**Algebra 2H Extra Notes 7-1: Combinations**

April 22

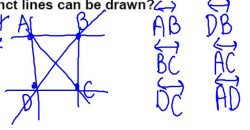
A *combination* is a grouping of objects in which order is not important.

**Example 1: Given 4 points, no 3 of which are collinear. How many distinct lines can be drawn?**

6

$_4C_2 = \frac{4!}{2!\,2!} = \frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1} = 6$

(4 total pts; 2 at a time)




$\overleftrightarrow{AB}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{DC}$ $\overleftrightarrow{AD}$

combinations of $n$ objects taken $r$ at a time is $_nC_r = \frac{n!}{r!(n-r)!}$

**Example 2:**

Standard Deck of 52 Playing Cards:

Diamonds (Red): 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦ A♦
Hearts (Red): 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥ A♥
Clubs (Black): 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣ A♣
Spades (Black): 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠ A♠

**a) How many different 7-card hands are possible?**

$_{52}C_7 = \frac{52!}{(7!\times45!)} = 133,784,560$

**b) How many 7-card hands with 4 kings and any other 3 cards are possible?**

$\underset{\text{4 Kings}}{_4C_4} \times \underset{\text{3 others}}{_{48}C_3} = 1 \cdot \frac{48!}{3!\,45!} = 17,296$

$\frac{8\cancel{48}\cdot47\cdot46\cdot\cancel{45}!}{\cancel{3\cdot2}\cdot\cancel{45}!}$

**Example 3: Twelve students applied for an ASB committee. In how many ways can committees of 2 or 3 or 4 students be chosen?**

mutually exclusive events (no overlap)

$_{12}C_2 + {}_{12}C_3 + {}_{12}C_4$

$\frac{12!}{2!\,10!} + \frac{12!}{3!\,9!} + \frac{12!}{4!\,8!}$

$\frac{\cancel{12}^{6}\cdot11}{\cancel{2}} + \frac{\cancel{12}^{2}\cdot11\cdot10}{\cancel{3\cdot2}} + \frac{\cancel{12}\cdot11\cdot\cancel{10}^{5}\cdot9}{\cancel{4\cdot3\cdot2}}$

$66 + 220 + 495 = 781$

286