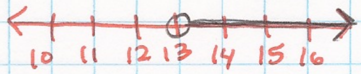
Algebra 1 Spring Semester

Final Study Guide

1) $x - 12 > 1$

$+12 \quad +12$

$\boxed{x > 13}$



2) $7 + z < 3$

$-7 \quad -7$

$\boxed{z < -4}$

3) $4x - 5 < 2x + 11$

$-2x \quad -2x$

$2x - 5 < 11$

$+5 \quad +5$

$\frac{2x}{2} < \frac{16}{2}$

$\boxed{x < 8}$

4) $5(p+2) - 2(p-1) \geq 7p + 4$

$5p + 10 - 2p + 2 \geq 7p + 4$

$3p + 12 \geq 7p + 4$

$3p + 12 - 7p \geq \cancel{7p} + 4 \cancel{-7p}$ 0

$-4p + 12 \geq 4$

$-4p + \cancel{12 - 12} \geq 4 - 12$ 0

$\frac{-4p}{-4} \geq \frac{-8}{-4} \rightarrow \boxed{p \leq 2}$

5) ~~Skip #5~~

~~Here's the solution, if you're interested~~

$|3-2x| \geq 1$

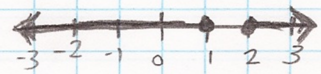
$3-2x \geq 1$ or $3-2x \leq -1$

$-3 \quad -3 \qquad -3 \quad -3$

$-2x \geq -2 \qquad -2x \leq -4$

$\overline{-2} \quad \overline{-2} \qquad \overline{-2} \quad \overline{-2}$

$\boxed{x \leq 1 \quad \text{or} \quad x \geq 2}$



6) $3y^5 \cdot y^3$

$3y^{5+3} = \boxed{3y^8}$

7) $(9m^3n^5)(-2mn^2)$

$-18m^{3+1}n^{5+2}$

$\boxed{-18m^4n^7}$

8) $(w^5y^4)^3 = (w^5)^3 \cdot (y^4)^3$

$= w^{5\cdot3}\, y^{4\cdot3}$

$= \boxed{w^{15}y^{12}}$

9) $\frac{16r^3s^{-5}}{4r^{-1}s^2} = 4r^{3-(-1)}s^{-5-2}$

$= 4r^4s^{-7} = \boxed{\frac{4r^4}{s^7}}$

10) $\frac{(-8x^2y^2)^2}{(4x^3y)^3} = \frac{\cancel{64}^{1}x^4y^4}{\cancel{64}x^9y^3} = x^{-5}y^1 = \boxed{\frac{y}{x^5}}$

11) Find the degree of $2x^3y^3 + 4xy - 10x^3y$

* the degree of a monomial is the sum of the exponents of the variables
* the degree of a polynomial is the largest degree of one of its monomials.

$2x^3y^3$: degree is $3+3=6$

$4xy$: degree is $1+1=2$

$-10x^3y$: degree is $3+1=4$

$2x^3y^3+4xy-10x^3y$: degree is $\boxed{6}$ because 6 is greater than 2 & 4

12) Arrange in descending order $4+3x^3y^3-x^5y+xy$

* the degrees of the terms should go in order from greatest to least

4: degree is 0
$3x^3y^3$: degree is $3+3=6$
$-x^5y$: degree is $5+1=6$
xy : degree is $1+1=2$

$\boxed{-x^5y+3x^3y^3+xy+4}$ or $\boxed{3x^3y^3-x^5y+xy+4}$

13) $(5n^2-2ny+3y^2)-(9n^2-8ny-10y^2)$

$5n^2-2ny-3y^2-9n^2-(-8ny)-(-10y^2)$ * Distributive Property

$5n^2-2ny-3y^2-9n^2+8ny+10y^2$ * combine like terms

$\boxed{-4n^2+6ny+7y^2}$

14) $(11m^2-2mn+8n^2)+(8m^2-4mn-2n^2)$

$11m^2-2mn+8n^2+8m^2-4mn-2n^2$ * With addition, ( ) don't matter
→ Associative property

