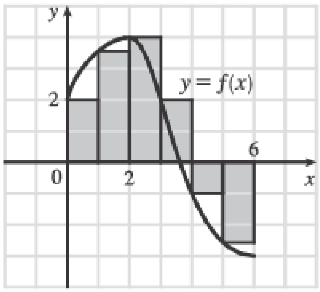
1. (a)



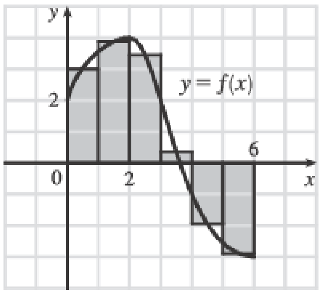

$$L_6 = \sum_{i=1}^{6} f(x_{i-1})\,\Delta x \quad [\Delta x = \tfrac{6-0}{6} = 1]$$

$$= f(x_0)\cdot 1 + f(x_1)\cdot 1 + f(x_2)\cdot 1 + f(x_3)\cdot 1 + f(x_4)\cdot 1 + f(x_5)\cdot 1$$

$$\approx 2 + 3.5 + 4 + 2 + (-1) + (-2.5) = 8$$

The Riemann sum represents the sum of the areas of the four rectangles above the $x$-axis minus the sum of the areas of the two rectangles below the $x$-axis.

(b)



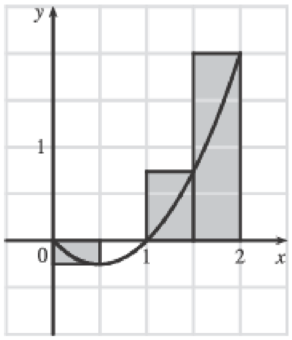

$$M_6 = \sum_{i=1}^{6} f(\overline{x}_i)\,\Delta x \quad [\Delta x = \tfrac{6-0}{6} = 1]$$

$$= f(\overline{x}_1)\cdot 1 + f(\overline{x}_2)\cdot 1 + f(\overline{x}_3)\cdot 1 + f(\overline{x}_4)\cdot 1 + f(\overline{x}_5)\cdot 1 + f(\overline{x}_6)\cdot 1$$

$$= f(0.5) + f(1.5) + f(2.5) + f(3.5) + f(4.5) + f(5.5)$$

$$\approx 3 + 3.9 + 3.4 + 0.3 + (-2) + (-2.9) = 5.7$$

**2.** (a)




$f(x) = x^2 - x$ and $\Delta x = \frac{2-0}{4} = 0.5 \quad \Rightarrow$

$$R_4 = 0.5f(0.5) + 0.5f(1) + 0.5f(1.5) + 0.5f(2)$$
$$= 0.5(-0.25 + 0 + 0.75 + 2) = 1.25$$

The Riemann sum represents the sum of the areas of the two rectangles above the $x$-axis minus the area of the rectangle below the $x$-axis. (The second rectangle vanishes.)

(b) $\int_0^2 (x^2 - x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad [\Delta x = 2/n \text{ and } x_i = 2i/n]$

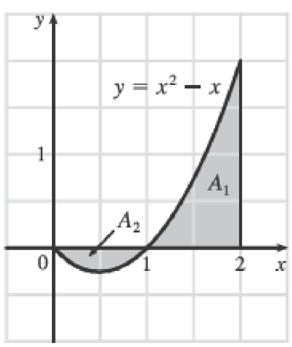
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} - \frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\sum_{i=1}^{n} i^2 - \frac{2}{n}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty} \left[\frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{4}{n^2} \cdot \frac{n(n+1)}{2}\right] = \lim_{n\to\infty} \left[\frac{4}{3} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n} - 2 \cdot \frac{n+1}{n}\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{3}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) - 2\left(1 + \frac{1}{n}\right)\right] = \tfrac{4}{3} \cdot 1 \cdot 2 - 2 \cdot 1 = \tfrac{2}{3}$$

(c) $\int_0^2 (x^2 - x)\,dx = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_0^2 = \left(\frac{8}{3} - 2\right) = \frac{2}{3}$

(d)

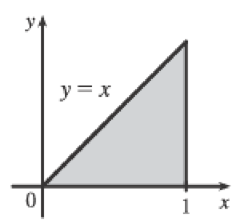


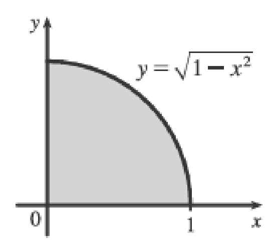

$\int_0^2 (x^2 - x)\,dx = A_1 - A_2$, where $A_1$ and $A_2$ are the areas shown in the diagram.

