45. $P(\text{3 or more})$
$= 1 - P(0, 1, \text{ or } 2)$
$= 1 - \left[{}_{10}C_0(0.188)^0(0.812)^{10} + {}_{10}C_1(0.188)^1(0.812)^9 + {}_{10}C_2(0.188)^2(0.812)^8 \right]$
≈ 0.29

CHALLENGE AND EXTEND

46a. 65; number of people × probability left-handed

b. standard deviation $= \sqrt{npq} = \sqrt{650(0.1)(0.9)} \approx 7.6485$
So, n is approximately between 57.3515 and 72.6485
So, $\{ n \mid 58 \le n \le 72 \}$.

47a. $P(\text{at least one 1 in 6 rolls})$
$= 1 - P(\text{zero 1s})$
$= 1 - {}_6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6$
≈ 0.67

b. $P(\text{at least two 1s in 12 rolls})$
$= 1 - P(\text{zero or one 1})$
$= 1 - \left[{}_{12}C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} + {}_{12}C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11}\right]$
≈ 0.62

48. $P(\text{at least 4})$
$= 1 - P(\text{at most 3})$
enter: 1 − binomcdf(20, 0.4, 3) ≈ 0.984

49. ${}_nC_r + {}_nC_{r+1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)![n-(r+1)]!}$$
$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$
$$= \frac{n!}{r!(n-r)(n-r-1)!} + \frac{n!}{(r+1)r!\,(n-r-1)!}$$
$$= \left(\frac{r+1}{r+1}\right)\frac{n!}{r!(n-r)(n-r-1)!} + \left(\frac{n-r}{n-r}\right)\frac{n!}{(r+1)r!(n-r-1)!}$$
$$= \frac{n!(r+1)}{(r+1)!(n-r)(n-r-1)!} + \frac{n!(n-r)}{(r+1)r!(n-r)!}$$
$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$
$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$
$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$
$$= {}_{n+1}C_{r+1}$$

50a. $P(1) = {}_2C_1p^1(1-p)^1$
The equation is: $2p(1-p) = 0.4$
$2p - 2p^2 = 0.4$
$-2\left(p^2 - p + 0.2\right) = 0.$
$p = \dfrac{1 \pm \sqrt{1 - 4(0.2)}}{2} \approx 0.72 \text{ or } \approx 0.28$
So, $p \approx 0.72$ or ≈ 0.28

b. $P(2) = {}_2C_2(0.72)^2(0.28)^0$
≈ 0.52 or ≈ 0.076 if $p \approx 0.28$.

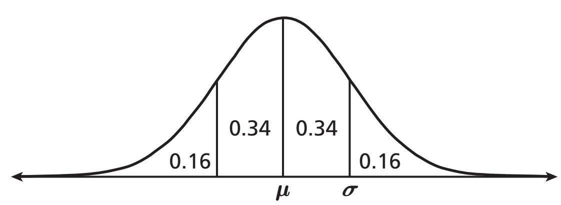
8-7 FITTING TO A NORMAL DISTRIBUTION

CHECK IT OUT!

1. Looking at the graph, there approximately 19 blocks under the curve that are less than 400. Therefore, the probability is about 0.19.
2. The probability that $x > 106$ is desired. The following is the z score:
$z = \frac{106 - 142}{18} = \frac{-36}{18} = -2$
Use the Standard Normal Values table to find the area under the curve for all values greater than −2, which is 1 − 0.02 = 0.98. The probability of scoring more than 106 is about 0.98.
3. Because of symmetry of the normal distribution, there should be approximately the same number of values above and below the mean. However, the table has 14 values below the mean and 4 values above the mean. Therefore, the data does not appear to be normally distributed. The table below confirms that the data values are not normally distributed.

z	Area	X	Values below x: Projected	Values below x: Actual
−2	0.02	5	0	0
−1	0.16	21	3	0
0	0.5	37	9	14
1	0.84	53	15	15
2	0.98	69	18	18

THINK AND DISCUSS

1. Subtract 50 from x, and divide the result by 5.
2. The total area under the normal curve is 1, so if the area for x-values less than a is p, then the area for x-values greater than a is $1 - p$.
3. 

EXERCISES

GUIDED PRACTICE

1. The standard normal value of a statistic is found by subtracting the mean from the statistic and dividing the result by the standard deviation.
2. There are approximately 83 boxes underneath the curve and to the right of 6. Therefore, the probability is approximately 0.83.

3. Find the standard normal value of 65, using $\mu = 70$ and $\sigma = 10$.
$z = \frac{65 - 70}{10} = \frac{-5}{10} = -0.5$
Use the Standard Normal Values table to find the area under the curve for all values less than −0.5, which is 0.31. The probability of scoring less than 65 is about 0.31.

