We can be 99% confident that between 63% and 69% of all adults favor the amendment.

$n = 1664$

$p = .66$

so $\sigma_{\hat{p}} = \sqrt{\dfrac{.66(.34)}{1664}}$

$m = .03$ so

$.03 = z^*\left(\sqrt{\dfrac{.66(.34)}{1664}}\right)$

$2.586 \approx z^*$

This z^* is associated with approximately 99% confidence.

AP Stats 10.20, 24

$\boxed{10.20}$

a) The intended population is hotel managers (of hotels between 200 and 500 rooms). The sample came from Chicago and Detroit so it may not represent all hotel managers. Also, the problem of voluntary response might make the sample non-representative of the entire population.

b) $\bar{x} = 5.396$ $\qquad$ $5.396 \pm 1.960(1.75/\sqrt{135})$

$\sigma = 1.75$ $\qquad$ $5.396 \pm .2952$

z^* for 95% = 1.960 $\qquad$ $(5.1008, 5.6912)$

$n = 135$

c) $\bar{x} = 4.398$ $\qquad$ $4.398 \pm 2.576(1.75/\sqrt{135})$

z^* for 99% = 2.576 $\qquad$ $4.398 \pm .38799$

$(4.0100, 4.7860)$

d) Because the sample is large enough (135) the Central Limit Theorem assures us of valid results (if we assume an SRS.).

$\boxed{10.24}$ $n_w = 1025$ $\quad m = \pm 3$

$n_m = 472$ $\quad m = \pm 4$

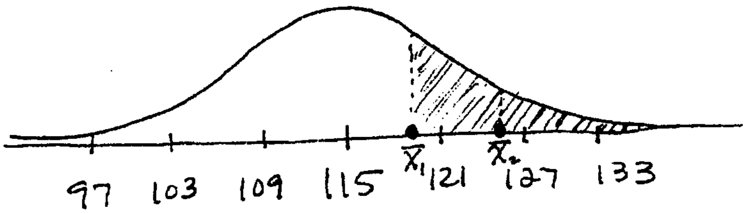
The sample size for men is smaller so the margin of error is larger. (with the same confidence level)

AP Stats $\boxed{10.27, 29-32}$

$\boxed{10.27}$

a) $\sigma_{\bar{x}} = \frac{30}{\sqrt{25}} = 6$

$N(115, 6)$



b) $\bar{x}_1 = 118.6$
$\bar{x}_2 = 125.7$

118.6 is fairly close to the center. A result like this would not be surprising (within 1 st. deviation of μ)
125.7 is less likely to occur if the true mean is 115. This provides evidence against $H_0 = 115$.

c) Shaded

$\boxed{10.29}$ $H_0: \mu = 5$
$H_a: \mu \neq 5$
where μ is the mean diameter of a spindle. (in mm)

$\boxed{10.30}$ $H_0: \mu = 42500$
$H_a: \mu > 42500$
where μ is the mean household income in the area of interest ($)

$\boxed{10.31}$ $H_0: \mu = 50$
$H_a: \mu < 50$
where μ is the mean score on an exam in accounting class.

$\boxed{10.32}$ $H_0: \mu = 2.6$
$H_a: \mu \neq 2.6$
where μ is the average time (hrs) it takes a service tech to respond to trouble calls from business customers.

AP Stats $\boxed{10.72, 73}$

$\boxed{10.72}$ $H_0: \mu = .86$
$H_a: \mu \neq .86$
$\alpha = .01$
$n = 3$
$\sigma = .0068$

a) 99% $z^* = 2.576$

$z = \dfrac{\bar{x} - .86}{.0068/\sqrt{3}}$

Reject Ho if

$\dfrac{\bar{x} - .86}{.0068/\sqrt{3}} \geq 2.576$

$\bar{x} \geq .8701$

or $\dfrac{\bar{x} - .86}{.0068/\sqrt{3}} \leq -2.576$

$x \leq .8499$

if $\mu = .845$ b) $P(\bar{x} \geq .8701) = P\left(z \geq \dfrac{.8701 - .845}{.0068/\sqrt{3}}\right)$