$\boxed{19m^2-6mn+6n^2}$

15) $(x^2+5y)-(2x^2+6y)$

$x^2+5y-2x^2-6y$

$\underline{x^2}+\underline{5y}-\underline{2x^2}-\underline{6y}$

$\boxed{-x^2-y}$

16) $5hk^2(2h^2k-hk^3+4h^2k^2)$

$5hk^2(2h^2k)+5hk^2(-hk^3)+5hk^2(4h^2k^2)$

$\boxed{10h^3k^3-5h^2k^5+20h^3k^4}$

17) $(4x^2+2y^2)(2x^2-y^2)$

$4x^2(2x^2)+4x^2(-y^2)+2y^2(2x^2)+2y^2(-y^2)$

$8x^4-4x^2y^2+4x^2y^2-2y^4 = \boxed{8x^4-2y^4}$

18) $(3s+5)(2s^2-8s+6)$

$3s(2s^2)+3s(-8s)+3s(6)+5(2s^2)+5(-8s)+5(6)$

$6s^3-24s^2+18s+10s^2-40s+30$

$\boxed{6s^3-14s^2-22s+30}$

19) $(5c-4)^2=(5c-4)(5c-4)$

$=25c^2-20c-20c+16$

$=\boxed{25c^2-40c+16}$

20) Find the GCF $12x^3y^2, 44xy^3$

$12x^3y^2=2\cdot2\cdot3\cdot x\cdot x\cdot x\cdot y\cdot y$

$44xy^3=2\cdot2\cdot11\cdot x\cdot y\cdot y\cdot y$

GCF $=2\cdot2\cdot x\cdot y\cdot y$

$=\boxed{4xy^2}$

Factor

21) $35a^3bc^2 - 45a^2b^2c$

$\boxed{5a^2bc(7ac - 9b)}$

22) $t^2 - 11t + 24$

$\boxed{(t-3)(t-8)}$

23) $n^2 + n - 42$

$\boxed{(n+7)(n-6)}$

24) $10y^2 - 31y + 15$

$(10y \quad)(y \quad)$

$+$

$= -31y$

$(5y-3)(2y-5)$

$+$

$= -31y$

$\boxed{(5y-3)(2y-5)}$

25) $36m^2 - 49$

$(6m)^2 - 7^2 \rightarrow \boxed{(6m+7)(6m-7)}$

26) $49w^2 - 25 = 0$ (solve)