4. Find the standard normal value of 80, using $\mu = 70$ and $\sigma = 10$.
$z = \frac{80 - 70}{10} = \frac{10}{10} = 1$
Use the Standard Normal Values table to find the area under the curve for all values more than 1, which is $1 - 0.84 = 0.16$. The probability of scoring more than 80 is about 0.16.

5. Find the standard normal values of 50 and 60, using $\mu = 70$ and $\sigma = 10$. Use the Standard Normal Values table to find the areas under the curve for all values less than *z*.
$z = \frac{50 - 70}{10} = \frac{-20}{10} = -2$ Area ≈ 0.02
$z = \frac{60 - 70}{10} = \frac{-10}{10} = -1$ Area ≈ 0.16
Subtract the areas to eliminate where the regions overlap. The probability of scoring between 50 and 60 is about $0.16 - 0.02 = 0.14$.

6. Find the standard normal values of 70 and 90, using $\mu = 70$ and $\sigma = 10$. Use the Standard Normal Values table to find the areas under the curve for all values less than *z*.
$z = \frac{70 - 70}{10} = 0$ Area ≈ 0.5
$z = \frac{90 - 70}{10} = \frac{20}{10} = 2$ Area ≈ 0.98
Subtract the areas to eliminate where the regions overlap. The probability of scoring between 70 and 90 is about $0.98 - 0.5 = 0.48$.

7. Yes; see the table below.

z	Area	X	Values below x	
			Projected	Actual
−2	0.02	0.3	1	0
−1	0.16	0.4	5	3
0	0.5	0.5	15	17
1	0.84	0.6	25	24
2	0.98	0.7	29	29

The projected number of data values below each *z*-value is close to the actual number. The data appears to be normally distributed.

PRACTICE AND PROBLEM SOLVING

8. Looking at the graph, there are approximately 50 blocks under the curve that are between 11.9 and 12.1. Therefore, the probability is about 0.5.

9. Find the standard normal value of 64, using $\mu = 76$ and $\sigma = 6$.
$z = \frac{64 - 76}{6} = \frac{-12}{6} = -2$
Use the Standard Normal Values table to find the area under the curve for all values less than −2, which is 0.02. The probability of scoring less than 64 is about 0.02.

10. Find the standard normal value of 70, using $\mu = 76$ and $\sigma = 6$.
$z = \frac{70 - 76}{6} = \frac{-6}{6} = -1$
Use the Standard Normal Values table to find the area under the curve for all values more than 0, which is $1 - 0.16 = 0.84$. The probability of scoring more than 70 is about 0.84.

11. Find the standard normal values of 85 and 91, using $\mu = 76$ and $\sigma = 6$. Use the Standard Normal Values table to find the areas under the curve for all values less than *z*.
$z = \frac{85 - 76}{6} = \frac{9}{6} = 1.5$ Area ≈ 0.93
$z = \frac{91 - 76}{6} = \frac{15}{6} = 2.5$ Area ≈ 0.99
Subtract the areas to eliminate where the regions overlap. The probability of scoring between 85 and 91 is about $0.99 - 0.93 = 0.06$.

12. Find the standard normal values of 73 and 79, using $\mu = 76$ and $\sigma = 6$. Use the Standard Normal Values table to find the areas under the curve for all values less than *z*.
$z = \frac{73 - 76}{6} = \frac{-3}{6} = -0.5$ Area ≈ 0.31
$z = \frac{79 - 76}{6} = \frac{3}{6} = 0.5$ Area ≈ 0.69
Subtract the areas to eliminate where the regions overlap. The probability of scoring between 73 and 79 is about $0.69 - 0.31 = 0.38$.

13. No; see the table below. There should be more values under 45 inches.

z	Area	X	Values below x	
			Projected	Actual
−2	0.02	33	0	0
−1	0.16	39	4	0
0	0.5	45	12	3
1	0.84	51	20	17
2	0.98	57	24	24

14. Find the standard normal value of 100, using $\mu = 100$ and $\sigma = 16$.
$z = \frac{100 - 100}{16} = \frac{0}{16} = 0$
Use the Standard Normal Values table to find the area under the curve for all values greater than 0, which is $1 - 0.5 = 0.5$. The probability of scoring greater than 100 is about 0.5.

15. Find the standard normal value of 140, using $\mu = 100$ and $\sigma = 16$.

$z = \frac{140 - 100}{16} = \frac{40}{16} = 2.5$

Use the Standard Normal Values table to find the area under the curve for all values less than 2.5, which is 0.99. The probability of scoring less than 140 is about 0.99.