$P(z \geq 6.393) \approx 0$

or $P(\bar{x} \leq .8499) = P\left(z \leq \dfrac{.8499 - .845}{.0068/\sqrt{3}}\right)$

$P(z \leq 1.248) = .89398$

Power of the test $\approx \boxed{.8940}$ for $\mu = .845$

c) $1 - \beta$ = power so $\beta = 1 -$ power $\Rightarrow 1 - .8940 \approx \boxed{.106}$

$\boxed{10.73}$ $H_0: \mu = 128$
$H_a: \mu \neq 128$
$\alpha = .05$
$n = 72$
$\sigma = 15$
$z^* = 1.96$

a) $\dfrac{\bar{x} - 128}{15/\sqrt{72}} \geq 1.96$

or $\dfrac{x - 128}{15/\sqrt{72}} \leq -1.96$

Reject Ho if
$\bar{x} \geq 131.465$
or
$\bar{x} \leq 124.535$

For $\mu = 134$

$P(\bar{x} \geq 131.465) = P\left(z \geq \dfrac{131.465 - 134}{15\sqrt{72}}\right)$

$= P(z \geq -1.43) = .9242$

$P(\bar{x} \leq 124.535) = P\left(z \leq \dfrac{124.535 - 134}{15/\sqrt{72}}\right)$

$= P(z \leq -5.354) \approx 0$

6 more than μ * Power of the test ($\mu = 134$) $\approx \boxed{.9242}$

6 less than μ b) for $\mu = 122$ $P\left(z \geq \dfrac{131.465 - 122}{15/\sqrt{72}}\right) = P(z \geq 5.35) \approx 0$

$P\left(z \leq \dfrac{124535 - 122}{15/\sqrt{72}}\right) = P(z \leq 1.43) = \boxed{.9236}$

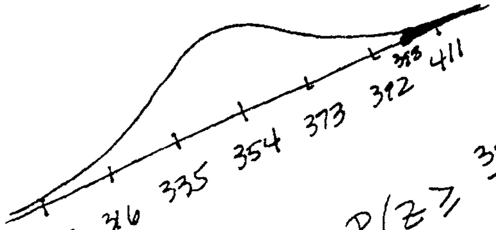
c) The power would be higher. It is easier to detect larger differences from μ.

$> 118.6) = P(z$.

$= P(z > .6)$

$(\bar{X} > 125.7) = P(z > \frac{125.7 - 115}{30/\sqrt{25}})$

$= P(z > 1.783) = \boxed{.0373}$

b) 

$\sigma = 33/\sqrt{3}$

411

c) $P(\bar{X} \geq 398) = P(z \geq \frac{398 - 354}{19.05}) =$

$= P(z \geq 2.31) =$

value of .0105 is significant at the

$(p < \alpha)$ but not quite at the $\alpha = .01$ le

chance of getting a sales average of

mean is still 354 is about 105 out

ut is pretty convincing evidence tha

sales not 354, but higher.

1] Getting a result as extreme as that

sample would occur less than 1 out

repeated samples if church atten

ethnocentric than non attender

there is a difference betwee

goers are more ethnocentri

$\boxed{10.39}$ $\sigma = 0.060$ $H_0: \mu = 224$ where μ is the mean of a critical dimension of auto crankshafts

$\mu = 224$ $H_a: \mu \neq 224$

$\bar{x} = 224.0019$

b) $$z = \frac{224.0019 - 224}{.060/\sqrt{16}} = \boxed{.1292}$$

* two-tailed test

c) P-value $= 2 \cdot P(z > .1292) = 2(.4486) = \boxed{.8972}$

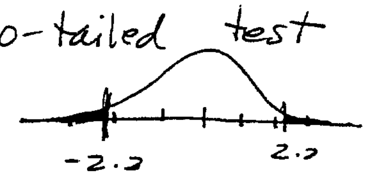
$\boxed{\text{Fail to reject Ho.}}$ A result as extreme as the one noted ($\bar{x} = 224.0019$) would be expected to occur by chance in nearly 90% of samples.

