10ReviewB

1. $\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cos 45° - \sin 60° \sin 45°)$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

2. $\sin 195° = \sin(150° + 45°)$

$= \sin 150° \cos 45° + \cos 150° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

3. $\tan 165° = \tan(120° + 45°)$

$= \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°} = \frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)}$

$= \frac{-\sqrt{3}+1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{-\sqrt{3}+3+1-\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3} = \frac{4-2\sqrt{3}}{-2}$

$= -2 + \sqrt{3}$

4. $\tan \frac{\pi}{12} = \tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+(\sqrt{3})(1)} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{\sqrt{3}-3-1+\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3} = \frac{2\sqrt{3}-4}{1-3} = \frac{2\sqrt{3}-4}{-2} = -\sqrt{3}+2$

5. $\sin \frac{7\pi}{12} = \sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

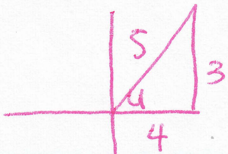
$\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4}$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

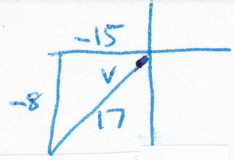
6. $\cos \frac{11\pi}{12} = \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$

$-\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}-\sqrt{6}}{4}$



$\sin u = \frac{3}{5}$
$\cos u = \frac{4}{5}$
$\tan u = \frac{3}{4}$



$\sin v = \frac{-8}{17}$
$\cos v = \frac{-15}{17}$
$\tan v = \frac{8}{15}$

7 $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$$\frac{3}{5}\left(-\frac{15}{17}\right) + \frac{4}{5}\left(-\frac{8}{17}\right) = \frac{-45-32}{85} = \frac{-77}{85}$$

8 $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$$= \frac{4}{5}\left(\frac{-15}{17}\right) - \frac{3}{5}\left(\frac{-8}{17}\right) = \frac{-60+24}{85} = -\frac{36}{85}$$

9 $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ $\quad \frac{\frac{15}{15}\frac{3}{4} + \frac{8}{15}\frac{4}{4}}{1 - \frac{3}{4}\cdot\frac{8}{15}} = \frac{\frac{45}{60} + \frac{32}{60}}{1 - \frac{24}{60}} = \frac{\frac{77}{60}}{\frac{36}{60}}$

$$\frac{77}{60} \div \frac{60}{36} = \frac{77}{36}$$

10 $\sin(u-v) = \sin u \cos v - \cos u \sin v$

$$\frac{3}{5}\left(\frac{-15}{17}\right) - \frac{4}{5}\left(\frac{-8}{17}\right) = \frac{-45+32}{85} = \frac{-13}{85}$$

11 $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$$\frac{4}{5}\left(\frac{-15}{17}\right) + \frac{3}{5}\left(\frac{-8}{17}\right) = \frac{-60-24}{85} = \frac{-84}{85}$$

12. $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$ $= \frac{\frac{15}{15}\frac{3}{4} - \frac{8}{15}\frac{4}{4}}{1 + \frac{3}{4}\left(\frac{8}{15}\right)} = \frac{\frac{45-32}{60}}{1 + \frac{24}{60}} = \frac{13/60}{84/60}$

$$\frac{13}{60} \cdot \frac{60}{84} = \frac{13}{84}$$

13 $\sin(x - 2\pi) = \sin x \cos 2\pi - \cos x \sin 2\pi$

$\sin x (1) - \cos x (0)$

$\sin x$

14 $\tan\left(x - \frac{3\pi}{4}\right) = \frac{\tan x - \tan\frac{3\pi}{4}}{1 + \tan x \tan\frac{3\pi}{4}} = \frac{\tan x - (-1)}{1 + \tan x(-1)} = \frac{\tan x + 1}{1 - \tan x}$

15 $\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3} \to \cos$ formula; change sign

$$= \cos\left(\frac{\pi}{4}\frac{3}{3} + \frac{\pi}{3}\frac{4}{4}\right) = \cos\frac{7\pi}{12}$$

16 $\sin 20° \cos 50° + \cos 20° \sin 50° \to$ sine formula; keep sign

$$\sin(20° + 50°) = \sin 70°$$

17 $\frac{\tan 78^\circ - \tan 42^\circ}{1 + \tan 78^\circ \tan 42^\circ}$ → tan formula ; keep top

$\tan(78^\circ - 42^\circ) = \tan 36^\circ$

18 $\frac{\tan \frac{5\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{5\pi}{3} \tan \frac{\pi}{4}}$ → tan formula; keep top

$\tan\left(\frac{4 \cdot 5\pi}{4 \cdot 3} + \frac{\pi \cdot 3}{4 \cdot 3}\right) = \tan \frac{23\pi}{12}$

