14.7: Half-Angle Identities ★ identities 15-19

derive $\cos\frac{\theta}{2}$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2\left(\frac{\theta}{2}\right) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$$

$$1+\cos\theta = 2\cos^2\frac{\theta}{2}$$

$$\pm\sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\cos^2\frac{\theta}{2}}$$

$$\pm\sqrt{\frac{1+\cos\theta}{2}} = \cos\frac{\theta}{2}$$

examples: find exact values

1 sin 112.5° Q2+

$$\sin\left(\frac{225^\circ}{2}\right)$$

$$= \sqrt{\frac{1-\cos 225^\circ}{2}}$$

$$= \sqrt{\frac{\left[1-\left(\frac{-\sqrt{2}}{2}\right)\right]^2}{[2]^2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

2 tan 105°

$$\tan\left(\frac{210^\circ}{2}\right) = \frac{1-\cos 210^\circ}{\sin 210^\circ}$$

$$= \frac{\left[1-\left(\frac{-\sqrt{3}}{2}\right)\right](-2)}{\left[\left(-\frac{1}{2}\right)\right](-2)}$$

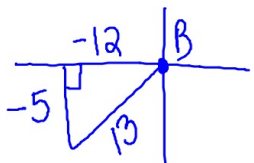
$$= -2-\sqrt{3}$$

3 Given: $\tan B = \frac{5}{12}$, $\pi \le B < \frac{3\pi}{2}$ (Q3)

$\frac{\pi}{2} < \frac{B}{2} < \frac{3\pi}{4}$ (Q2)

find $\cos\frac{B}{2}$ (Q2)

$= -\sqrt{\frac{1+\cos B}{2}}$



$= -\sqrt{\frac{1-12/13}{2}} = -\sqrt{1/13 \cdot 1/2}$

$= -\sqrt{\frac{1}{26}} = -\frac{1}{\sqrt{26}}$

$= -\frac{\sqrt{26}}{26}$