

Warm-Up

Thomas collected data on 25 randomly selected 17-year-olds at his school, and summarized the results in a table.

		Has a Driver's License	
		Yes	No
Has a Job	Yes	9	3
	No	8	5

Handwritten notes: $\frac{8}{25}$ with an arrow pointing to the 'No' column of the 'Has a Driver's License' table.

a. Make a table of the joint relative frequencies and marginal relative frequencies. Round to the nearest hundredth where appropriate. The table has been started for you.

		Has a Driver's License		
		Yes	No	Total
Has a Job	Yes	.36	0.12	0.48
	No	.32	0.20	0.52
Total		0.68	0.32	1

Handwritten notes: $\frac{.32}{1}$ with an arrow pointing to the 'No' total cell.

b. If you are given that a 17-year-old has a job, what is the probability that the 17-year-old also has a driver's license? Divide a joint relative frequency by a marginal relative frequency to find the answer. Round your answer to the nearest hundredth.

Handwritten calculation: $\frac{.36}{.48} = \frac{36}{48} = \frac{6}{8} = \frac{3}{4}$

c. If you are given that a 17-year-old has a driver's license, what is the probability that the 17-year-old also has a job? Divide a joint relative frequency by a marginal relative frequency to find the answer. Round your answer to the nearest hundredth.

Handwritten calculation: $\frac{.36}{.68} = \frac{36}{68} = \frac{9}{17} \approx .53$

7-5 Compound Events
Going Deeper

Essential question: How do you find the probability of mutually exclusive events and overlapping events?

COMMON CORE Standards for Mathematical Content

CC.9-12.5.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.*

One card is drawn from the deck. Find each probability.

- selecting a two: $\frac{4}{52} = \frac{1}{13}$
- selecting a face card: $\frac{12}{52} = \frac{3}{13}$

Two cards are drawn from the deck. Find each probability.

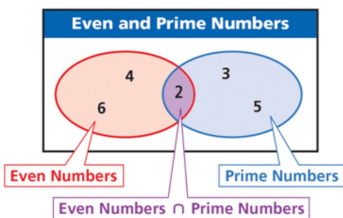
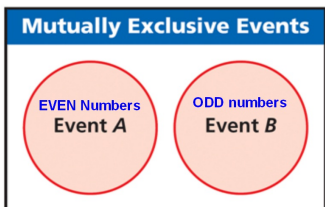
- selecting two kings when the first card is replaced: $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$
- selecting two hearts when the first card is not replaced: $\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

A **simple event** is an event that describes a single outcome.

A **compound event** is an event made up of two or more simple events.

Mutually exclusive events are events that cannot both occur in the same trial of an experiment.

Inclusive events are events that have one or more outcomes in common.



Mutually Exclusive Events

WORDS	ALGEBRA	EXAMPLE
The probability of two mutually exclusive events A or B occurring is the sum of their individual probabilities.	For two mutually exclusive events A and B, $P(A \cup B) = P(A) + P(B)$ U or	When a number cube is rolled, $P(\text{less than } 3) = P(1 \text{ or } 2) = P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

Inclusive Events

WORDS	ALGEBRA	EXAMPLE
The probability of two inclusive events A or B occurring is the sum of their individual probabilities minus the probability of both occurring.	For two inclusive events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ or	When you roll a number cube, $P(\text{even or prime}) = P(\text{even}) + P(\text{prime}) - P(\text{even and prime}) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$.

Remember!

Recall that the union symbol \cup means "or."

Remember!

Recall that the intersection symbol \cap means "and."

✦ A group of students is donating blood during a blood drive. A student has a $\frac{9}{20}$ probability of having type O blood and a $\frac{2}{5}$ probability of having type A blood.

Explain why the events "type O" and "type A" blood are mutually exclusive.

A person can only have one blood type.

A group of students is donating blood during a blood drive. A student has a $\frac{9}{20}$ probability of having type O blood and a $\frac{2}{5}$ probability of having type A blood.

✦ What is the probability that a student has type O or type A blood?

$$P(O \text{ or } A) = P(O) + P(A) \\ \frac{9}{20} + \frac{2}{5} = \frac{17}{20}$$

✦ Each student cast one vote for senior class president. Of the students, 25% voted for Hunt, 20% for Kline, and 55% for Vila. A student from the senior class is selected at random.

Explain why the events "voted for Hunt," "voted for Kline," and "voted for Vila" are mutually exclusive.

Each student can vote only once.

Each student cast one vote for senior class president. Of the students, 25% voted for Hunt, 20% for Kline, and 55% for Vila. A student from the senior class is selected at random.

✦ What is the probability that a student voted for Kline or Vila?

$$P(\text{Kline} \cup \text{Vila}) = P(\text{Kline}) + P(\text{Vila}) \\ = 20\% + 55\% = 75\%$$

✦ A dodecahedral number cube has 12 sides numbered 1 through 12. What is the probability that you roll the cube and the result is an even number or a 7?

$$P(\text{even or } 7) = P(\text{even}) + P(7) \\ \frac{6}{12} + \frac{1}{12} = \frac{7}{12}$$

Find the probability on a number cube.

★ rolling a 4 or an even number

$$\begin{aligned}
 P(4 \text{ or even}) &= P(4) + P(\text{even}) - P(4 \text{ and even}) \\
 &= \frac{1}{6} + \frac{3}{6} - \frac{1}{6} \\
 &= \frac{3}{6} \\
 &= \frac{1}{2}
 \end{aligned}$$

Find the probability on a number cube.

