SECTION 3.5

1) Suppose $h(x) = f(g(x))$, $f'(7) = 3$, $g(4) = 7$, and $g'(4) = 5$. Find $h'(4)$.

$h'(4) = f'(g(4)) \cdot g'(4) = f'(7) \cdot 5$
$= 3 \cdot 5 = 15.$

2) Find $\frac{dy}{dx}$ for:

a) $y = \sqrt{x^3 + 6x}$

Since $y = (x^3 + 6x)^{1/2}$,
$y' = \frac{1}{2}(x^3 + 6x)^{-1/2}(3x^2 + 6) = \dfrac{3x^2 + 6}{2\sqrt{x^3 + 6x}}.$

b) $y = \sec x^2$

$y' = (\sec x^2)(\tan x^2)(2x).$

c) $y = \sec^2 x$

Since $y = [\sec x]^2$,
$y' = 2[\sec x]^1(\sec x \tan x) = 2 \sec^2 x \tan x.$

d) $y = \cos^3 x^2$

Since $y = [\cos x^2]^3$,
$y' = 3[\cos x^2]^2((-\sin x^2)(2x))$
$= -6x(\cos^2 x^2)\sin x^2.$

e) $y = \sin 2x \cos 3x$

y is a product of $\sin 2x$ and $\cos 3x$, so
$y' = (\sin 2x)[-\sin 3x \cdot 3]$
$+ (\cos 3x)[\cos 2x \cdot 2]$
$= -3 \sin 2x \sin 3x + 2 \cos 3x \cos 2x.$

f) $y = \sin(\sec x)$

$y' = \cos(\sec x)(\sec x \tan x).$

g) $y = \tan(x^2 + 1)^4$

$y' = \sec^2(x^2 + 1)^4(4(x^2 + 1)^3(2x))$
$= 8x(x^2 + 1)^3 \sec^2(x^2 + 1)^4.$

h) $y = \dfrac{1}{(x^2 - 1)^4}$

Since $y = (x^2 - 1)^{-4}$,
$y' = -4(x^2 - 1)^{-5}(2x) = \dfrac{-8x}{(x^2 - 1)^5}.$

i) $y = x^3\sqrt{x^2 + 1}$

Since $y = x^3(x^2 + 1)^{1/2}$,
$y' = x^3\left(\frac{1}{2}(x^2 + 1)^{-1/2} \cdot 2x\right)$
$+ (x^2 + 1)^{1/2} \cdot 3x^2$
$= \dfrac{x^4}{\sqrt{x^2 + 1}} + 3x^2\sqrt{x^2 + 1} = \dfrac{4x^4 + 3x^2}{\sqrt{x^2 + 1}}.$

4) The temperature at noon each day x at the top of Mt. Arthur is approximately

$$50 + 30 \sin \frac{\pi(x - 200)}{180}$$

degrees Fahrenheit. (Assume a 360 day year.) Find the rate of change of the daily temperature for January 15.

<*Find the derivative with respect to x and substitute* $x = 15$.>

$$T(x) = 50 + 30 \sin \frac{\pi(x - 200)}{180}$$

$$T'(x) = 30 \cos \frac{\pi(x - 200)}{180} \cdot \frac{\pi}{180}$$

At $x = 15$, $T'(15) = 30 \cos \dfrac{\pi(-185)}{180} \cdot \dfrac{\pi}{180}$

$$\approx -.52 \text{ F}^\circ/\text{day}.$$

On January 15, the noon temperature is decreasing about a half a degree per day.

SECTION 3.6

3) Find $\frac{dy}{dx}$ implicitly:

a) $x^2y^3 = 2x + 1$

Step 1) Differentiate with respect to x (using the Product Rule because both x^2 and y^3 are functions of x):
$x^2(3y^2y') + y^3(2x) = 2.$

Step 2) Solve for y': $3x^2y^2y' = 2 - 2xy^3$
$$y' = \frac{2 - 2xy^3}{3x^2y^2}.$$

b) $3x^2 - 5xy + y^2 = 10$

Step 1) Differentiate:
$6x - (5x(y') + y \cdot 5) + 2y \cdot y' = 0.$

Step 2) Solve for y': $-5xy' + 2yy' = 5y - 6x$

$(2y - 5x)y' = 5y - 6x$
$$y' = \frac{5y - 6x}{2y - 5x}.$$

4) Find the slope of the tangent line to the curve defined by $x^2 + 2xy - y^2 = 41$ at the point $(5, 2)$.

The slope is $\frac{dy}{dx}$ at $(5, 2)$. First find $\frac{dy}{dx}$ implicitly:

Step 1) $2x + (2xy' + y \cdot 2) - 2yy' = 0$

Step 2) $2y'(x - y) = -2(x + y)$
$$y' = -\frac{x + y}{x - y} = \frac{x + y}{y - x}.$$

Now use $x = 5$, $y = 2$:
$$\frac{dy}{dx} = \frac{5 + 2}{2 - 5} = \frac{7}{-3} = -\frac{7}{3}.$$

5) For $x^2y = 1$ find y' both explicitly and implicitly.

Explicitly:

From $x^2y = 1$, $y = x^{-2}$ so $y' = -2x^{-3}$.

Implicitly:

Step 1) $x^2y' + 2xy = 0$

Step 2) $x^2y' = -2xy$
$$y' = -\frac{2y}{x}$$

These are the same because (remember $y = x^{-2}$)
$$\frac{-2y}{x} = \frac{-2x^{-2}}{x} = -2x^{-3}.$$

SECTION 3.7

1) Water is flowing out of a tank in such a fashion that after t minutes there are $10000 - 10t - t^3$ gallons of water in the tank. How fast is the water flowing after 2 minutes?