$(7w)^2 - 5^2 = 0$

$(7w+5)(7w-5) = 0$

$7w + 5 = 0$
$-5 \quad -5$

$\frac{7w}{7} = \frac{-5}{7}$

$w = -5/7$

$7w - 5 = 0$
$+5 \quad +5$

$\frac{7w}{7} = \frac{5}{7}$

$w = 5/7$

$\boxed{w = \pm 5/7}$

Alternate Method

$49w^2 - \cancel{25} = 0$
$\cancel{+25} \quad +25$

$\frac{49w^2}{49} = \frac{25}{49}$

$\sqrt{w^2} = \pm\sqrt{\frac{25}{49}}$

$\boxed{w = \pm \frac{5}{7}}$

27) $m^2+12m-28$

$\boxed{(m+14)(m-2)}$

28) $5t^2+17t-12$

$(5t-3)(t+4)$

$+$

$=17t$

-12

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$\boxed{(5t-3)(t+4)}$

29) $6p^2-20p+16$

$2(3p^2-10p+8)$

$2(3p-2)(p-4)$

$+$

$=-10p$

$\boxed{2(3p-2)(p-4)}$

30) $49a^2-169$ ← that's a difference of 2 squares

$(7a)^2-13^2$

$a^2-b^2=(a+b)(a-b)$

$\boxed{(7a+13)(7a-13)}$

31) Use table to graph. * Axis of Symmetry is $x=\frac{-b}{2a}$

$y=-x^2+3x+10$

* Plug in that x to find the y-coordinate of the vertex

$x=\frac{-b}{2a}=\frac{-3}{2(-1)}=\frac{-3}{-2}=\frac{3}{2}=1\frac{1}{2}$

* pick x's & find the y's using the formula to graph

$y=-\left(\frac{3}{2}\right)^2+3\left(\frac{3}{2}\right)+10$

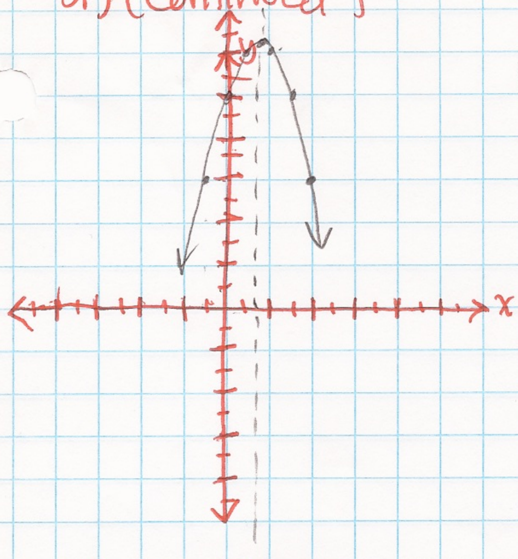
$=-\frac{9}{4}+\frac{9}{2}\cdot\frac{2}{2}+\frac{10}{1}\cdot\frac{4}{4}$

$=-\frac{9}{4}+\frac{18}{4}+\frac{40}{4}=\frac{9}{4}+\frac{40}{4}=\frac{49}{4}=12\frac{1}{4}$

axis of symmetry: $x=3/2$

Vertex: $\left(\frac{3}{2},\frac{49}{4}\right)$

31) (continued)



x	y
$1\frac{1}{2} = 3/2$	$49/4 = 12\frac{1}{4}$
1	12
0	10
-1	6

$y = -x^2 + 3x + 10$

$y = -(1)^2 + 3(1) + 10$
$= -1 + 3 + 10$
$= 2 + 10$
$= 12$

$y = -(0)^2 + 3(0) + 10$
$= 0 + 0 + 10$
$= 10$

$y = -(-1)^2 + 3(-1) + 10$
$= -1 - 3 + 10$
$= -4 + 10$
$= 6$

32) $y = 2x^2 - 3x$

axis of symmetry

$x = \frac{-b}{2a} = \frac{-(-3)}{2(2)} = \frac{3}{4}$

$x = 3/4$

Vertex: $y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right)$

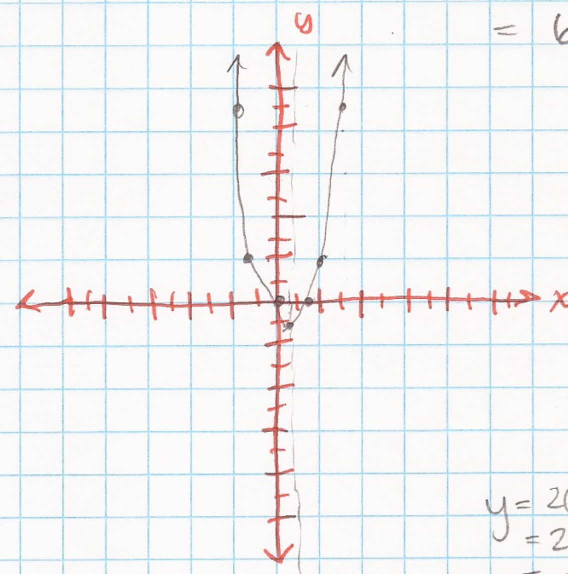
$= \cancel{2}\left(\frac{9}{\cancel{16}_8}\right) - \frac{9}{4}$

$= \frac{9}{8} - \frac{9}{4} \cdot \frac{2}{2}$

$= \frac{9}{8} - \frac{18}{8} = \frac{-9}{8}$

$(3/4, -9/8)$

$-1\frac{1}{8}$



x	y
3/4	$-1\frac{1}{8}$
0	0
2	2
3	9

$y = 2(2)^2 - 3(2)$
$= 2(4) - 6$
$= 8 - 6$
$= 2$

$y = 2(3)^2 - 3(3)$
$= 2(9) - 9$
$= 18 - 9$
$= 9$

wrong problem!

