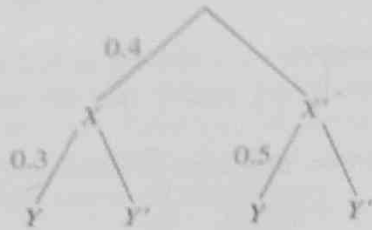
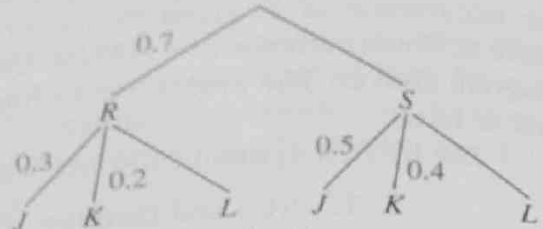


WRITTEN EXERCISES

- A** 1. Use the tree diagram at the left below to find each probability.
- | | | |
|---------------------------|----------------|--------------------------|
| a. $P(X')$ | b. $P(Y' X)$ | c. $P(Y' X')$ |
| e. $P(X' \text{ and } Y)$ | f. $P(Y)$ | d. $P(X \text{ and } Y)$ |
| | | g. $P(X Y)$ |
| | | h. $P(X' Y)$ |

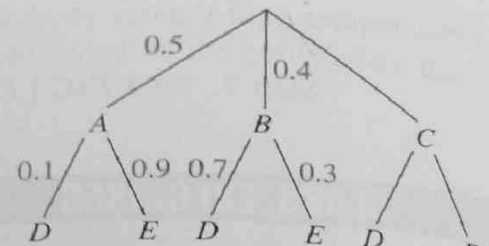


Ex. 1



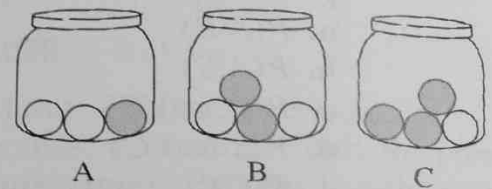
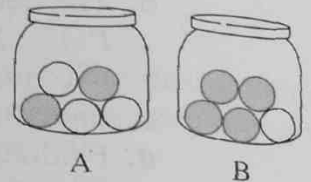
Ex. 2

2. Use the tree diagram at the right above to find each probability.
- | | | |
|--------------------------|---------------|--------------------------|
| a. $P(S)$ | b. $P(L R)$ | c. $P(L S)$ |
| e. $P(J \text{ and } S)$ | f. $P(J)$ | d. $P(J \text{ and } R)$ |
| | | g. $P(R J)$ |
| | | h. $P(S J)$ |
3. Use the tree diagram at the right to find each sum of probabilities.
- $P(A \text{ and } D) + P(A \text{ and } E)$
 - $P(B \text{ and } D) + P(B \text{ and } E)$
 - $P(C \text{ and } D) + P(C \text{ and } E)$



Exs. 3, 4

4. Refer to the tree in Exercise 3 and suppose that event D cannot possibly happen if event C happens. Find $P(A | D)$, $P(B | D)$, and $P(C | D)$.
5. Jar A contains 2 red balls and 3 white balls. Jar B contains 4 red balls and 1 white ball. A coin is tossed. If it shows "heads," a ball is randomly picked from Jar A; if it shows "tails," a ball is randomly picked from Jar B.
- Draw a tree diagram showing the probabilities of each jar and then the probabilities of picking a red ball or a white ball.
 - Find the probability of picking a red ball.
 - If a red ball is picked, find the probability that it came from Jar A.
6. Jars A, B, and C contain red and white balls as shown. A die is rolled. If an even number comes up, a ball is randomly picked from Jar A. If a "1" or a "3" comes up, a ball is randomly picked from Jar B. If a "5" comes up, a ball is randomly picked from Jar C.
- Incorporate the facts given above into a tree diagram.
 - Find the probability of picking a red ball.
 - If a red ball is picked, what is the probability that it came from Jar A? from Jar B? from Jar C?



7. **Manufacturing** Machine A produces 60% of the ball bearings manufactured by a factory and Machine B produces the rest. Five percent of Machine A's bearings fail to have the required precision, and two percent of Machine B's bearings fail.

- Incorporate the facts given above into a tree diagram.
- What percent of the bearings fail to have the required precision?
- If a bearing is inspected and fails to have the required precision, what is the probability that it was produced by Machine A?