AP Stats $\boxed{10.41, 43, 44, 58-63}$

$\boxed{10.41}$ $H_0: \mu = .5$ where μ is the mean of the 100 random #s

$n = 100$ $H_a: \mu \neq .5$

two-tailed test



$\bar{x} = .4365$

$\sigma = .2887$ a) $z = \dfrac{.4365 - .5}{.2887/\sqrt{100}} = -2.1995$

$2 \cdot P(z < -2.1995) = 2(.0139) = .0278$

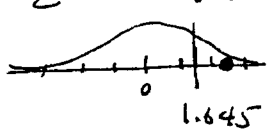
b) 5% $z^* = 1.960$ $\quad 2.1995 > 1.960$ $\boxed{\text{Yes}}$

c) 1% $z^* = 2.576$ $\quad 2.1995 \not> 2.576$ $\boxed{\text{No}}$

$\boxed{10.43}$ $H_0: \mu = 1.4$ where μ is the mean nicotine content of a certain brand of cigarette.

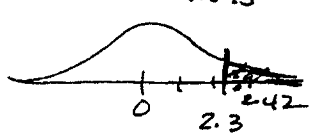
$H_a: \mu > 1.4$

$z = 2.42$



a) z^* for one-sided 5% $= 1.645$

$2.42 > 1.645$ $\boxed{\text{yes}}$



b) z^* for one-sided 1% $= 2.326$

$2.42 > 2.326$ $\boxed{\text{yes}}$

$\boxed{10.44}$ $n = 12$ $\quad \bar{x} = 104.13$ $\quad$ a) $104.13 \pm 1.645(9/\sqrt{12})$

$\sigma = 9$ $\quad z^*_{90\%} = 1.645$ $\quad 104.13 \pm 4.2738$

$(99.86, 108.40)$

b)

$H_0: \mu = 105$ where μ is the mean reading given by

$H_a: \mu \neq 105$ detectors of Radon gas.

We <u>cannot</u> reject H_0 because 105 falls within the 90% confidence interval.

$\boxed{10.58}$ $\mu = 475$ $\quad$ Ho: $\mu = 475$ Where μ is the mean SATM score of the students who

$\sigma = 100$

$n = 100$ $\quad$ Ha: $\mu > 475$ went through training program

$\bar{x} = 491.4$

$\alpha = .05$

z^* for 90% $(5\% \ \alpha) = 1.645$

(a) $z = \dfrac{491.4 - 475}{100/\sqrt{100}} = 1.64$

$1.64 \not> 1.645$ $\boxed{No}$

The resuet does not allow us to reject Ho.
The result is not significant at the 5% level.

(b) $\bar{x} = 491.5$ $\quad z = \dfrac{491.5 - 475}{100/\sqrt{100}} = 1.65$

$1.65 > 1.645$ $\boxed{yes}$

This result is significant at the 5% level.

* This illustrates the fact that we shouldn't be too strict in interpreting tests at given α levels.

$\boxed{10.59}$ Ho: $\mu = 475$
Ha: $\mu > 475$

$\sigma = 100$ $\quad \alpha = .05$

a) $n = 100$ $\quad z = \dfrac{478 - 475}{100/\sqrt{100}} = .3$

$\bar{x} = 478$ $\quad$ p-value $= \underline{.3821}$

$p \not< \alpha$

b) $n = 1000$ $\quad z = \dfrac{478 - 475}{100/\sqrt{1000}} = .9487$ $\quad$ p-value $= \underline{.1714}$

$p \not< \alpha$

c) $n = 10000$ $\quad z = \dfrac{478 - 475}{100/\sqrt{10000}} = 3$ $\quad$ p-value $= .0013$

