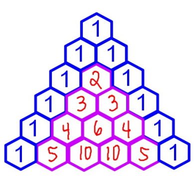
8-6
Binomial Distributions
Day 1
Binomial Theorem

Pascal's Triangle

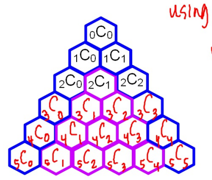
If you arrange the values of n C r in a triangular pattern in which each row corresponds to a value of n, you get what is called Pascal's triangle. It is named after the famous French mathematician Blaise Pascal (1623 - 1662).

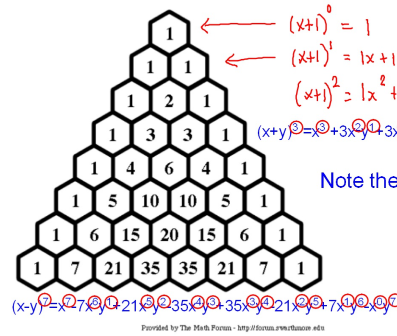
Pascal's Triangle



Binomial Theorem △

using $_nC_r$





$(x+1)^0 = 1$

$(x+1)^1 = 1x + 1$

$(x+1)^2 = 1x^2 + 2x + 1$

$(x+y)^3 = x^3 + 3x^2y^1 + 3x^1y^2 + x^0y^3$ $(x+1)^3$

Note the sum of the exponents!

$(x-y)^7 = x^7 - 7x^6y^1 + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7x^1y^6 - x^0y^7$

$(x+1)^4$

$1x^4 + 4x^3 + 6x^2 + 4x + 1$

Binomial Expansion:

$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + \ldots + {}_nC_n a^0 b^n$

$$\sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

$(x+2)^4$

${}_4C_0 (x)^{4-0} (2)^0 = 1x^4(1) = x^4$

${}_4C_1 (x)^{4-1} (2)^1 = 4x^3(2) = 8x^3$

${}_4C_2 (x)^{4-2} (2)^2 = 6x^2(4) = 24x^2$

${}_4C_3 (x)^{4-3} (2)^3 = 4x(8) = 32x$

${}_4C_4 (x)^{4-4} (2)^4 = 1(1)(16) = 16$

$x^4 + 8x^3 + 24x^2 + 32x + 16$

$(a+3)^5$

$${}_5C_0\,(a)^5(3)^0 = 1\,a^5(1)$$
$${}_5C_1\,(a)^4(3)^1 = 5\,a^4(3)$$
$${}_5C_2\,(a)^3(3)^2 = 10\,a^3(9)$$
$${}_5C_3\,(a)^2(3)^3 = 10\,a^2(27)$$
$${}_5C_4\,(a)^1(3)^4 = 5\,a(81)$$
$${}_5C_5\,(a)^0(3)^5 = 1(1)(243)$$

$$a^5+15a^4+90a^3+270a^2+405a+243$$

$(u+v^2)^3$

$${}_3C_0\,(u)^3(v^2)^0 = 1\,u^3(1)$$
$${}_3C_1\,(u)^2(v^2)^1 = 3\,u^2v^2$$
$${}_3C_2\,(u)^1(v^2)^2 = 3\,u\,v^4$$
$${}_3C_3\,(u)^0(v^2)^3 = 1(1)\,v^6$$

$$\boxed{u^3+3u^2v^2+3uv^4+v^6}$$

$(a+2b^3)^4$

$a^4+8a^3b^3+24a^2b^6+32ab^9+16b^{12}$

$(x-y)^5$

$${}_5C_0\,(x)^5(-y)^0 = 1\,x^5(1)$$
$${}_5C_1\,(x)^4(-y)^1 = 5\,x^4(-y)$$
$${}_5C_2\,(x)^3(-y)^2 = 10\,x^3(y^2)$$
$${}_5C_3\,(x)^2(-y)^3 = 10\,x^2(-y^3)$$
$${}_5C_4\,(x)^1(-y)^4 = 5\,x\,(y^4)$$
$${}_5C_5\,(x)^0(-y)^5 = 1(1)(-y^5)$$

$$x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$$

$(x-5)^4$

$x^4-20x^3+150x^2-500x+625$

$(5-2a)^4$

$${}_4C_0\,(5)^4(-2a)^0 = 1\,(625)(1) = 625$$
$${}_4C_1\,(5)^3(-2a)^1 = 4\,(125)(-2a) = -1000\,a$$
$${}_4C_2\,(5)^2(-2a)^2 = 6\,(25)(4a^2) = 600\,a^2$$
$${}_4C_3\,(5)^1(-2a)^3 = 4\,(5)(-8a^3) = -160\,a^3$$
$${}_4C_4\,(5)^0(-2a)^4 = 1(1)(16a^4) = 16a^4$$

$$625-1000a+600a^2-160a^3+16a^4$$