19 $\cos(u+v) + \cos(u-v) = 2\cos u \cos v$

$(\cos u \cos v - \sin u \sin v) + (\cos u \cos v + \sin u \sin v) =$

$\cos u \cos v - \sin \cancel{u \sin v} + \cos u \cos v + \sin \cancel{u \sin} v$

$2\cos u \cos v$

20 $\sin(u+v) - \sin(u-v) = 2\cos u \sin v$

$(\sin u \cos v + \cos u \sin v) - (\sin u \cos v - \cos u \sin v)$

$\sin \cancel{u \cos u} + \cos u \sin v - \sin \cancel{u \cos} v + \cos u \sin v$

$2 \cdot \cos u \sin v$

21. $(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x$

$\sin^2 x + \cos^2 x + 2\sin x \cos x$

$1 + \sin 2x$

22 $\frac{\sin 2x}{\sin x} = \frac{2 \cancel{\sin x} \cos x}{\cancel{\sin x}} = 2\cos x$

23. $4 \sin \frac{x}{2} \cos \frac{x}{2} = y$ let $\theta = \frac{x}{2}$

$4 \sin\theta \cos\theta = y$ Divide by 2

$2\sin\theta \cos\theta = y/2$

$\sin(2\theta) = y/2$

$\rightarrow y = 2\sin(2\theta) = 2\sin\left[2\left(\frac{x}{2}\right)\right] = 2\sin x$

24. $\cos^4 x - \sin^4 x = (\cos^2 x)^2 - (\sin^2 x)^2$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= 1 \quad (\cos 2x)$

$= \cos 2x$

25 $\frac{1}{2}\sin 3x \cos 3x = y$ let $\theta = 3x$

$\frac{1}{2}\sin\theta\cos\theta = y$; Multiply by 4

$2\sin\theta\cos\theta = 4y$

$\sin 2\theta = 4y \rightarrow y = \frac{\sin(2\theta)}{4}$

$y = \frac{1}{4}\sin[2(3x)] = \frac{1}{4}\sin 6x$

26 $\frac{\cos 2x}{\cos^2 x} = \frac{2\cos^2 x - 1}{\cos^2 x} = \frac{2\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = 2 - \sec^2 x$

27 $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$2\cos x \quad 1$

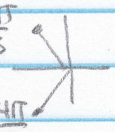
$\cos x \quad -1$

$\rightarrow (2\cos x + 1)(\cos x - 1) = 0$

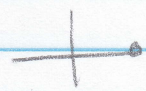
$2\cos x + 1 = 0$ $\qquad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$ $\qquad$ $(1,0)$

$\frac{2\pi}{3}$



$\frac{4\pi}{3}$



$x = \frac{2\pi}{3} + 2n\pi$; $x = \frac{4\pi}{3} + 2n\pi$ $\qquad$ $x = 0 + 2n\pi$

$n = 0$; $x = 0$

$n = 1$; $x = \cancel{2\pi}$

$n=0$ $\quad x = \frac{2\pi}{3}$; $\frac{4\pi}{3}$

$n=1$ $\quad x = \cancel{\frac{8\pi}{3}}$ $\quad \cancel{\frac{10\pi}{3}}$

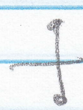
28 $\sin 2x \sin x = \cos x$

$2\sin x\cos x \sin x = \cos x \rightarrow 2\sin^2 x\cos x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin^2 x - 1 = 0$

$(0,1)$ $\quad \frac{\pi}{2}$



$(0,-1)$ $\quad \frac{3\pi}{2}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}}$

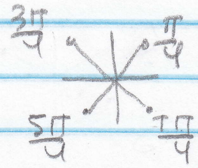
$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{2} + n\pi$

n=0 $x = \frac{\pi}{2}$

n=1 $x = \frac{3\pi}{2}$

n=2 $x = \cancel{\frac{5\pi}{2}}$



$x = \frac{\pi}{4} + \frac{n\pi}{2} = \frac{\pi}{4} + \frac{2n\pi}{4}$

$\frac{\pi}{4}$; $\frac{3\pi}{4}$; $\frac{5\pi}{4}$

$\frac{7\pi}{4}$; $\cancel{\frac{9\pi}{4}}$

29 $\cos 2x = -\sin x$

$1 - 2\sin^2 x = -\sin x \rightarrow \quad 2\sin^2 x - \sin x - 1 = 0$

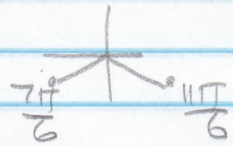
$2\sin x \quad +1$

$\sin x \quad -1$

$(2\sin x + 1)(\sin x - 1) = 0$

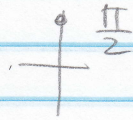
$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$