Solve Using quadratic formula

33) $7m^2 + 8m = 3$
$\quad -3 \quad -3$

$7m^2 + 8m - 3 = 0$

$a = 7$
$b = 8$
$c = -3$

*quadratic formula:
the solution(s) to an equation in the form
$ax^2 + bx + c = 0$
can be found using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

33) (Continued)

$$\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array}$$

$$m = \frac{-(8) \pm \sqrt{(8)^2 - 4(7)(-3)}}{2(7)} = \frac{-8 \pm \sqrt{64 - 28(-3)}}{14} = \frac{-8 \pm \sqrt{64 + 84}}{14}$$

$$= \frac{-8 \pm \sqrt{148}}{14} = \frac{-8 \pm \sqrt{4 \cdot 37}}{14} = \frac{-8 \pm 2\sqrt{37}}{14}$$

148
4 37

$$= \frac{\cancel{2}^{1}(-4 \pm \sqrt{37})}{\cancel{14}\,7} = \boxed{\frac{-4 \pm \sqrt{37}}{7}}$$

34) $r^2 + 16r + 21 = 0$

$a = 1$
$b = 16$
$c = 21$

$$r = \frac{-(16) \pm \sqrt{(16)^2 - 4(1)(21)}}{2(1)} = \frac{-16 \pm \sqrt{256 - 84}}{2}$$

$$\begin{array}{r} 256 \\ 84 \\ \hline 172 \end{array}$$

$$= \frac{-16 \pm \sqrt{172}}{2} = \frac{-16 \pm \sqrt{4 \cdot 43}}{2} = \frac{-16 \pm 2\sqrt{43}}{2}$$

$$= \frac{\cancel{2}^{1}(-8 \pm \sqrt{43})}{\cancel{2}} = \boxed{-8 \pm \sqrt{43}}$$

~~State the value of the discriminant & determine~~

~~35)~~ 33) $15n^2 - 3 = 4n$

$-4n \quad -4n$

$15n^2 - 4n - 3 = 0$

$a = 15$
$b = -4$
$c = -3$

$$n = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(15)(-3)}}{2(15)}$$

$$= \frac{4 \pm \sqrt{16 - 60(-3)}}{30} = \frac{4 \pm \sqrt{16 + 180}}{30}$$

$$= \frac{4 \pm \sqrt{196}}{30} = \frac{4 \pm 14}{30}$$

$$n = \frac{4 + 14}{30} = \frac{18}{30} = \frac{3}{5}$$

$$n = \frac{4 - 14}{30} = \frac{-10}{30} = -\frac{1}{3}$$

$$\boxed{-\frac{1}{3}, \frac{3}{5}}$$

35) $7m^2 + 8m = 3$
$\quad\quad\quad\quad -3 \quad -3$

$7m^2+8m-3=0$

$a=7$
$b=8$
$c=-3$

b^2-4ac
$(8)^2-4(7)(-3)$
$64-28(-3)$
$64+84$
$\boxed{148}$

$148>0$ so there are (2) roots

* Discriminant $= b^2-4ac$

→ if $b^2-4ac>0$, then 2 roots
→ if $b^2-4ac=0$, then 1 root
→ if $b^2-4ac<0$, then 0 real roots.

36) $4p^2 = 4p-1$
$-4p+1 \quad -4p+1$

$4p^2-4p+1=0$

$a=4$
$b=-4$
$c=1$

b^2-4ac
$=(-4)^2-4(4)(1)$
$= 16 - 16$
$= \boxed{0}$

$0=0$ so there is $\boxed{1}$ root

37) axis of symmetry, vertex, max or min.

$y=-2x^2+4x-5$

$a=-2$
$b=4$
$c=-5$

$x=\frac{-b}{2a} = \frac{-(4)}{2(-2)} = \frac{-4}{-4} = 1$

axis of symmetry is $\boxed{x=1}$

$y=-2(1)^2+4(1)-5$
$= -2+4-5$
$= 2-5$
$= -3$

Vertex: $\boxed{(1,-3)}$

Since $a=-2$ then the parabola opens downward & the vertex is the $\boxed{\text{maximum}}$ y-value

