1) $a_n = n^2 - 4$

$a_1 = 1^2 - 4 = -3$

$a_2 = 2^2 - 4 = 0$

$a_3 = 3^2 - 4 = 5$

$a_4 = 4^2 - 4 = 12$

$a_5 = 5^2 - 4 = 21$

$-3, 0, 5, 12, 21$

2) $a_1 = 48 \qquad a_n = \frac{1}{2}(a_{n-1}) - 8$

current term ↑ previous term ↑

$a_1 = 48$

$a_2 = \frac{1}{2}(a_1) - 8 = \frac{1}{2}(48) - 8 = 16$

$a_3 = \frac{1}{2}(a_2) - 8 = \frac{1}{2}(16) - 8 = 0$

$a_3 = \frac{1}{2}(a_3) - 8 = \frac{1}{2}(0) - 8 = -8$

$a_4 = \frac{1}{2}(a_4) - 8 = \frac{1}{2}(-8) - 8 = -12$

$48, 16, 0, -8, 12$

3) $-4, -2, 0, 2, 4, \ldots$

$+2 \quad +2 \quad +2 \quad +2$ ⇒ arithmetic, $d = 2$

$a_n = a_1 + (n-1)(d)$

$a_n = -4 + (n-1)(2)$

$a_n = -4 + 2n - 2$

$a_n = 2n - 6$

4) $54, 18, 6, 2, \frac{2}{3}, \ldots$

$\cdot(\frac{1}{3}) \quad \cdot(\frac{1}{3}) \quad \cdot(\frac{1}{3})$ Geometric $r = \frac{1}{3}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 54\left(\frac{1}{3}\right)^{n-1}$

5) $\sum_{k=1}^{4} 5k^3$

$k=1 \quad k=2 \quad k=3 \quad k=4$

$5 + 40 + 135 + 320$

$= 500$

6) $\sum_{k=1}^{7} (-1)^{k+1}(k)$

$k=1 \quad k=2 \quad k=3 \quad k=4 \quad k=5 \quad k=6 \quad k=7$

$1 + (-2) + 3 + (-4) + 5 + (-6) + 7$

$= 4$

7) $-19, -13, -7, -1, \ldots\ldots$

$+6 \quad +6 \quad +6$ Arithmetic, $d = 6$

$a_n = a_1 + (n-1)(d)$

$a_n = -19 + (n-1)(6)$

$a_n = -19 + 6n - 6$

$a_n = 6n - 25$

$a_9 = 6(9) - 25$

$a_9 = 54 - 25$

$\boxed{a_9 = 29}$

8) $a_2 = 11.6$ and $a_5 = 5$

$-, 11.6, -, -, 5, -, \ldots\ldots$

$a_n = a_1 + (n-1)d$

$5 = 11.6 + (5-2)d$

$-6.6 = 3d$

$d = -2.2$

$a_2 = a_1 + (2-1)d$

$11.6 = a_1 + (1)(-2.2)$

$11.6 = a_1 - 2.2$

$a_1 = 13.8$

$a_n = 13.8 + (n-1)(-2.2)$

$a_n = 13.8 - 2.2n + 2.2$

$a_n = 16 - 2.2n$ ← rule

$a_9 = 16 - 2.2(9)$

$a_9 = 16 - 19.8$

$\boxed{a_9 = -3.8}$

9) $125, -, -, 65$

$a_n = a_1 + (n-1)d$

$a_4 = a_1 + (4-1)d$

$65 = 125 + 3d$

$-60 = 3d$

$d = -20$

$125, \underline{105}, \underline{85}, 65$

105 and 85

10) $4 + 7 + 10 + 13 + \cdots\cdots$

$+3 \quad +3 \quad +3$

Arithmetic, $d = 3$

S_{20} (find the sum of the first 20 terms)

$a_n = 4 + (n-1)(3)$

$a_n = 4 + 3n - 3$

$a_n = 3n + 1$

$a_{20} = 3(20) + 1$

$a_{20} = 61$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$

$S_{20} = 20\left(\frac{a_1 + a_{20}}{2}\right)$

$S_{20} = 20\left(\frac{4 + 61}{2}\right)$

$= 650$

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11) $\sum_{k=1}^{12} (-9k + 8)$ $\quad -1 + (-10) + (-19) + \cdots\cdots + (-100)$