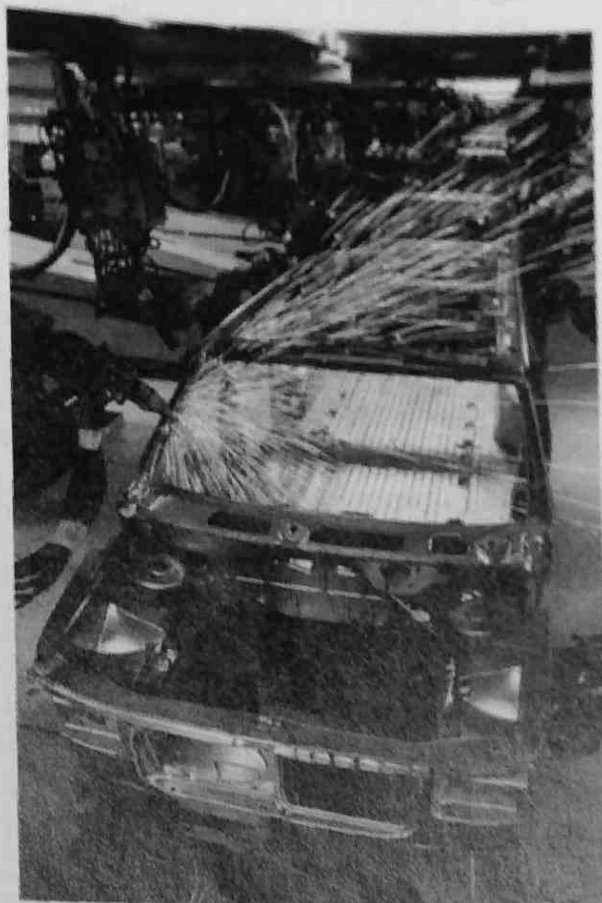
8. **Manufacturing** Five percent of the welds on an automobile assembly line are defective. The defective welds are found using an X-ray machine. The machine correctly rejects 92% of the defective welds and correctly accepts all of the good welds.

- Incorporate the facts given above into a tree diagram.
- What percent of the welds are accepted by the machine?
- Find the probability that an accepted weld is defective.

9. **Insurance** An auto insurance company charges younger drivers a higher premium than it does older drivers because younger drivers as a group tend to have more accidents. The company has 3 age groups: Group A includes those under 25 years old, 22% of all its policyholders. Group B includes those 25–39 years old, 43% of all of its policyholders. Group C includes those 40 years old or older. Company records show that in any given one-year period, 11% of its Group A policyholders have an accident. The percentages for groups B and C are 3% and 2%, respectively.

- What percent of the company's policyholders are expected to have an accident during the next 12 months?
- Suppose Mr. X has just had a car accident. If he is one of the company's policyholders, what is the probability that he is under 25?

10. **Insurance** Suppose the insurance company of Exercise 9 not only classifies drivers by age, but in the case of drivers under 25 years old, it also notes whether they have had a driver's education course. One quarter of its policyholders under 25 have had driver's education and 5% of these have an accident in a one-year period. Of those under 25 who have not had driver's education, 13% have an accident within a one year period. A 20-year-old woman takes out a policy with this company and within one year she has an accident. What is the probability that she did *not* have a driver's education course?

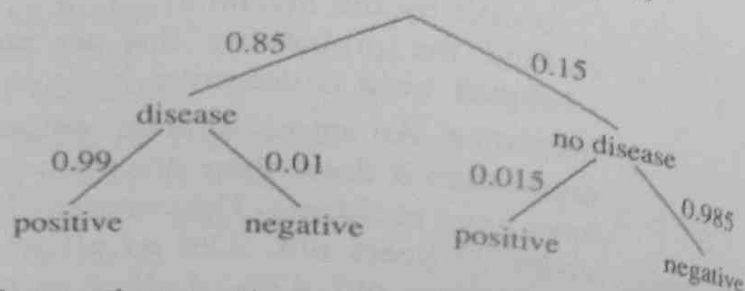


B 11. Medicine A medical research lab proposes a screening test for a disease. In order to try out this test, it is given to 100 people, 60 of whom are known to have the disease and 40 of whom are known not to have the disease. A positive test indicates the disease and a negative test indicates no disease. Unfortunately, such medical tests can produce two kinds of errors:

- (1) A *false negative* test: For the 60 people who do have the disease, this screening test indicates that 2 do *not* have it.
- (2) A *false positive* test: For the 40 people who do not have the disease, this screening test indicates that 10 *do* have it.