* if $a>0$ → & vertex is minimum

* if $a<0$ → & vertex is maximum

38) $y = -x + 4$ ①
$y = x - 4$ ②

*$y = mx + b$

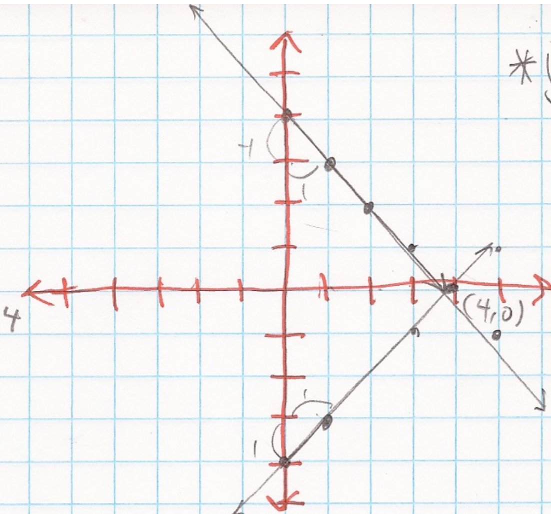
m: slope
b: y-intercept

① $y = -x + 4$

$m = -1$
$b = 4$

② $y = x - 4$

$m = 1$
$b = -4$



$\boxed{\text{One solution, } (4,0)}$

39) $2x - y = -3$ ①
$6x - 3y = -9$ ②

① $2x - y = -3$
$\quad -2x \qquad -2x$

$-y = -2x - 3$

$-1(-y) = -1(-2x - 3)$
$y = 2x + 3$

$m = 2 \quad b = 3$

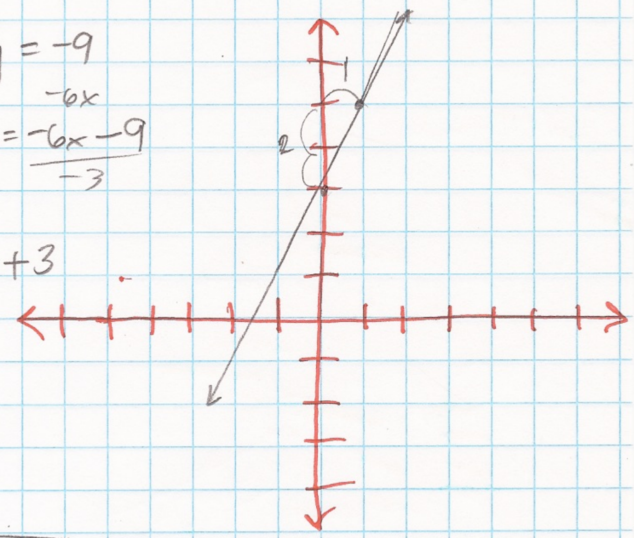
② $6x - 3y = -9$
$-6x \qquad -6x$

$\frac{-3y}{-3} = \frac{-6x - 9}{-3}$

$y = 2x + 3$

$m = 2$

$b = 3$



$\boxed{\text{infinitely many solutions}}$

it's the same line

40) $y = 3x$ ①
$x + y = 4$ ②

*Solve one equation for either x or y
* replace x or y in the second equation with the expression it's equal to.

② $x + y = 4$

$x + (3x) = 4$

$\frac{4x}{4} = \frac{4}{4}$

$x = 1$

① $y = 3x$

$y = 3(1)$

$y = 3$

$\boxed{(1,3)}$

41) $x + 4y = -8$
$x - 4y = -8$ +

$2x + 0 = -16$

$\frac{2x}{2} = \frac{-16}{2}$

$x = -8$

$x + 4y = -8$

$-8 + 4y = -8$
$+8 \qquad +8$

$\frac{4y}{4} = \frac{0}{4}$

$y = 0$

$\boxed{(-8, 0)}$

42) (2,5) and (3,6)

$m = \frac{6-5}{3-2} = \frac{1}{1} = \boxed{1}$

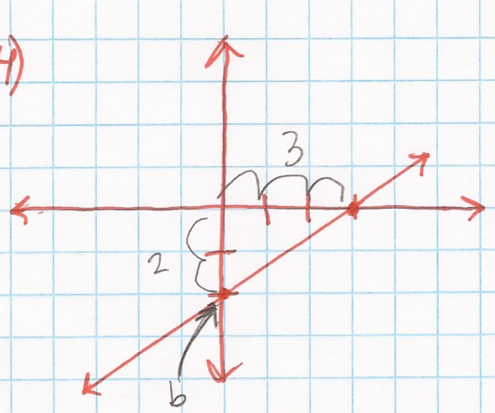
* to find the slope between two points (x_1, y_1) & (x_2, y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