$p < \alpha$

The same $\bar{x}$ is significant from a very large sample
The last test indicates a very significant resuet, but, in reality, an increase from 475 to 478 is not significant in a practical sense

$\boxed{10.60}$ 99% $z^* = 2.576$

$n=100$ $\quad 478 \pm 2.576\left(100/\sqrt{100}\right) = (452.24,\ 503.76)$

$n=1000$ $\quad 478 \pm 2.576\left(100/\sqrt{1000}\right) = (469.85,\ 486.15)$

$n=10000$ $\quad 478 \pm 2.576\left(100/\sqrt{10000}\right) = (475.424,\ 480.576)$

The third interval shows us that the mean is greater than 475 but only by a small amount.

$\boxed{10.61}$ Since the data come from a voluntary response sample, we cannot assume that it is representative of the entire population of all viewers. This result is probably more representative of people with strong negative opinions.

$\boxed{10.62}$ a) at $\alpha = .01$ we would expect 5 out of 500 people to have results as extreme as these just by chance variation. Are these 4 people just some of those 5?

b) The researcher should repeat the test procedure on these 4 people to see if they again perform well.

$\boxed{10.63}$ A significance test addresses the question of whether the result of the experiment could be due to chance alone. (b)

AP Stats $\boxed{10.66-69,\ 71}$

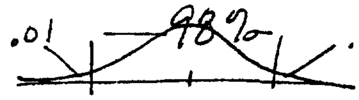
$\boxed{10.66}$ a) H_0: the patient is ill (<u>should</u> see a doctor)
H_a: the patient is healthy (doesn't need a doctor)

Type I error: false negative - clearing a patient who should be referred to a doctor

Type II error: false positive - sending a healthy to the doctor.

b) You might want to lower the probability of a type I error so that most ill patients will be treated.

If you are an insurance administrator you might want to save money by reducing type II errors and therefore sending fewer patients to see the doctor unnecessisarily.



$\boxed{10.67}$ a) We will reject H_0 if $z < -2.326$

b) Prob. (type I error) = 0.01

c) Reject if $\dfrac{\bar{x}-275}{60/\sqrt{840}} < -2.326$

$$\bar{x} < -2.326\,(60/\sqrt{840}) + 275$$
$$\bar{x} < 270.185$$

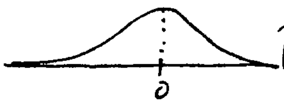
Accept H_0 if $\bar{x} > 270.185$

$$P(\bar{x} > 270.185) = P\left(z > \frac{270.185-275}{60/\sqrt{840}}\right)$$
$$= P(z > .0892) = .4644$$

Probability of type II error $= .4644 = \beta$

$\boxed{10.68}$ $H_0: \mu = 0$ $\quad n = 9, \sigma = 1$

$H_a: \mu > 0$ $\quad$ Reject H_0 if $\bar{x} > 0$

(α) a) 

P(type I error: rejecting Ho) = .5

b) Reject Ho if $\bar{x} > 0$. Accept Ho if $\bar{x} < 0$.

If $\mu = .3$ $\quad P(\bar{X} < 0) = P\left(z < \frac{0 - .3}{1/\sqrt{9}}\right) = P(z < -.9)$

$= \boxed{.1841}$

(β) Probability of a type II error is .1841 if $\mu = .3$

c) If $\mu = 1$ $\quad P(\bar{X} < 0) = P\left(z < \frac{0 - 1}{1/\sqrt{9}}\right) = P(z < -3)$

$= \boxed{.0013}$

Probability of a type II error is .0013 if $\mu = 1$

$\boxed{10.69}$ $\alpha = 5\% \Rightarrow z = 1.645$ $\quad$ Reject Ho if $z > 1.645$

