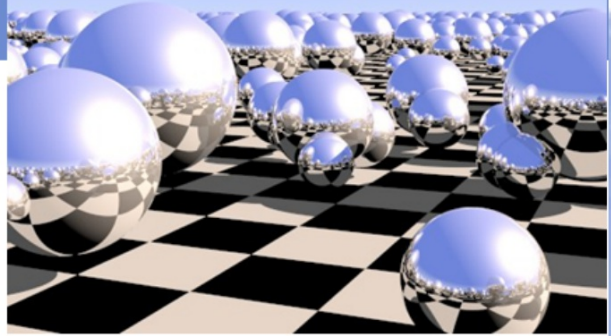
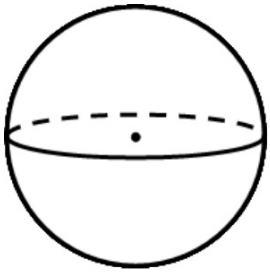


Spheres



$$\text{Surface Area} = 4 \pi r^2$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$



$$r=10$$

Find SA and V

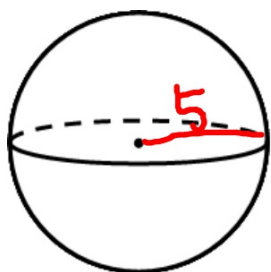
$$A = 4\pi R^2 = 4\pi \cdot 10^2$$

$$\boxed{400\pi}$$

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi 10^3$$

$$\frac{4}{3} (1000)\pi$$

$$\boxed{\frac{4000\pi}{3}}$$



$$\text{area} = 100\pi$$

Find V

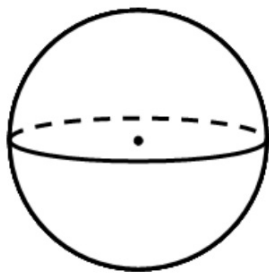
$$\frac{4}{3}\pi(5)^3$$

$$\frac{100\pi}{4\pi} = \frac{4\pi}{4\pi} R^2$$

$$\frac{4}{3}(125)\pi$$

$$25 = R^2$$

radius = $\frac{1}{4}$. Find A and V.



$$A = 4\pi \left(\frac{1}{4}\right)^2$$

$$4 \cdot \frac{1}{16} \pi$$

$$A = \boxed{\frac{1}{4} \pi}$$

$$V = \frac{4}{3} \pi \left(\frac{1}{4}\right)^3$$

$$V = \frac{4}{3} \cdot \frac{1}{64} \pi$$

$$\frac{4}{3 \cdot 64} = \frac{1}{48} \pi$$

12-4 Spheres

Objective: Find the area and volume of a sphere.

The area of a sphere equals 4π times the square of the radius. $A = 4\pi r^2$

The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. $V = \frac{4}{3}\pi r^3$



Notice that the formula for volume includes cubes of numbers. Some common cubes are listed. These will be useful when solving some exercises, such as Example 1(b) below.

$$1^3 = 1 \quad 2^3 = 8 \quad 3^3 = 27 \quad 4^3 = 64 \quad 5^3 = 125 \quad 6^3 = 216 \quad 7^3 = 343$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8} \quad \left(\frac{1}{3}\right)^3 = \frac{1}{27} \quad \left(\frac{1}{4}\right)^3 = \frac{1}{64} \quad 8^3 = 512 \quad 9^3 = 729 \quad 10^3 = 1000$$

Example 1

a. Find the area and volume of a sphere with radius 5.

Solution

$$\begin{aligned} \text{a. } A &= 4\pi r^2 \\ &= 4\pi \cdot 5^2 = 4\pi \cdot 25 = 100\pi \\ V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi \end{aligned}$$

b. The volume of a sphere is 972π . Find its area.

$$\begin{aligned} \text{b. } V &= \frac{4}{3}\pi r^3 & 972\pi &= \frac{4}{3}\pi r^3 \\ & & \left(\frac{3}{4}\right)(972) &= r^3 \\ & & 729 &= r^3 \\ & & 9 &= r \\ A &= 4\pi r^2 = 4\pi \cdot 9^2 = 324\pi \end{aligned}$$

Complete the table for spheres.