Let $V(t) = 10000 - 10t - t^3$ be the volume at time t. The question asks for the rate of change of V after 2 minutes.

$V'(t) = -10 - 3t^2$. At $t = 2$,

$V'(2) = -10 - 3(2)^2 = -22$ gal/min.

Note: The answer is negative, indicating that the volume of water is decreasing.

2) A space shuttle is $16t + t^3$ meters from its launch pad t seconds after liftoff. What is its velocity after 3 seconds?

$d(t) = 16t + t^3$, $d'(t) = 16 + 3t^2$.

$d'(3) = 16 + 3(3)^2 = 43$ m/s.

3) A particle is moving along an axis so that at time t its position is
$f(t) = t^3 - 6t^2 + 6$ feet.

a) What is its velocity at time t?

$f'(t) = 3t^2 - 12t.$

b) What is the velocity at 3 seconds?

$f'(3) = 3(3)^2 - 12(3) = -9$ ft/s.

c) Is the particle moving left or right at 3 seconds?

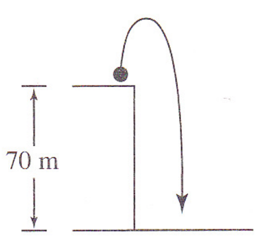
Left, since $f'(3) = -9$ is negative.

d) At what time(s) is the particle (instantaneously) motionless?

<Motionless means zero velocity. Solve the equation $f'(t) = 0$ for t.>

$f'(t) = 0$
$3t^2 - 12t = 0$
$3t(t - 4) = 0$
At $t = 0$ and $t = 4$ the particle has zero velocity.

4) A stone is thrown upward from a 70 m cliff so that its height above ground is $f(t) = 70 + 3t - t^2$. What is the velocity of the stone as it hits the ground?



The time when the stone hits the ground is when $f(t) = 0$.
$70 + 3t - t^2 = 0$
$(10 - t)(7 + t) = 0$
$t = 10$ or $t = -7$ (Disregard $t = -7$.)
$f'(t) = 3 - 2t.$
$f'(10) = 3 - 20 = -17$ m/s.

5) The number of yellow perch in a bay in Lake Michigan was measured annually over a six year period. See the table. Estimate the rate of change of the perch population in the bay in 2004.

t	$P(t)$ (thousands of fish)
2001	420,000
2002	400,000
2003	370,000
2004	360,000
2005	330,000
2006	310,000

<Because the data is given in discrete values, average the slopes of the secant lines.>

The secant line for the period 2003–2004 has slope

$$\frac{360{,}000 - 370{,}000}{2004 - 2003} = \frac{-10{,}000}{1} = -10{,}000.$$

The secant line for the period 2004–2005 has slope

$$\frac{330{,}000 - 360{,}000}{2005 - 2004} = \frac{-30{,}000}{1} = -30{,}000.$$

We estimate $P'(2004)$ to be the average:

$$\frac{-10{,}000 + (-30{,}000)}{2} = \frac{-40{,}000}{2} = -20{,}000.$$

In 2004, the fish population was declining at a rate of about 20,000 fish per year.

SECTION 3.9

1) Find the linear approximation to $f(x) = 5x^3 + 6x$ at $x = 2$.

<*What you are asked to do is find the equation of the tangent line.*>
$f(2) = 5(2)^3 + 6(2) = 52$.
$f'(x) = 15x^2 + 6$, so
$f'(2) = 15(2)^2 + 6 = 66$.
Thus, $L(x) = 52 + 66(x - 2)$.

2) Approximate $f(1.98)$ for the function in question 1.

We use $f(1.98) \approx L(1.98)$.
$L(1.98) = 52 + 66(1.98 - 2) = 50.68$.
(*Note:* $f(1.98) = 50.69196$, so $L(1.98)$ is rather close.

3) Use a calculator to find $\sqrt{66}$, then approximate $\sqrt{66}$ using a linear approximation.

By calculator $\sqrt{66} \approx 8.1240384$.
Choose $f(x) = \sqrt{x}$ and $a = 64$ (64 is near 66 and $f(64) = 8$ is easy to calculate).
Then $f(64) = 8, f'(x) = \frac{1}{2\sqrt{x}}$,
and $f'(64) = \frac{1}{2\sqrt{64}} = \frac{1}{16}$.
The linear approximation is:
$L(x) = 8 + \frac{1}{16}(x - 64)$.
At $x = 66$,
$L(66) = 8 + \frac{1}{16}(66 - 64) = 8.125$.

4) Find the linear approximation of $f(x) = \sin x$ at $a = 0$. Use it to estimate $\sin\left(\frac{\pi}{15}\right)$.

$f(x) = \sin x$
$f(0) = \sin 0 = 0$.
$f'(x) = \cos x$
$f'(0) = \cos 0 = 1$.
So $L(x) = f(0) + f'(0)(x - 0)$
$= 0 + 1(x - 0) = x$.
Thus, $\sin x \approx x$, for x near 0. (This is a common approximation used in physics and other sciences.)

For $x = \frac{\pi}{15}$, $\sin\left(\frac{\pi}{15}\right) \approx \frac{\pi}{15}$. (Using a calculator, $\sin \frac{\pi}{15} \approx .2079$ and $\frac{\pi}{15} \approx .2094$.)