$\bar{x} = 1.02$ $\quad$ if $\frac{\bar{x} - 0}{1/\sqrt{10}} > 1.645$

$\sigma = 1$

$n = 10$ $\quad \boxed{\bar{x} > .5202}$

If $\mu = 1.1$ $\quad P(\bar{x} > .5202) = P\left(z > \frac{.5202 - 1.1}{1/\sqrt{10}}\right)$

$= P(z > -1.833)$

The power of the test with $\mu = 1.1$ = .9667 = $(1 - \beta)$

$\boxed{10.71}$ a) $\frac{\bar{x} - 300}{3\sqrt{25}} \le -1.645$ $\quad$ b) $\frac{\bar{x} - 300}{3/\sqrt{100}} \le -1.645$

$\bar{x} \le 299.013$ $\quad \bar{x} \le 299.5065$

a) If $\mu = 299$, $n = 25$ $\quad P(\bar{x} \le 299.013) = P\left(z \le \frac{299.013 - 299}{3/\sqrt{25}}\right) = P(z \le .0217)$

$= \boxed{.5086}$

b) If $\mu = 299$, $n = 100$ $\quad P(\bar{x} \le 299.5065) = P\left(z \le \frac{299.5065 - 299}{3/\sqrt{100}}\right) = P(z < 1.688)$

$= \boxed{.9543}$

AP Stats 10.77 - 82

$\boxed{10.77}$

2	0 3 4
2	
3	0 1 1 2 4
3	6
4	3

a) The plot is reasonably symmetric considering such a small sample.

b) $(26.061, 34.739)$

30.4 ± 4.339

c) $H_0: \mu = 25$ where μ is the mean odor threshold
$H_a: \mu > 25$ for students

$$z = \frac{30.4 - 25}{7/\sqrt{10}} = 2.4395 \quad \text{p-value} = .0074$$

Reject H_0

We have strong evidence to conclude that the mean odor threshold for students is higher than the published threshold.

$\boxed{10.78}$

$\sigma = 8$
$n = 15$
$\bar{x} = 145$

a) $145 \pm 1.645(8/\sqrt{15})$

145 ± 3.4

$(141.6, 148.4)$

b) $H_0: \mu = 140$ where μ is the mean cellulose content
$H_a: \mu > 140$ of a variety of alfalfa hay.

$$z = \frac{145 - 140}{8/\sqrt{15}} = 2.4206 \quad \text{p-value} = .0077$$

We have strong evidence to conclude that the mean cellulose content of this variety of alfalfa hay is more than 140 mg/g.

c) We must assume that the 15 cuttings in the sample are an SRS. Since our sample is not very large, we must assume that the population is normally distributed (or at least not extremely non-normal.)

$\boxed{10.79}$ $\bar{x} = 12.9$ $\quad 12.9 \pm 1.960(1.6/\sqrt{26})$

$\sigma = 1.6$ $\quad 12.9 \pm .615$

$n = 26$ $\quad (12.28, 13.52)$

This assumes that the babies are an SRS from the population. The population should not be too non-normal. The sample size of 26 is fairly large.

$\boxed{10.80}$ a) $H_0: \mu = 32$ where μ is the mean score of 3rd graders in this district

$H_a: \mu > 32$

$\sigma = 11$

$n = 44$

$z = \dfrac{35.09 - 32}{11/\sqrt{44}} = 1.86$ $\quad$ p-value = .031

Reject H_0

There is strong evidence that the mean score of 3rd graders in this district is higher than the national mean (32). A result as extreme as that observed ($\bar{x} = 35.09$) would occur only about 3 times out of 100 if the district mean was 32.

$\boxed{10.81}$ a) 95% confidence interval would be <u>narrower</u> than a 99% interval. Lowering confidence decreases the width of the interval

b) $32264 \pm 397 \Rightarrow (31867, 32661)$

\$33000 does not fall within the interval indicating that $p < .01$. Reject the hypothesis.

$\boxed{10.82}$ a) Sample size increases — margin of error decreases

b) " — p-value decreases

c) " — power increases

(α fixed, n increases $\Rightarrow$ β decreases)