31. $\lim_{x \to 0} \frac{x}{\sqrt{1+3x} - 1}$

© $\lim_{x \to 0} \frac{x}{\sqrt{1+3x} - 1} \cdot \frac{\sqrt{1+3x} + 1}{\sqrt{1+3x} + 1}$

$= \lim_{x \to 0} \frac{x(\sqrt{1+3x} + 1)}{(1+3x) - 1}$

$= \lim_{x \to 0} \frac{x(\sqrt{1+3x} + 1)}{3x}$

$= \lim_{x \to 0} \frac{\sqrt{1+3x} + 1}{3}$

$= \frac{\sqrt{1+3(0)} + 1}{3}$

$= \frac{2}{3}$

32. $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

© $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}}$

$= \lim_{x \to 0} \frac{(3+x) - (3)}{x(\sqrt{3+x} + \sqrt{3})}$

$= \lim_{x \to 0} \frac{x}{x(\sqrt{3+x} + \sqrt{3})}$

$= \lim_{x \to 0} \frac{1}{\sqrt{3+x} + \sqrt{3}}$

$= \frac{1}{\sqrt{3} + \sqrt{3}}$

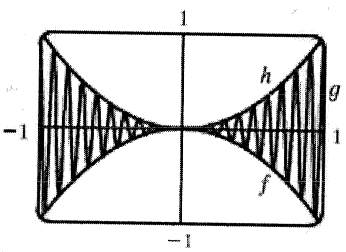
$= \frac{1}{2\sqrt{3}}$

33. show $\lim_{x\to 0}(x^2\cos(20\pi x))=0$

$$-1 \le \cos(20\pi x) \le 1$$
$$-x^2 \le x^2\cos(20\pi x) \le x^2 \qquad \text{since } x^2>0$$

$\lim_{x\to 0}(-x^2)=0$ and $\lim_{x\to 0} x^2=0$

so by the Squeeze Thm $\lim_{x\to 0} x^2\cos(20\pi x)=0$



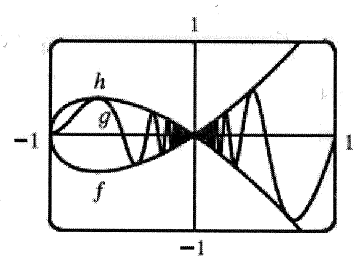
$f(x)=-x^2$
$g(x)=x^2\cos(20\pi x)$
$h(x)=x^2$

34. show $\lim_{x\to 0}\sqrt{x^3+x^2}\,\sin(\frac{\pi}{x})=0$

$$-1\le \sin(\tfrac{\pi}{x})\le 1$$
$$-\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\,\sin(\tfrac{\pi}{x}) \le \sqrt{x^3+x^2}$$
since $\sqrt{x^3+x^2}>0$

$\lim_{x\to 0}(-\sqrt{x^3+x^2})=0$ and $\lim_{x\to 0}\sqrt{x^3+x^2}=0$

so by the Squeeze Theorem
$\lim_{x\to 0}\sqrt{x^3+x^2}\,\sin(\frac{\pi}{x})=0$



$f(x)=-\sqrt{x^3+x^2}$
$g(x)=\sqrt{x^3+x^2}\,\sin(\frac{\pi}{x})$
$h(x)=\sqrt{x^3+x^2}$

35. $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$

$\lim_{x\to 4}(4x-9) = 4(4)-9=7$

$\lim_{x\to 4}(x^2-4x+7) = (4)^2-4(4)+7=7$

so by the Squeeze Thm $\lim_{x\to 4} f(x)=7$

36. $2x \le g(x) \le x^4-x^2+2$ for all x

$\lim_{x\to 1}(2x) = 2(1)=2$

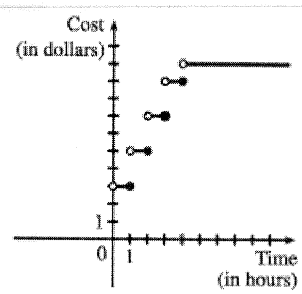
$\lim_{x\to 1}(x^4-x^2+2) = (1)^4-(1)^2+2=2$

so by the Squeeze Thm $\lim_{x\to 1} g(x)=2$

1. $\lim_{x \to 4} f(x) = f(4)$

2. The graph has no hole, jump, or vertical asymptote

7. (a)



(b) Discontinuities at $t = 1, 2, 3,$ and 4
A person should remember the charge will jump at the beginning of each hour.

8. (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.

(b) Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.

(c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values—at a cliff, for example.

(d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.

(e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

10. $f(x) = x^2 + \sqrt{7-x}$ $\quad a=4$

$f(4) = 4^2 + \sqrt{7-4} = 16 + \sqrt{3}$

$\lim_{x \to 4} (x^2 + \sqrt{7-x}) = 4^2 + \sqrt{7-4} = 16 + \sqrt{3}$

f is continuous at $a=4$ since $\lim_{x \to 4} f(x) = f(4)$

11. $f(x) = (x+2x^3)^4$ $\quad a=-1$

$f(-1) = [(-1)+2(-1)^3]^4 = 81$

$\lim_{x \to -1} (x+2x^3)^4 = [(-1)+2(-1)^3]^4 = 81$

f is continuous at $a=-1$ since $\lim_{x \to -1} f(x) = f(-1)$

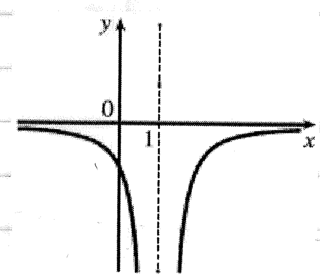
12. $h(t) = \dfrac{2t-3t^2}{1+t^3}$ $\quad a=1$

$h(1) = \dfrac{2(1)-3(1)^2}{1+(1)^3} = \dfrac{-1}{2}$

$\lim_{t \to 1} \dfrac{2t-3t^2}{1+t^3} = \dfrac{2(1)-3(1)^2}{1+(1)^3} = \dfrac{-1}{2}$

h is continuous at $a=-\frac{1}{2}$ since $\lim_{t \to 1} h(t) = h(1)$

15. $f(x) = \dfrac{-1}{(x-1)^2}$ $\quad a=1$



f is discontinuous at $x=1$ since $f(1)$ is undefined

16.

$$f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

$f(1) = 2$

$\lim\limits_{x\to1} f(x) = \lim\limits_{x\to1} \frac{1}{x-1}$ DNE since $\lim\limits_{x\to1^-} f(x) \neq \lim\limits_{x\to1^+} f(x)$

$$\lim_{x\to1^-} \frac{1}{x-1} = -\infty$$

$$\lim_{x\to1^+} \frac{1}{x-1} = \infty$$

f is discontinuous at $x=1$ since $\lim\limits_{x\to1} f(x)$ DNE