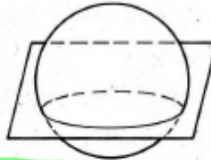
| | 1. | 2. | 3. | 4. | 5. | 6. | 7. |
|--------|----|----|----|---------------|------------|----------|---------------------|
| radius | 3 | 6 | 2 | $\frac{1}{3}$ | $\sqrt{3}$ | ? | ? |
| area | ? | ? | ? | ? | ? | 576π | ? |
| volume | ? | ? | ? | ? | ? | ? | $\frac{1372}{3}\pi$ |



8. The area of a sphere is $\frac{\pi}{4}$. Find its diameter.

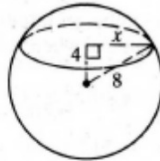
9. The area of a sphere is 9π . Find its volume.

When a sphere and a plane intersect, their intersection is either a tangent point or a circle called the **circle of intersection**.



Example 2

A plane passes 4 cm from the center of a sphere with radius 8 cm. Find the area of the circle of intersection.



Solution

Let x = the radius of the circle.
 $4^2 + x^2 = 8^2$
 $x^2 = 64 - 16 = 48$
 $A = \pi x^2 = 48\pi \text{ (cm}^2\text{)}$

A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

- 12. $r = 5$; $h = 3$
- 13. $r = 13$; $h = 12$
- 14. $r = 8$; $h = 2$
- 15. $r = 10$; $h = 6$
- 16. $r = 12$; $h = 10$



17. When a plane passes 5 cm from the center of a sphere, the radius of the circle of intersection is 12 cm. Find the volume of the sphere.

18. A scoop of ice cream with radius 4 cm is placed on an ice-cream cone with radius 3 cm and height 15 cm. Is the cone big enough to hold the ice cream if it melts?



19. A spherical fishbowl has diameter 24 cm. To fill the fishbowl three-fourths full, about how many liters of water will you need? Give your answer to the nearest 0.1 L. Use $\pi \approx 3.14$. ($1000 \text{ cm}^3 = 1 \text{ L}$)



| | | | | | | | |
|------------------------|--------------------------------------|----------------------------|-------------------|----------------------------------|------------------------------|---------------------|---------------------|
| | | $4 \cdot 36\pi$ | | | $\sqrt{144}$ | $\otimes \sqrt{\#}$ | |
| r | 3 | $4 \cdot 36$ 6 | 2 | 1/3 | $\sqrt{3}$ | 12 | 7 |
| A $4\pi r^2$ | $4 \cdot 9\pi$ 36π | 144π | 16π | $\frac{4}{9}\pi$ | 12π | 576π | |
| V $\frac{4}{3}\pi r^3$ | $\frac{4}{3} \cdot 27\pi$ 36π | 288π | $\frac{32\pi}{3}$ | $\frac{4}{81}\pi$ | $4\sqrt{3}\pi$ | 2304π | $\frac{1372}{3}\pi$ |
| | | $\frac{4}{3} \cdot 216\pi$ | | $\frac{4}{3} \cdot \frac{1}{27}$ | $\frac{4}{3} \cdot \sqrt{3}$ | | 3 |

Note what happened to the area and volume of a sphere when the radius is doubled and tripled.

$$\cancel{\frac{4}{3} \cdot 216\pi} \cdot \frac{8}{4} \cdot R^3 \quad \sqrt[3]{343}$$

The area of a sphere is $\pi/4$. Find its diameter.

$$\frac{\pi}{4} = \frac{4\pi R^2}{4\pi}$$

$$\sqrt{\frac{1}{16}} = R^2$$

$$\frac{1}{4} = R$$

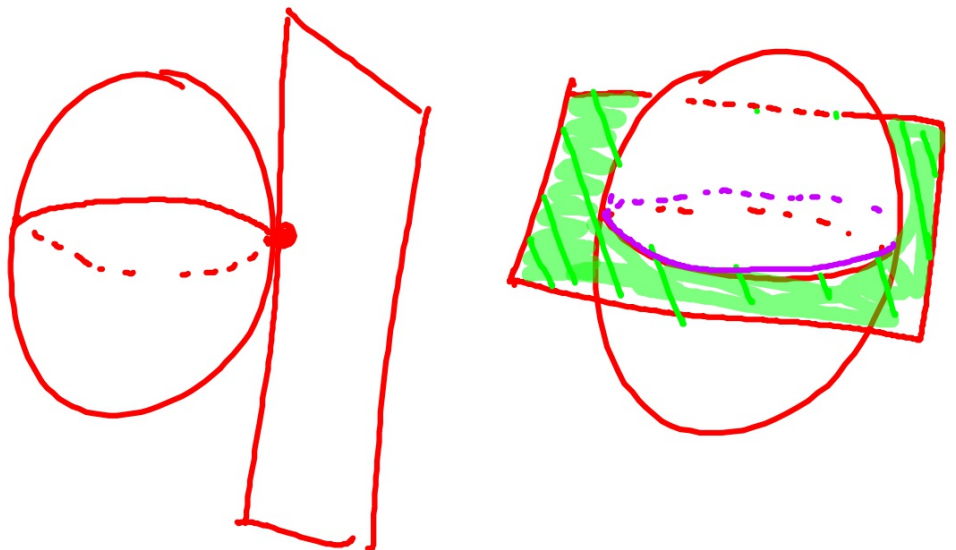
$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\frac{1}{4} \times 2 = \frac{1}{2}$$