**3.** $\int_0^1 \left(x + \sqrt{1 - x^2}\right)dx = \int_0^1 x\,dx + \int_0^1 \sqrt{1 - x^2}\,dx = I_1 + I_2.$

$I_1$ can be interpreted as the area of the triangle shown in the figure and $I_2$ can be interpreted as the area of the quarter-circle.

Area $= \frac{1}{2}(1)(1) + \frac{1}{4}(\pi)(1)^2 = \frac{1}{2} + \frac{\pi}{4}$.




**4.** On $[0, \pi]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \sin x_i\,\Delta x = \int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi} = -(-1) - (-1) = 2.$

**5.** $\int_0^6 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^6 f(x)\,dx \quad \Rightarrow \quad 10 = 7 + \int_4^6 f(x)\,dx \quad \Rightarrow \quad \int_4^6 f(x)\,dx = 10 - 7 = 3$

8. (a) By FTC2, we have $\int_0^{\pi/2} \frac{d}{dx}\left(\sin\frac{x}{2}\cos\frac{x}{3}\right)dx = \left[\sin\frac{x}{2}\cos\frac{x}{3}\right]_0^{\pi/2} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - 0\cdot 1 = \frac{\sqrt{6}}{4}$.

(b) $\frac{d}{dx}\int_0^{\pi/2} \sin\frac{x}{2}\cos\frac{x}{3}\,dx = 0$, since the definite integral is a constant.

(c) $\frac{d}{dx}\int_x^{\pi/2} \sin\frac{t}{2}\cos\frac{t}{3}\,dt = \frac{d}{dx}\left(-\int_{\pi/2}^{x} \sin\frac{t}{2}\cos\frac{t}{3}\,dt\right) = -\frac{d}{dx}\int_{\pi/2}^{x} \sin\frac{t}{2}\cos\frac{t}{2}\,dt = -\sin\frac{x}{2}\cos\frac{x}{3}$, by FTC1.

9. $\int_1^2 (8x^3+3x^2)\,dx = \left[8\cdot\frac{1}{4}x^4 + 3\cdot\frac{1}{3}x^3\right]_1^2 = \left[2x^4+x^3\right]_1^2 = (2\cdot 2^4 + 2^3) - (2+1) = 40 - 3 = 37$

10. $\int_0^T (x^4 - 8x + 7)\,dx = \left[\frac{1}{5}x^5 - 4x^2 + 7x\right]_0^T = \left(\frac{1}{5}T^5 - 4T^2 + 7T\right) - 0 = \frac{1}{5}T^5 - 4T^2 + 7T$

11. $\int_0^1 (1-x^9)\,dx = \left[x - \frac{1}{10}x^{10}\right]_0^1 = \left(1 - \frac{1}{10}\right) - 0 = \frac{9}{10}$

12. Let $u = 1-x$, so $du = -dx$ and $dx = -du$. When $x=0$, $u=1$; when $x=1$, $u=0$. Thus,

$\int_0^1 (1-x)^9\,dx = \int_1^0 u^9(-du) = \int_0^1 u^9\,du = \frac{1}{10}\left[u^{10}\right]_0^1 = \frac{1}{10}(1-0) = \frac{1}{10}$.

13. $\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du = \int_1^9 (u^{-1/2} - 2u)\,du = \left[2u^{1/2} - u^2\right]_1^9 = (6-81) - (2-1) = -76$

14. $\int_0^1 \left(\sqrt[4]{u}+1\right)^2\,du = \int_0^1 (u^{1/2} + 2u^{1/4} + 1)\,du = \left[\frac{2}{3}u^{3/2} + \frac{8}{5}u^{5/4} + u\right]_0^1 = \left(\frac{2}{3} + \frac{8}{5} + 1\right) - 0 = \frac{49}{15}$

15. Let $u = y^2+1$, so $du = 2y\,dy$ and $y\,dy = \frac{1}{2}\,du$. When $y=0$, $u=1$; when $y=1$, $u=2$. Thus,

$\int_0^1 y(y^2+1)^5\,dy = \int_1^2 u^5\left(\frac{1}{2}\,du\right) = \frac{1}{2}\left[\frac{1}{6}u^6\right]_1^2 = \frac{1}{12}(64-1) = \frac{63}{12} = \frac{21}{4}$.

