Name: ______________________ Date: ______________ Row: ______ Period: ____

NOTES SECTION 3.9: LINEAR APPROXIMATIONS & DIFFERENTIALS

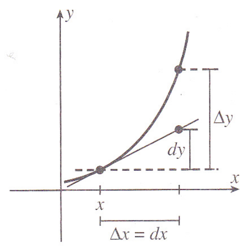
Let $y = f(x)$ be a differentiable function. The differential dx is an independent variable. The **differential** dy is defined as $dy = f'(x)\, dx$.

Note that dy is a function of two variables x (because of $f'(x)$) and dx.

If we let $dx = \Delta x$, then for small values of dx, the change in the function (Δy) is approximately the same as the change in the tangent line dy:

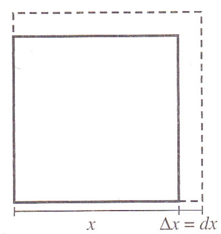
$$dy \approx \Delta y, \text{ when } dx \text{ is small.}$$

This is handy since dy may be easier to calculate than Δy.



dy may be thought of as the **error** in calculating a value for y if an error of dx is made in estimating x. $\frac{dy}{y}$ is the **relative error.**

Example: The sides of a square field are measured and found to be 50 m with a possible error of .02 m in the measurement. We calculate the area to be 2500 m^2. Estimate the maximum error and relative error in this calculation.



Let $x =$ the side of the field and A be the area.
We are given $\Delta x = dx = .02$ when $x = 50$.

$A = x^2$, so $dA = 2x\, dx$.

We estimate the maximum error ΔA with dA:

$$dA = 2(50)(.02) = 2 \text{ m}^2$$

and the relative error with $\frac{dA}{A}$:

$$\frac{dA}{A} = \frac{2\text{m}^2}{2500\text{m}^2} = .0008 = .08\%$$

5) True or False:

a) $\Delta x = dx$

True.

b) $\Delta y = dy$

False. (dy may be used to approximate Δy.)

6) Compute dy and Δy for $f(x) = x^2 + 3x$ at $x = 2$ with $\Delta x = dx = 0.1$.

$f(x) = x^2 + 3x$.
$f'(x) = 2x + 3$.
At $x = 2, f'(2) = 2(2) + 3 = 7$.
Thus, $dy = f'(x)\, dx = f'(2)(0.1)$
$= 7(0.1) = 0.7$.
At $x = 2, y = f(2) = 2^2 + 3(2) = 10$.
At $x = 2 + \Delta x = 2 + 0.1 = 2.1$,
$y = f(2.1) = (2.1)^2 + 3(2.1) = 10.71$.
Thus, $\Delta y = f(2.1) - f(2)$
$= 10.71 - 10 = 0.71$.
Note that $dy = 0.7$ and $\Delta y = 0.71$ are quite close but dy is easier to calculate.

7) A circle has a radius of 20 cm with a possible measurement error of 0.02 cm. Use differentials to estimate the maximum error and relative error for the area of the circle.

Let $x =$ radius of circle and $A =$ area of circle. We are given $\Delta x = dx = 0.02$ for $x = 20$.

The error is ΔA which we estimate with dA:
$A = \pi x^2$, so
$dA = 2\pi x\, dx = 2\pi(20) \cdot (0.02) = 0.8\pi$.
The relative error is
$\frac{dA}{A} = \frac{0.8\pi}{\pi(20)^2} \approx .002 = .2\%$.