Algebra 2H Extra Notes 7-1: Counting and Permutations

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

Example 1: California license plates have 1 digit, 3 letters, and 3 digits 175,760,000

a) How many license plates are possible if there are no restrictions on digits and letters and repeated letters or digits are allowed?

digits 0-9: 10 (1 digit) x 26 x 26 x 26 (letters) x 10 x 10 x 10 (digits)

b) How many license plates beginning with an "8" are possible if no letter can be repeated?

15,600,000 = 1 ("8") x 26 x 25 x 24 (letters) x 10 x 10 x 10 (digits)

Example 2: You are taking a five-question multiple choice quiz, with A,B,C,D as answer choices. How many ways can the quiz be answered?

4 x 4 x 4 x 4 x 4 = 4^5 = 1024

Permutations

A *permutation* is a selection or arrangement of objects where order is important.

Example 3: You have homework in 5 classes tonight.

a) In how many ways can you complete all of the homework?

5! = 5 (1st) x 4 (2nd) x 3 (3rd) x 2 (4th) x 1 (5th) = 120

5 factorial

n! PRB menu (!) math → PRB

permutations of n objects taken n at a time is ${}_nP_n = n!$

${}_5P_5 = 120$

b) In how many ways can you complete the homework for only 3 of the classes?

5 (1st) x 4 (2nd) x 3 (3rd) = 60 = $\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(5-3)!}$ = $\frac{5!}{2!}$

$\cdot 5!$

permutations of n objects taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$

${}_5P_3$

Example 4: Evaluate each expression.

a) ${}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!}$ (10 objects, 4 at a time) = 10·9·8·7 = 5040

b) $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!} = n^2 + 3n + 2$

Example 5: How many distinct permutations of the letters in the word WOW?

WOW OWW WWO 3

$\frac{6}{2} = \frac{3!}{2!}$ (2! = W's) all permutations / permutations of W's

permutations of n objects with numbers of repeated objects $r_1, r_2, r_3, \ldots$

$\frac{n!}{r_1! \cdot r_2! \cdot r_3! \cdots}$ ← permutations of n objects ← product of (repeats!)

Example 6: A dog has a litter of 8 puppies, 5 males and 3 females. One possible birth order is MMMMMFFF. How many distinguishable birth orders are possible?

56 = $\frac{8!}{(5! \times 3!)}$ = $\frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1}$

M F