16. Let $u = 1+y^3$, so $du = 3y^2\,dy$ and $y^2\,dy = \frac{1}{3}\,du$. When $y=0$, $u=1$; when $y=2$, $u=9$. Thus,

$\int_0^2 y^2\sqrt{1+y^3}\,dy = \int_1^9 u^{1/2}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_1^9 = \frac{2}{9}(27-1) = \frac{52}{9}$.

17. $\int_1^5 \frac{dt}{(t-4)^2}$ does not exist because the function $f(t) = \frac{1}{(t-4)^2}$ has an infinite discontinuity at $t=4$; that is, $f$ is discontinuous on the interval $[1,5]$.

18. Let $u = 3\pi t$, so $du = 3\pi\,dt$. When $t=0$, $u=1$; when $t=1$, $u=3\pi$. Thus,

$\int_0^1 \sin(3\pi t)\,dt = \int_0^{3\pi} \sin u\left(\frac{1}{3\pi}\,du\right) = \frac{1}{3\pi}\left[-\cos u\right]_0^{3\pi} = -\frac{1}{3\pi}(-1-1) = \frac{2}{3\pi}$.

19. Let $u = v^3$, so $du = 3v^2\,dv$. When $v=0$, $u=0$; when $v=1$, $u=1$. Thus,

$\int_0^1 v^2\cos(v^3)\,dv = \int_0^1 \cos u\left(\frac{1}{3}\,du\right) = \frac{1}{3}\left[\sin u\right]_0^1 = \frac{1}{3}(\sin 1 - 0) = \frac{1}{3}\sin 1$.

**20.** $\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx = 0$ by Theorem 5.5.6(b), since $f(x) = \frac{\sin x}{1+x^2}$ is an odd function.

**21.** $\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2+\cos t}\,dt = 0$ by Theorem 5.5.6(b), since $f(t) = \frac{t^4 \tan t}{2+\cos t}$ is an odd function.

**22.** Let $u = x^2 + 4x$. Then $du = (2x+4)\,dx = 2(x+2)\,dx$, so

$$\int \frac{x+2}{\sqrt{x^2+4x}}\,dx = \int u^{-1/2}\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\cdot 2u^{1/2} + C = \sqrt{u} + C = \sqrt{x^2+4x} + C.$$

**23.** Let $u = \sin \pi t$. Then $du = \pi \cos \pi t\,dt$, so $\int \sin \pi t \cos \pi t\,dt = \int u\left(\frac{1}{\pi}\,du\right) = \frac{1}{\pi}\cdot\frac{1}{2}u^2 + C = \frac{1}{2\pi}(\sin \pi t)^2 + C$.

**24.** Let $u = \cos x$. Then $du = -\sin x\,dx$, so $\int \sin x \cos(\cos x)\,dx = -\int \cos u\,du = -\sin u + C = -\sin(\cos x) + C$.

**25.** Let $u = 2\theta$. Then $du = 2\,d\theta$, so

$$\int_0^{\pi/8} \sec 2\theta \tan 2\theta\,d\theta = \int_0^{\pi/4} \sec u \tan u\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\left[\sec u\right]_0^{\pi/4} = \tfrac{1}{2}\left(\sec\tfrac{\pi}{4} - \sec 0\right) = \tfrac{1}{2}\left(\sqrt{2}-1\right) = \tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}.$$

**26.** Let $u = 1 + \tan t$, so $du = \sec^2 t\,dt$. When $t = 0$, $u = 1$; when $t = \frac{\pi}{4}$, $u = 2$. Thus,

$$\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt = \int_1^2 u^3\,du = \left[\tfrac{1}{4}u^4\right]_1^2 = \tfrac{1}{4}(2^4 - 1^4) = \tfrac{1}{4}(16-1) = \tfrac{15}{4}.$$