- a. Which of the false tests do you think is more serious? Why?
- b. Incorporate the facts given above into a tree diagram. (Be sure to convert the given integers into probabilities.)
- c. Suppose the test is given to a person not in the original group of 100 people. It is not known whether this person has the disease, but the test result is positive. What is the probability that the person really does have the disease?
- d. Suppose the test is given to a person whose disease status is unknown. If the test result is negative, what is the probability that the person does *not* have the disease?

12. Medicine Part (c) of Exercise 11 indicates that about 85% of those who test positive really do have the disease, so that 15% of those who test positive do not have it. This 15% error may seem high, but people with a positive screening test are usually given a more thorough diagnostic test. Even the diagnostic test can yield errors but they are much less likely than the screening test, as the diagram shows.



- a. What is the probability that the diagnostic test gives:
 - (1) a false negative result?
 - (2) a false positive result?
- b. What is the probability that:
 - (1) the diagnostic test gives the correct result?
 - (2) a person with a positive diagnostic test has the disease?
 - (3) a person with a negative diagnostic test does not have the disease?

13. The children of a math professor play two games that use dice. In one game, two dice are rolled and the sum of the numbers on the dice is called out. In the other game, a single die is rolled and its number is called out. The professor hears the children in another room call out the number 2, and knowing that they play the two games about equally often, the professor is able to calculate the probability they are playing the two-dice game. What is this probability?

14. Solve Exercise 13 if the children call out:

a. the number 4

b. the number 7

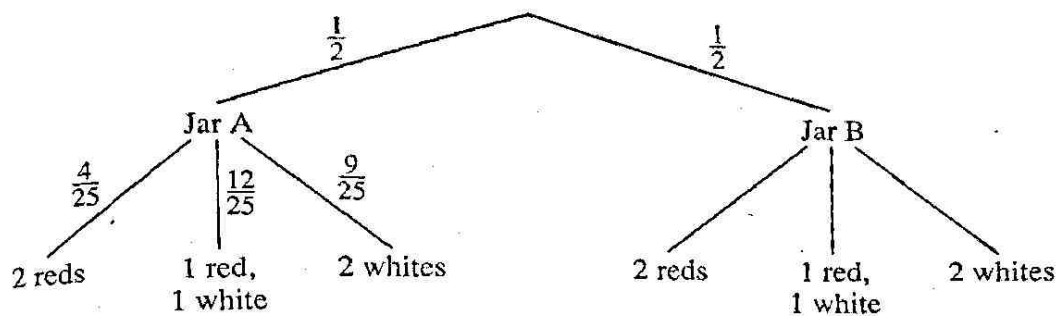
c. the number 1

15. Suppose you have two pairs of dice. One pair is fair, but each die of the other pair is weighted so that a six comes up with probability $\frac{1}{4}$ instead of the usual $\frac{1}{6}$. If you randomly choose one pair of dice, roll them, and obtain two sixes, what is the probability that you rolled the weighted dice?
16. Suppose you have 2 coins. One of them is fair, but the other has two "heads."
 a. You choose one coin at random, flip it n times, and get "heads" each time. Find the probability that the coin is two-headed for $n = 1$, $n = 2$, and $n = 10$.
 b. In terms of n , find the probability that the coin is two-headed.
17. Refer to the two jars pictured in Exercise 5. Suppose that someone randomly picks one of the jars, but you don't know which one. Before guessing which jar the "mystery jar" is, you have your choice of doing either of the following experiments.

Experiment 1: Pick 2 balls from the "mystery jar" and note their colors. Replace the first ball before choosing the second.

Experiment 2: Pick 2 balls from the "mystery jar" and note their colors. Do *not* replace the first ball before choosing the second.

The tree diagram for Experiment 1 looks like this:



- Copy and complete the diagram.
- Suppose you performed Experiment 1 and got 2 red balls. Then you would no doubt guess that they came from Jar B. Which jar would you guess if you got 2 white balls? 1 red and 1 white ball? What is the probability that you would guess the correct jar if you performed the first experiment?
- Make a tree diagram for Experiment 2.
- If you performed Experiment 2, which jar would you guess if you got 2 red balls? 2 white balls? 1 red and 1 white ball? What is the probability that you would guess correctly if you performed the second experiment?
- Which experiment gives you the better chance of guessing correctly?
- Suppose a third experiment allows you to pick just one ball and then guess the jar. How likely are you to guess correctly in this experiment? Compare your answer with the probability of being correct in the first or second experiment. Are you surprised?