16. Find the standard normal values of 92 and 108, using $\mu = 100$ and $\sigma = 16$. Use the Standard Normal Values table to find the areas under the curve for all values less than z.

$z = \frac{92 - 100}{16} = \frac{-8}{16} = -0.5$ Area ≈ 0.31

$z = \frac{108 - 100}{16} = \frac{8}{16} = 0.5$
Area ≈ 0.69

Subtract the areas to eliminate where the regions overlap. The probability of scoring between 92 and 108 is about $0.69 - 0.31 = 0.38$.

17. Find the standard normal value of 84, using $\mu = 100$ and $\sigma = 16$.

$z = \frac{84 - 100}{16} = \frac{-16}{16} = -1$

Use the Standard Normal Values table to find the area under the curve for all values less than -1, which is 0.16. The probability of scoring less than 84 is about 0.16.

18. Find the standard normal values of 60 and 124, using $\mu = 100$ and $\sigma = 16$. Use the Standard Normal Values table to find the areas under the curve for all values less than z.

$z = \frac{60 - 100}{16} = \frac{-40}{16} = -2.5$ Area ≈ 0.01

$z = \frac{124 - 100}{16} = \frac{24}{16} = 1.5$
Area ≈ 0.93

Subtract the areas to eliminate where the regions overlap. The probability of scoring between 60 and 124 is about $0.93 - 0.01 = 0.92$.

19. Find the standard normal values of 68 using $\mu = 100$ and $\sigma = 16$. Use the Standard Normal Values table to find the area under the curve for all values less than z.

$z = \frac{68 - 100}{16} = \frac{-32}{16} = -2$ Area ≈ 0.02

Find the standard normal values of 132 using $\mu = 100$ and $\sigma = 16$. Use the Standard Normal Values table to find the area under the curve for all values greater than z.

$z = \frac{132 - 100}{16} = \frac{32}{16} = 2$ Area $\approx 1 - 0.98 = 0.02$

Add the areas together because the regions do not overlap. The probability of scoring below 68 and above 132 is about $0.02 + 0.02 = 0.04$.

20. The original shape of the distribution is a normal curve centered at 5. After the bags that weigh less than 5 pounds are rejected, the graph of the weights of the remaining bags would be shaped like half of a normal curve, with a maximum at 5 and no values below 5, and a tail stretching out to the right, so the data would be skewed right.

TEST PREP

21. Find the standard normal value of $x = \mu$.

$z = \frac{\mu - \mu}{\sigma} = \frac{0}{\sigma} = 0$

Use the Standard Normal Values table to find the area under the curve for all values less than 0, which is 0.5. The probability of scoring less than μ is about 0.5. The answer is B.

22. Find the standard normal values of 63, 69, 72, 78, 81, and 90 using $\mu = 78$ and $\sigma = 6$. Use the Standard Normal Values table to find the areas under the curve for all values less than z.

$z = \frac{63 - 78}{6} = \frac{-15}{6} = -2.5$ Area ≈ 0.01

$z = \frac{69 - 78}{6} = \frac{-9}{6} = -1.5$ Area ≈ 0.07

$z = \frac{72 - 78}{6} = \frac{-6}{6} = -1$ Area ≈ 0.16

$z = \frac{78 - 78}{6} = \frac{0}{6} = 0$ Area ≈ 0.5

$z = \frac{81 - 78}{6} = \frac{3}{6} = 0.5$ Area ≈ 0.69

$z = \frac{90 - 78}{6} = \frac{12}{6} = 2$ Area ≈ 0.98

For F, the probability that a student's score is greater than 90 is about $1 - 0.98 = 0.02$.
For G, the probability that a student's score is less than 69 is about 0.07.
For H, the probability that a student's score is between 78 and 81 is found by subtracting the areas to eliminate where the regions overlap. The probability of scoring between 78 and 81 is about $0.69 - 0.5 = 0.19$.
For J, the probability that a student's score is between 63 and 72 is found by subtracting the areas to eliminate where the regions overlap. The probability of scoring between 63 and 72 is about $0.16 - 0.01 = 0.15$.

Letter H has the greatest probability.

CHALLENGE AND EXTEND

23. $P(x > 70) = 0.31$ means that the $P(x < 70) = 0.69$. The z value corresponding to this probability is 0.5.

$0.5 = \frac{70 - 65}{\sigma}$

$0.5 = \frac{5}{\sigma}$

$0.5\sigma = 5$

$\sigma = 10$

24. $P(x < 30) = 0.07$ means the z value corresponding to this probability is -1.5.

$-1.5 = \frac{30 - \mu}{12}$

$-1.5(12) = 30 - \mu$

$-18 - 30 = 30 - \mu - 30$

$-48 = -\mu$

$48 = \mu$