**27.** Since $x^2 - 4 < 0$ for $0 \le x < 2$ and $x^2 - 4 > 0$ for $2 < x \le 3$, we have $|x^2-4| = -(x^2-4) = 4 - x^2$ for $0 \le x < 2$ and $|x^2-4| = x^2 - 4$ for $2 < x \le 3$. Thus,

$$\begin{aligned}\int_0^3 |x^2-4|\,dx &= \int_0^2 (4-x^2)\,dx + \int_2^3 (x^2-4)\,dx = \left[4x - \frac{x^3}{3}\right]_0^2 + \left[\frac{x^3}{3} - 4x\right]_2^3 \\ &= \left(8 - \tfrac{8}{3}\right) - 0 + (9-12) - \left(\tfrac{8}{3} - 8\right) = \tfrac{16}{3} - 3 + \tfrac{16}{3} = \tfrac{32}{3} - \tfrac{9}{3} = \tfrac{23}{3}\end{aligned}$$

**28.** Since $\sqrt{x} - 1 < 0$ for $0 \le x < 1$ and $\sqrt{x} - 1 > 0$ for $1 < x \le 4$, we have $\left|\sqrt{x}-1\right| = -\left(\sqrt{x}-1\right) = 1 - \sqrt{x}$ for $0 \le x < 1$ and $\left|\sqrt{x}-1\right| = \sqrt{x} - 1$ for $1 < x \le 4$. Thus,

$$\begin{aligned}\int_0^4 \left|\sqrt{x}-1\right|\,dx &= \int_0^1 \left(1-\sqrt{x}\right)dx + \int_1^4 \left(\sqrt{x}-1\right)dx = \left[x - \tfrac{2}{3}x^{3/2}\right]_0^1 + \left[\tfrac{2}{3}x^{3/2} - x\right]_1^4 \\ &= \left(1 - \tfrac{2}{3}\right) - 0 + \left(\tfrac{16}{3} - 4\right) - \left(\tfrac{2}{3} - 1\right) = \tfrac{1}{3} + \tfrac{16}{3} - 4 + \tfrac{1}{3} = 6 - 4 = 2\end{aligned}$$

**33.** $F(x) = \int_0^x \frac{t^2}{1+t^3}\,dt \quad\Rightarrow\quad F'(x) = \frac{d}{dx}\int_0^x \frac{t^2}{1+t^3}\,dt = \frac{x^2}{1+x^3}$

**34.** $F(x) = \int_x^1 \sqrt{t+\sin t}\,dt = -\int_1^x \sqrt{t+\sin t}\,dt \quad\Rightarrow\quad F'(x) = -\frac{d}{dx}\int_1^x \sqrt{t+\sin t}\,dt = -\sqrt{x+\sin x}$

**35.** Let $u = x^4$. Then $\frac{du}{dx} = 4x^3$. Also, $\frac{dg}{dx} = \frac{dg}{du}\frac{du}{dx}$, so

$$g'(x) = \frac{d}{dx}\int_0^{x^4} \cos(t^2)\,dt = \frac{d}{du}\int_0^{u} \cos(t^2)\,dt \cdot \frac{du}{dx} = \cos(u^2)\,\frac{du}{dx} = 4x^3\cos(x^8).$$

**36.** Let $u = \sin x$. Then $\frac{du}{dx} = \cos x$. Also, $\frac{dg}{dx} = \frac{dg}{du}\frac{du}{dx}$, so

$$g'(x) = \frac{d}{dx}\int_1^{\sin x} \frac{1-t^2}{1+t^4}\,dt = \frac{d}{du}\int_1^{u} \frac{1-t^2}{1+t^4}\,dt \cdot \frac{du}{dx} = \frac{1-u^2}{1+u^4}\cdot\frac{du}{dx} = \frac{1-\sin^2 x}{1+\sin^4 x}\cdot\cos x = \frac{\cos^3 x}{1+\sin^4 x}$$

**37.** $y = \int_{\sqrt{x}}^{x} \frac{\cos\theta}{\theta}\,d\theta = \int_1^{x} \frac{\cos\theta}{\theta}\,d\theta + \int_{\sqrt{x}}^{1} \frac{\cos\theta}{\theta}\,d\theta = \int_1^{x} \frac{\cos\theta}{\theta}\,d\theta - \int_1^{\sqrt{x}} \frac{\cos\theta}{\theta}\,d\theta \quad\Rightarrow$

$$y' = \frac{\cos x}{x} - \frac{\cos\sqrt{x}}{\sqrt{x}}\frac{1}{2\sqrt{x}} = \frac{2\cos x - \cos\sqrt{x}}{2x}$$