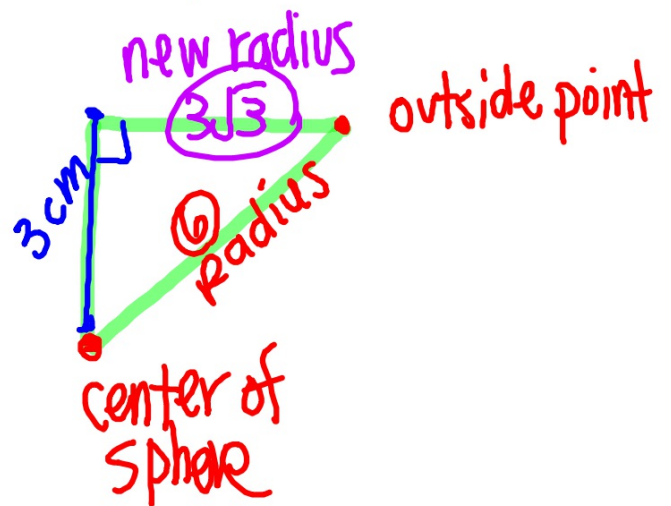
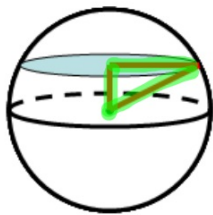
The area of a sphere is 9π . Find its volume.

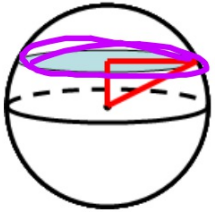
Compare the volume of a sphere with radius 3 and a hemisphere (half a sphere) with radius 6.

When a sphere and a plane intersect, their intersection is either a tangent point or a circle called the circle of intersection.



Find the area of a circle formed when a plane passes 3 cm from the center of a sphere with radius 6 cm.

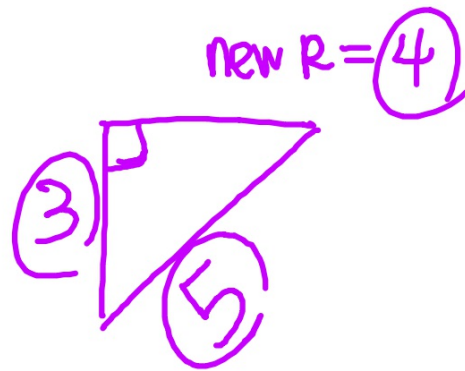


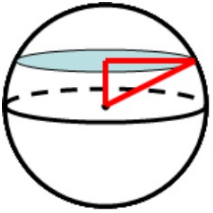


A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

$$r=5; h=3$$

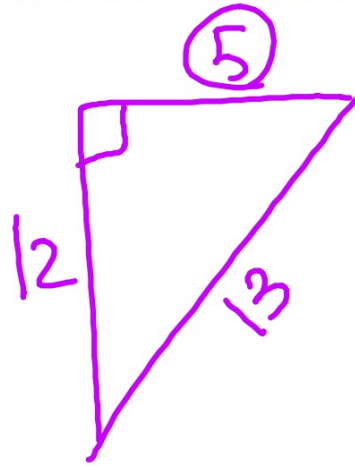
$$\pi R^2$$
$$4^2$$
$$16\pi$$

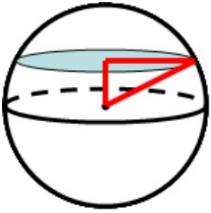




A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

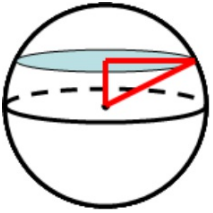
$$r=13; h = 12$$





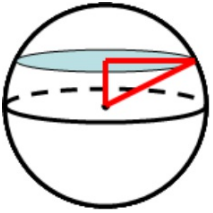
A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