Arith. $d = -9$

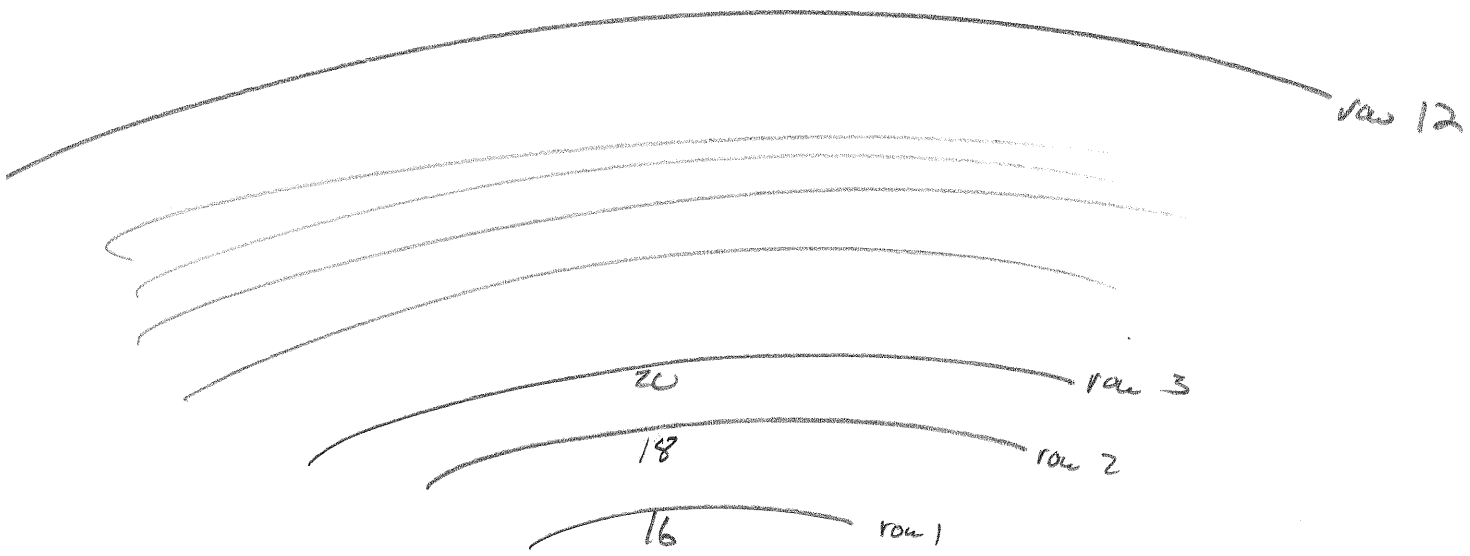
$S_{12} = 12\left(\frac{a_1 + a_{12}}{2}\right)$

$S_{12} = 12\left(\frac{(-1) + (-100)}{2}\right)$

$= 12\left(\frac{-101}{2}\right)$

$= -606$

12)



16, 18, 20, ...

arithmetic, $d=2$

$$a_n = a_1 + (n-1)(d)$$
$$= 16 + (n-1)(2)$$
$$= 16 + 2n - 2$$

rule → $a_n = 2n + 14$

$$a_{12} = 2(12) + 14$$
$$a_{12} = 24 + 14$$
$$a_{12} = 38$$

there are 38 seats in row 12

$$S_{12} = 12\left(\frac{a_1 + a_{12}}{2}\right)$$
$$= 12\left(\frac{16 + 38}{2}\right)$$
$$= 12\left(\frac{54}{2}\right)$$
$$= 324$$