43) (-1,3) and (6,3)

$m = \frac{3-3}{6-(-1)} = \frac{0}{6+1} = \frac{0}{7} = \boxed{0}$

44)

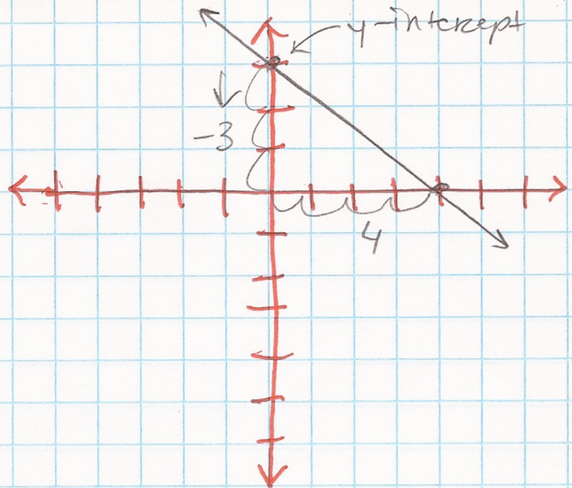


$y = mx + b$

$m = \frac{rise}{run} = \frac{2}{3}$ $\qquad b = -2$

$\boxed{y = \frac{2}{3}x - 2}$

45) y-int 3
slope $-3/4$



46) $(-1,-2)$ $(3,4)$

$y=mx+b$ $\quad m=\dfrac{4-(-2)}{3-(-1)}=\dfrac{4+2}{3+1}=\dfrac{6}{4}=\dfrac{3}{2}$

$y=\frac{3}{2}x+b$

use one of the points' coordinates

$(3,4) \rightarrow x=3, y=4$

$4=\frac{3}{2}(3)+b$

$4=\frac{9}{2}+b$

$\frac{4}{1}-\frac{9}{2}=\cancel{\frac{9}{2}}+b\cancel{-\frac{9}{2}}$

$\frac{8}{2}-\frac{9}{2}=b=-\frac{1}{2}$

$\boxed{y=\frac{3}{2}x-\frac{1}{2}}$

47) undefined slope $(-6,4)$

x y

$\boxed{x=-6}$

*Standard form: $Ax+By=C$

*a vertical line has an undefined slope

*a vertical line has an equation ~~of~~ $x=$____

48) Domain = $\boxed{\{-3, 0, 4\}}$

*Domain is all the x-values of the relation

All the points of the relation are: $\{(-3,-1), (-3,2), (0,1), (4,3)\}$

the x-values

49) Range = $\boxed{\{-1, 1, 2, 3\}}$

*the range is all the y-values of the relation

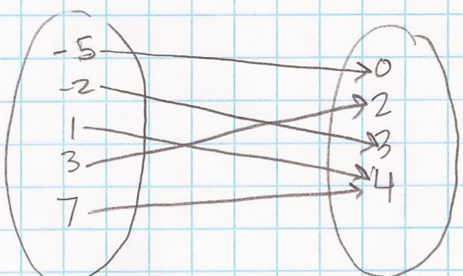
50) the inverse:

the relation: $\{(-3,-1), (-3,2), (0,1), (4,3)\}$

the inverse: $\boxed{\{(-1,-3), (2,-3), (1,0), (3,4)\}}$

*the inverse of a relation contains the same points, but with the x & y-coordinates switched.

