Chapter 11 **Mathematical Induction** Date____________________

***Mathematical Induction*** is a method of proof used to show that an algebraic statement is true for all positive integers.

To prove a statement is true for all positive integers *n* :

1. Show that the statement is true for $n = 1$.
2. Assume that the statement is true for $n = k - 1$, where $k - 1$ is any positive integer
3. Show that the statement is true for the next positive integer, $n = k$.

*Example 1.* **Prove by induction:** $1 + 4 + 7 + \ldots + (3n - 2) = \dfrac{n(3n-1)}{2}$

1 For $n=1$, $(3 \cdot 1 - 2) = \dfrac{1(3 \cdot 1 - 1)}{2}$

$$1 = 1\left(\frac{2}{2}\right)$$

$$1 = 1 \checkmark$$

② Assume that $1+4+7+\cdots+\big(3(k-1)-2\big) = \dfrac{(k-1)\big(3(k-1)-1\big)}{2}$

③ Show that $1+4+7+\cdots+(3k-2) = \dfrac{k(3k-1)}{2}$

Proof:

$$1+4+7+\cdots+\big(3(k-1)-2\big) = \frac{(k-1)(3k-4)}{2}$$

$$1+4+7+\cdots+\big(3(k-1)-2\big)+(3k-2) = \frac{(k-1)(3k-4)}{2}+(3k-2)$$

$$= \frac{3k^2-7k+4+6k-4}{2}$$

$$= \frac{3k^2-k}{2}$$

$$= \frac{k(3k-1)}{2}$$

Ex2: Prove by Induction: $\sum_{i=1}^{n} 5^i = \frac{5^{n+1} - 5}{4}$

(1) For $n=1$, $5^1 = \frac{5^{1+1} - 5}{4}$

$$5 = \frac{20}{4}$$

$$5 = 5$$

(2) Assume $5^1 + 5^2 + 5^3 + \cdots + 5^{k-1} = \dfrac{5^{(k-1)+1} - 5}{4}$

(3) Show $5^1 + 5^2 + 5^3 + \cdots + 5^{k} = \dfrac{5^{k+1} - 5}{4}$ (Simplify)

Proof: $5^1 + 5^2 + 5^3 + \cdots + 5^{k-1} = \dfrac{5^{k} - 5}{4}$

$$5^1 + 5^2 + 5^3 + \cdots + 5^{k-1} + 5^k = \frac{5^k - 5}{4} + \frac{5^k}{1}$$

$$= \frac{5^k - 5 + 4 \cdot 5^k}{4}$$

$$= \frac{5^1 \cdot 5^k - 5}{4}$$

$$= \frac{5^{k+1} - 5}{4}$$

HW 37

Algebra 2 Honors Math Induction Problems

**Using the model from the class notes, prove by math induction:**

1. $\sum_{i=1}^{n} 2i = n^2 + n$

2. $\sum_{i=1}^{n} (2i - 1) = n^2$

**3.** $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$