**38.** $y = \int_{2x}^{3x+1} \sin(t^4)\,dt = \int_{2x}^{0} \sin(t^4)\,dt + \int_0^{3x+1} \sin(t^4)\,dt = \int_0^{3x+1} \sin(t^4)\,dt - \int_0^{2x} \sin(t^4)\,dt \quad\Rightarrow$

$$y' = \sin\left[(3x+1)^4\right]\cdot\frac{d}{dx}(3x+1) - \sin\left[(2x)^4\right]\cdot\frac{d}{dx}(2x) = 3\sin\left[(3x+1)^4\right] - 2\sin\left[(2x)^4\right]$$

**39.** If $1 \le x \le 3$, then $\sqrt{1^2+3} \le \sqrt{x^2+3} \le \sqrt{3^2+3} \quad\Rightarrow\quad 2 \le \sqrt{x^2+3} \le 2\sqrt{3}$, so

$2(3-1) \le \int_1^3 \sqrt{x^2+3}\,dx \le 2\sqrt{3}(3-1)$; that is, $4 \le \int_1^3 \sqrt{x^2+3}\,dx \le 4\sqrt{3}$.

**40.** If $3 \le x \le 5$, then $4 \le x+1 \le 6$ and $\frac{1}{6} \le \frac{1}{x+1} \le \frac{1}{4}$, so $\frac{1}{6}(5-3) \le \int_3^5 \frac{1}{x+1}\,dx \le \frac{1}{4}(5-3)$;

that is, $\frac{1}{3} \le \int_3^5 \frac{1}{x+1}\,dx \le \frac{1}{2}$.

**41.** $0 \le x \le 1 \quad\Rightarrow\quad 0 \le \cos x \le 1 \quad\Rightarrow\quad x^2\cos x \le x^2 \quad\Rightarrow\quad \int_0^1 x^2\cos x\,dx \le \int_0^1 x^2\,dx = \frac{1}{3}\left[x^3\right]_0^1 = \frac{1}{3}$ [Property 7].

**42.** On the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$, $x$ is increasing and $\sin x$ is decreasing, so $\frac{\sin x}{x}$ is decreasing. Therefore, the largest value of $\frac{\sin x}{x}$ on $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ is $\frac{\sin(\pi/4)}{\pi/4} = \frac{\sqrt{2}/2}{\pi/4} = \frac{2\sqrt{2}}{\pi}$. By Property 8 with $M = \frac{2\sqrt{2}}{\pi}$ we get $\int_{\pi/4}^{\pi/2} \frac{\sin x}{x}\,dx \le \frac{2\sqrt{2}}{\pi}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

**43.** $\Delta x = (3-0)/6 = \frac{1}{2}$, so the endpoints are $0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$, and $3$, and the midpoints are $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$, and $\frac{11}{4}$.

The Midpoint Rule gives

$$\int_0^3 \sin(x^3)\,dx \approx \sum_{i=1}^{6} f(\overline{x}_i)\,\Delta x = \frac{1}{2}\left[\sin\left(\tfrac{1}{4}\right)^3 + \sin\left(\tfrac{3}{4}\right)^3 + \sin\left(\tfrac{5}{4}\right)^3 + \sin\left(\tfrac{7}{4}\right)^3 + \sin\left(\tfrac{9}{4}\right)^3 + \sin\left(\tfrac{11}{4}\right)^3\right] \approx 0.280981.$$

**44.** (a) Displacement $= \int_0^5 (t^2 - t)\,dt = \left[\frac{1}{3}t^3 - \frac{1}{2}t^2\right]_0^5 = \frac{125}{3} - \frac{25}{2} = \frac{175}{6} = 29.1\overline{6}$ meters

(b) Distance traveled $= \int_0^5 |t^2 - t|\,dt = \int_0^5 |t(t-1)|\,dt = \int_0^1 (t - t^2)\,dt + \int_1^5 (t^2 - t)\,dt$

$= \left[\frac{1}{2}t^2 - \frac{1}{3}t^3\right]_0^1 + \left[\frac{1}{3}t^3 - \frac{1}{2}t^2\right]_1^5 = \frac{1}{2} - \frac{1}{3} - 0 + \left(\frac{125}{3} - \frac{25}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{177}{6} = 29.5$ meters