51) $\{(1,4), (-2,3), (-5,0), (7,4), (3,2)\}$



52) $x - 2y = 9$

*an x-intercept is where the graph crosses the x-axis

*the y-coordinate is zero

$x - 2(0) = 9$

$x - 0 = 9$

$\boxed{x = 9}$ or $\boxed{(9,0)}$

53) $12 + r = 3$

$\cancel{12} + r \cancel{-12} = 3 - 12$

$\boxed{r = -9}$

54) $-12 = p - 7$

$-12 + 7 = p \cancel{-7 + 7}$

$\boxed{-5 = p}$

55) $31 = -\frac{n}{6}$

$(-6)31 = -\frac{n}{6}\left(\frac{-6}{1}\right)$

$\boxed{-186 = n}$

56) $\frac{9}{25} = \frac{p}{125}$

$\left(\frac{125}{1}\right)\frac{9}{25} = \frac{p}{125}\left(\frac{125}{1}\right)$

$5(9) = p = \boxed{45}$

57) $-3a + 4 = -14$

$-3a \cancel{+4 - 4} = -14 - 4$

$-3a = -18$

$\frac{-3a}{-3} = \frac{-18}{-3}$

$\boxed{a = 6}$

58) $\frac{x}{6} = \frac{2}{9}$

$\left(\frac{6}{1}\right)\left(\frac{x}{6}\right) = \frac{2}{9}\left(\frac{6}{1}\right)$

$x = \frac{2}{3}\left(\frac{2}{1}\right) = \boxed{\frac{4}{3}}$

59) $5n + 7 = 7(n+1) - 2n$

$5n + 7 = 7n + 7 - 2n$

$5n + 7 = 5n + 7$

$\boxed{\text{Identity, all real numbers}}$

60) $-4(p+2) + 8 = 2(p-1) - 7p + 15$

$4p \cancel{-8 + 8} = 2p - 2 - 7p + 15$

$4p = -5p + 13$

$4p + 5p = \cancel{-5p} + 13 + \cancel{5p}$

$9p = 13$

$\frac{9p}{9} = \frac{13}{9}$ $\boxed{p = \frac{13}{9}}$

61) $-\frac{7}{9}y = -6$

$-\frac{9}{7}\left(-\frac{7}{9}y\right) = -\frac{6}{1}\left(\frac{-9}{7}\right)$

$\boxed{y = \frac{54}{7}}$

62) $\frac{10}{27} = \frac{a}{135}$

$\left(\frac{\cancel{135}^{\cancel{15}\,5}}{1}\right)\frac{10}{\cancel{27}_{\cancel{3}\,1}} = \frac{a}{\cancel{135}}\left(\frac{\cancel{135}^{1}}{1}\right)$

$\frac{5}{1}\left(\frac{10}{1}\right) = a$ $\boxed{a=50}$

63) $9-t=t+3$

$9-\cancel{t}+\cancel{t}^{\,0} = t+3+t$

$9 = 2t+3$

$9-3 = 2t+\cancel{3}-\cancel{3}^{\,0}$

$6=2t$

$\frac{6}{2} = \frac{2t}{2} \rightarrow \boxed{t=3}$

64) $2(y-6) = 3y+12-y$

$2y-12 = 2y+12$

$\cancel{2y}-12-\cancel{2y} = \cancel{2y}+12-\cancel{2y}^{\,0}$

$-12=12$ X

$\boxed{\text{no solution}}$

65) $17+3(z-2)-11z = -7(z+2)+14$

$17+3z-6-11z = -7z-14+14$

$-8z+11 = -7z$

$\cancel{-8z}+11\cancel{+8z}^{\,0} = -7z+8z$

$\boxed{11=z}$

66) $3w+(8-v)t$ $w=4$ $v=5$ $t=2$

$3(4)+(8-5)(2)$

$12+(3)(2)$

$12+6$

$\boxed{18}$

67) $4(5\cdot 1\div 20)$

$4(5\div 20)$ Identity

$4\left(\frac{1}{4}\right)$ substitution

$\boxed{1}$ inverse

68) $7(2y+1)+3y$

$14y+7+3y$

$\boxed{17y+7}$

69) $\frac{1}{3}n+27$ *sum → add

70) $4n^2$ *product → multiply

71) $4w+(v-5)t$ $w=2,\ v=8,\ t=4$

$4(2)+(8-5)(4)$

$8+(3)(4)=8+12=\boxed{20}$

72) $\frac{6+4^2\cdot 3}{10-1}=y=\frac{6+16\cdot 3}{9}=\frac{6+48}{9}=\frac{54}{9}=\boxed{6}$

73) $6(6\cdot 1\div 36)$

$6(6\div 36)$ identity

$6\left(\frac{1}{6}\right)$ substitution

$\boxed{1}$ inverse