★ rolling an odd number or a number greater than 2

$$\begin{aligned}
 P(\text{odd or greater than 2}) &= P(\text{odd}) + P(>2) - P(\text{odd and } >2) \\
 &= \frac{3}{6} + \frac{4}{6} - \frac{2}{6} \\
 &= \frac{5}{6}
 \end{aligned}$$

A card is drawn from a deck of 52. Find the probability of each.

★ drawing a king or a heart

$$\begin{aligned}
 P(K \text{ or } \heartsuit) &= P(K) + P(\heartsuit) - P(K \text{ and } \heartsuit) \\
 &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\
 &= \frac{16}{52} = \left(\frac{4}{13}\right)
 \end{aligned}$$

A card is drawn from a deck of 52. Find the probability of each.

★ drawing a red card (hearts or diamonds) or a face card (jack, queen, or king)

$$\begin{aligned}
 P(R \text{ or Face}) &= P(R) + P(F) - P(R \text{ and } F) \\
 &= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} \\
 &= \frac{32}{52} = \left(\frac{8}{13}\right)
 \end{aligned}$$

★ What is the probability that you roll a dodecahedral number cube and the result is an even number or a number greater than 7

$$\begin{aligned}
 P(\text{even or } >7) &= P(\text{even}) + P(>7) - P(\text{even and } >7) \\
 &= \frac{6}{12} + \frac{5}{12} - \frac{3}{12}
 \end{aligned}$$

$$P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{8}{12} = \frac{2}{3}$$

Of 1560 students surveyed, 840 were seniors and 630 read a daily paper. The rest of the students were juniors. Only 215 of the paper readers were juniors. What is the probability that a student was a senior or read a daily paper?

★

	sr	jr	
DP	415	215	630
no DP	425	505	930
	840	720	1560

$$= P(\text{senior}) + P(\text{reads paper}) - P(\text{senior AND reads paper})$$

$$= \frac{840}{1560} + \frac{630}{1560} - \frac{415}{1560} = \frac{1055}{1560} \approx 0.676$$

The probability that the student was a senior or read the daily paper is about 67.6%.

You shuffle a standard deck of playing cards and choose a card at random. What is the probability that you choose a king or a heart



$$\begin{aligned}
 P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
 &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \quad \text{Substitute.} \\
 &= \frac{16}{52} \text{ or } \frac{4}{13} \quad \text{Simplify.}
 \end{aligned}$$

So, the probability of choosing a king or a heart is $\frac{4}{13}$.

Of 160 beauty spa customers, 96 had a hair styling and 61 had a manicure. There were 28 customers who had only a manicure. What is the probability that a customer had a hair styling or a manicure?



	hair	no hair	
mani	33	28	61
no mani	63	36	99
	96	64	160 customers

$$P(\text{hair}) + P(\text{manicure}) - P(\text{hair and manicure})$$

$$= \frac{96}{160} + \frac{61}{160} - \frac{33}{160} = \frac{124}{160} = \frac{31}{40}$$

The probability that a customer had a hair styling or manicure is 77.5%.

Each of 6 students randomly chooses a butterfly from a list of 8 types. What is the probability that at least 2 students choose the same butterfly?



$$P(\text{at least 2 students choose same}) = 1 - P(\text{all choose different})$$

$$\begin{aligned}
 P(\text{all choose different}) &= \frac{\text{number of ways 6 students can choose different butterflies}}{\text{total number of ways 6 students can choose butterflies}} \\
 &= \frac{{}^8P_6}{8^6} \\
 &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8} = \frac{20,160}{262,144} \approx 0.0769
 \end{aligned}$$

$$P(\text{at least 2 students choose same}) = 1 - 0.0769 \approx 0.9231$$

The probability that at least 2 students choose the same butterfly is about 0.9231, or 92.31%.

In one day, 5 different customers bought earrings from the same jewelry store. The store offers 62 different styles. Find the probability that at least 2 customers bought the same style.



$$\begin{aligned}
 P(\text{all choose different}) &= \frac{\text{number of ways 5 customers can choose different earrings}}{\text{total number of ways 5 customers can choose earrings}} \\
 &= \frac{{}^{62}P_5}{62^5} \\
 &= \frac{62 \cdot 61 \cdot 60 \cdot 59 \cdot 58}{62 \cdot 62 \cdot 62 \cdot 62 \cdot 62} \approx 0.8476
 \end{aligned}$$

$$P(\text{at least 2 choose the same}) \square 1 - 0.8476 \square 0.1524$$

The probability that at least 2 customers buy the same style is about 0.1524, or 15.24%.

INDEPENDENT

DEPENDENT

MUTUALLY EXCLUSIVE

$$P(A) = P(A|B) \quad P(A) \neq P(A|B) \quad P(A|B) = 0$$

$$P(A \text{ AND } B) = P(A) P(B)$$

$$P(A \text{ AND } B) = P(A|B) P(B)$$

$$P(A \text{ AND } B) = 0$$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$$

$$P(A \text{ OR } B) = P(A) + P(B)$$

INDEPENDENT VS DEPENDENT: to see if we can use $P(A \text{ and } B) = P(A) P(B)$

MUTUALLY EXCLUSIVE VS INCLUSIVE: to see if we can use $P(A \text{ OR } B) = P(A) + P(B)$