14-7: Double Angle Identities

★ identities 10-14

derive: $\sin 2A = \sin(A+A) = \sin A\cos A + \cos A\sin A$

$$\boxed{\sin 2A = 2\sin A\cos A}$$

$\cos 2A = \cos(A+A) = \cos A\cos A - \sin A\sin A$

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$$

$\cos^2 A - \sin^2 A$
$\cos^2 A - (1-\cos^2 A)$
$2\cos^2 A - 1$

$\tan 2A = \tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A\tan A} = \dfrac{2\tan A}{1-\tan^2 A}$

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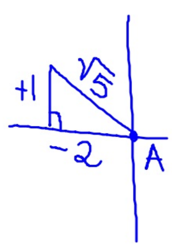
ex. 1: Given: $\tan A = \dfrac{-1}{2},\ \dfrac{\pi}{2} < A \le \pi$

Q2

find $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A} = \dfrac{2\left(-\frac{1}{2}\right)}{1-\frac{1}{4}} = \dfrac{-1}{\frac{3}{4}} = \dfrac{-4}{3}$

find $\cos 2A = 1 - 2\sin^2 A = 1 - 2\left(\dfrac{1}{\sqrt{5}}\right)^2$

$1 - 2\left(\dfrac{1}{5}\right) = \dfrac{3}{5}$



ex. 2: simplify to a trig function of a single angle and find the exact value

$$2\sin \overset{\alpha}{157.5^\circ} \cos \overset{\alpha}{157.5^\circ} = \sin 2(157.5^\circ) = \boxed{\sin 315^\circ = -\frac{\sqrt{2}}{2}}$$

$$\frac{2\tan \overset{\alpha}{105^\circ}}{1-\tan^2 \underset{\alpha}{105^\circ}} = \tan 2(105^\circ) = \boxed{\tan 210^\circ = \frac{\sqrt{3}}{3}}$$