$$r=8; h = 2$$



A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

$$r=10; h = 6$$

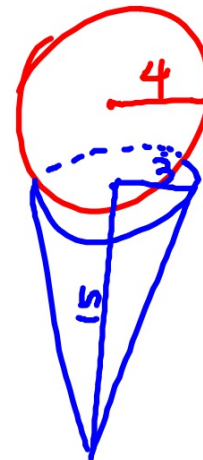


A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

$$r=12; h = 10$$

When a plane passes 5 km from the center of a sphere, the radius of the circle of intersection is 12 km. Find the volume of the sphere.

A scoop of ice cream with radius 4 cm is placed on an ice-cream cone with radius 3cm and height 15cm. Is the cone big enough to hold the ice cream if it melts?

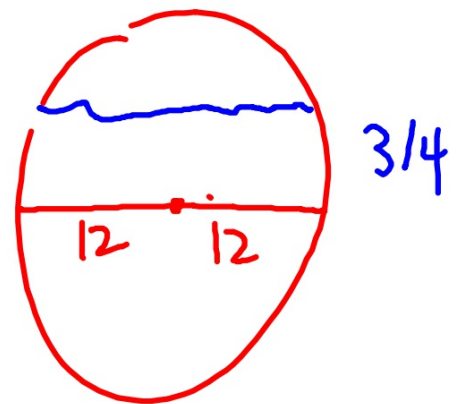


A spherical fishbowl has diameter 24cm. To fill the fishbowl three-fourths full, about how many liters of water will you need?

Give your answer to the nearest 0.1 L.

Use $\pi=3.14$. $1000\text{cm}^3=1\text{L}$

$(\frac{3}{4})$ Volume whole bowl



If the radius of a sphere is halved, the area is multiplied by _____ and the volume is multiplied by _____

If the Surface Area of a sphere is 16π , find the diameter and volume.

Find the area of the circle formed when a plane passes 9 cm from the center of a sphere with radius 15 cm.

Betty made two wax candles, one in the shape of a sphere with radius 5 cm and another in the shape of a cylinder with radius 5 cm and height 6. Which candle required more wax?

The number of square centimeters in the area of a sphere is the same as the number of cubic centimeters in the volume of a sphere.
Find the radius of this sphere.

HOMEWORK 12.4

- *keep* Finish 12.4 Handout, skip #18
- Page 500 WE #5-8, 11-14
- Page 496 #1, 2, 4, 6

Written Exercises

Copy and complete the table for spheres.

A

| | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. |
|--------|----|----|---------------|----------------|---------|----------|------------|----------|
| Radius | 7 | 5 | $\frac{1}{2}$ | $\frac{3}{4}k$ | ? | ? | $\sqrt{2}$ | ? |
| Area | ? | ? | ? | ? | 64π | 324π | ? | ? |
| Volume | ? | ? | ? | ? | ? | ? | ? | 288π |

- If the radius of a sphere is doubled, the area of the sphere is multiplied by $\underline{\quad?}$ and the volume is multiplied by $\underline{\quad?}$.
- Repeat Exercise 9 if the radius of the sphere is tripled.
- The area of a sphere is $\pi \text{ cm}^2$. Find the diameter of the sphere.
- The volume of a sphere is $36\pi \text{ m}^3$. Find its area.
- Find the area of the circle formed when a plane passes 2 cm from the center of a sphere with radius 5 cm.
- Find the area of the circle formed when a plane passes 7 cm from the center of a sphere with radius 8 cm.

Self-Test 1

For Exercises 1–5 find the lateral area, total area, and volume of each solid.

1. A rectangular solid with length 10, width 8, and height 4.5
2. A regular square pyramid with base edge 24 and slant height 13
3. A cylinder with radius 10 in. and height 7 in.
4. A right hexagonal prism with height 5 cm and base edge 6 cm
5. A cone with height 12 and radius 9
6. The total area of a cube is 2400 m^2 . Find the volume.
7. A solid metal cylinder with radius 2 and height 2 is recast as a solid cone with radius 2. Find the height of the cone.
8. A prism with height 2 m and a pyramid with height 5 m have congruent triangular bases. Find the ratio of their volumes.