$\sin x - 1 = 0$

$\sin x = 1 \quad (0,1)$



$x = \frac{7\pi}{6} + 2n\pi$; $x = \frac{11\pi}{6} + 2n\pi$

$x = \frac{\pi}{2} + 2n\pi$

n=0 $x = \frac{7\pi}{6}$; $\frac{11\pi}{6}$

n=1 too big

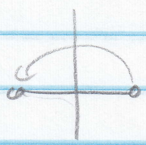
$n=0 \rightarrow x = \frac{\pi}{2}$

$n=1 \rightarrow$ too big

30 $\sin 2x + \sqrt{2}\sin x = 0$

$2\sin x \cos x + \sqrt{2}\sin x = 0$

$\sin x \,(2\cos x + \sqrt{2}) = 0$

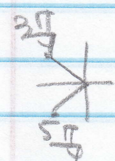


$\sin x = 0 \rightarrow (\pm 1, 0)$

$x = 0 + n\pi$

$2\cos x + \sqrt{2} = 0$

$\cos x = -\frac{\sqrt{2}}{2}$



n=0 0

n=1 π

n=2 too big

$x = \frac{3\pi}{4} + 2n\pi$

$x = \frac{5\pi}{4} + 2n\pi$

$n=0 \rightarrow \frac{3\pi}{4}$; $\frac{5\pi}{4}$

n=1 too big

31\. $\cos 2x - \cos x = 2$

$2\cos^2 x - 1 - \cos x = 2$

$2\cos^2 x - \cos x - 3 = 0$

$2\cos x \quad -3$

$\cos x \quad +1$

$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0$

$\cos x + 1 = 0$

$\cos x = \frac{3}{2}$

not possible

$\cos x = -1 \rightarrow (-1, 0)$

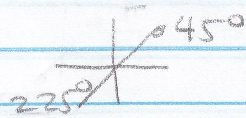
$x = \pi + 2n\pi$

$n = 0 \rightarrow x = \pi$

$n = 1 \rightarrow$ too big

32 $\tan(x - 10^\circ) = 1$ $\quad \theta = x - 10^\circ$

$\tan\theta = 1$



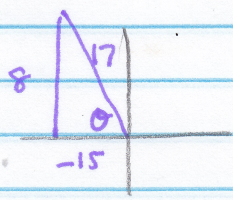
$\theta = 45^\circ + n \cdot 180^\circ$

$x - 10^\circ = 45^\circ + n \cdot 180^\circ$

$n = 0 \rightarrow x - 10^\circ = 45^\circ \rightarrow x = 55^\circ$

$n = 1 \rightarrow x - 10^\circ = 225^\circ \rightarrow x = 235^\circ$

$n = 2 \rightarrow x - 10^\circ = 405^\circ \rightarrow x = \cancel{415^\circ}$



$\sin\theta = 8/17$

$\cos\theta = -15/17$

$\tan\theta = -8/15$

$90^\circ < \theta < 180^\circ$

$45^\circ < \frac{\theta}{2} < 90^\circ$

$\frac{\theta}{2}$ is in Q I

33 $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(-\frac{15}{17}\right)^2 - 1 = \frac{161}{289}$

34 $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{8}{17}\right)\left(-\frac{15}{17}\right) = -\frac{240}{289}$

35 $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2(-8/15)}{1 - (-8/15)^2} = \frac{-16/15}{1 - 64/225} = \frac{-16/15}{161/225}$

$= \frac{-16}{15} \cdot \frac{225}{161} = - \frac{240}{161}$

36 $\sin\frac{\theta}{2} = +\sqrt{\frac{1 - \cos\theta}{2}} = +\sqrt{\frac{1 - (-15/17)}{2}} = +\sqrt{\frac{32}{17} \cdot \frac{1}{2}} = +\sqrt{\frac{32}{34} \cdot \frac{34}{34}}$

$= +\frac{8\sqrt{17}}{34} = +\frac{4\sqrt{17}}{17}$

37 $\cos\frac{\theta}{2} = \overset{QI}{+}\sqrt{\frac{1+\cos\theta}{2}} = +\sqrt{\frac{1+(-15/17)}{2}} = +\sqrt{\frac{2/17}{2}} = +\sqrt{\frac{1}{17}}$

$= +\frac{\sqrt{17}}{17}$

38 $\tan\frac{\theta}{2} = \overset{QI}{+}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = +\sqrt{\frac{1-(-15/17)}{1+(-15/17)}} = +\sqrt{\frac{32/17}{2/17}}$

$+\sqrt{\frac{32}{17}\cdot\frac{17}{2}} = +\sqrt{16} \pm 4$

OR $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{\frac{8}{17}}{1+(-15/17)} = \frac{8/17}{2/17} = \frac{8}{17}\cdot\frac{17}{2} = 4$