there are 324 total seats in the auditorium

13) $\frac{3}{256}, \frac{3}{64}, \frac{3}{16}, \frac{3}{4}, \ldots\ldots$

$\cdot(4)$ $\cdot(4)$ $\cdot(4)$

Geometric $r = 4$

$a_n = a_1 \cdot r^{n-1}$

$a_n = \frac{3}{256}(4)^{n-1}$

$a_{10} = \frac{3}{256}(4)^{10-1}$

$= \frac{3}{256}(4)^9$

$= \frac{3}{256}(262144)$

$= 3(1024)$

$= 3072$

14) $a_4 = 2$ and $a_5 = 8$

$-, -, -, 2, 8$

$a_n = a_1 \cdot r^{n-1}$

$8 = 2 \cdot r^{5-4}$

$8 = 2 \cdot r^1$

$4 = r$

$a_5 = a_1 \cdot r^{5-1}$

$8 = a_1(4)^4$

$8 = 256\, a_1$

$a_1 = \frac{1}{32}$

$a_n = \frac{1}{32}(4)^{n-1}$

$a_{10} = \frac{1}{32}(4)^{10-1}$

$a_{10} = \frac{1}{32}(262144)$

$a_{10} = 8192$

15) find the geometric mean of 4 and 25

$x^2 = 4 \cdot 25$

$x = \sqrt{4 \cdot 25}$

$x = \sqrt{100}$

$x = 10$

16) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

$\cdot(\frac{1}{2}) \quad \cdot(\frac{1}{2})$

Geo. $r = \frac{1}{2}$

Find S_6

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$S_6 = a_1\left(\frac{1-r^6}{1-r}\right)$

$S_6 = 2\left(\frac{1-(\frac{1}{2})^6}{1-\frac{1}{2}}\right)$

$S_6 = 2\left(\frac{1-\frac{1}{64}}{\frac{1}{2}}\right)$

$S_6 = 2\left(\frac{\frac{63}{64}}{\frac{1}{2}}\right)$

$S_6 = 2\left(\frac{63}{64} \cdot \frac{2}{1}\right)$

$S_6 = \frac{63}{16}$

17) $\sum_{k=1}^{6} 250\left(-\frac{1}{5}\right)^{k-1}$

$250 - 50 + 10 - \cdots$

Geo. $r = -\frac{1}{5}$

$S_6 = a_1\left(\frac{1-r^6}{1-r}\right)$

$S_6 = 250\left(\frac{1-(-\frac{1}{5})^6}{1-(-\frac{1}{5})}\right)$

$S_6 = 250\left(\frac{1-\frac{1}{15625}}{\frac{6}{5}}\right)$

$S_6 = 250\left(\frac{15624}{15625} \cdot \frac{5}{6}\right)$

$S_6 = \frac{5208}{25}$ or 208.32

18) 1000

$1000(1.05) = 1050$ $(n=1)$

$1000(1.05)^2 = 1102.5$ $(n=2)$

$1000(1.05)^3 = 1157.625$ $(n=3)$

1050 , 1102.5 , 1157.625, ...

Geo. $r = 1.05$

$a_{10} = 1050(1.05)^{10-1}$

$= 1050(1.05)^9$

$= \$1628.89$

$S_{10} = 1050\left(\frac{1-(1.05)^{10}}{1-(1.05)}\right)$

$S_{10} = 1050\left(\frac{-0.628894626 8}{-0.05}\right)$

$S_{10} = \$13206.79$

19) $200 - 100 + 50 - 25 + \cdots\cdots$

$\cdot(-\frac{1}{2})$ $\cdot(-\frac{1}{2})$ $\cdot(-\frac{1}{2})$ Geo. $r = -\frac{1}{2}$

$S = \frac{a_1}{1-r}$

$S = \frac{200}{1-(-\frac{1}{2})}$

$S = \frac{200}{\frac{3}{2}}$ $S = 200 \cdot \frac{2}{3}$ $S = \frac{400}{3}$

OR $S = 133.\overline{3}$

geometric ✓

infinite ✓

series/sum ✓

$-1 < r < 1$ ✓

20) $\sum_{k=1}^{\infty} 2\left(\frac{7}{8}\right)^k$ Geo. $r = \frac{7}{8}$

$\frac{7}{4} + \frac{49}{32} + \cdots\cdots$

$S = \frac{\frac{7}{4}}{1-\frac{7}{8}}$

$S = \frac{\frac{7}{4}}{\frac{1}{8}}$

$S = \frac{7}{4} \cdot \frac{8}{1}$

$S = 14$

geo ✓

inf. ✓

sum ✓

$-1 < r < 1$ ✓