1.7 Absolute Value Equations and Inequalities

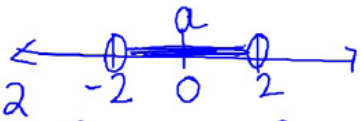
What does the absolute value of a number show? distance from 0
nonnegative #

3 types of absolute value equations or inequalities:

1. $|a| = 2$

a=-2 or a=2

2. $|a| < 2$



-2 < a < 2

3. $|a| > 2$

a<-2 OR a>2

2.4 Equations of Lines

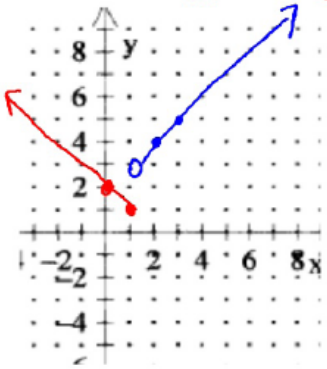
slope-intercept form:	$y = mx + b$
point-slope form:	$y - y_1 = m(x - x_1)$
standard form:	$Ax + By = C$ (A, B, and C are integers)

Algebra 2/Trig H
Sec. 2.7 Piecewise Functions

Ex. 2: Graph each function.

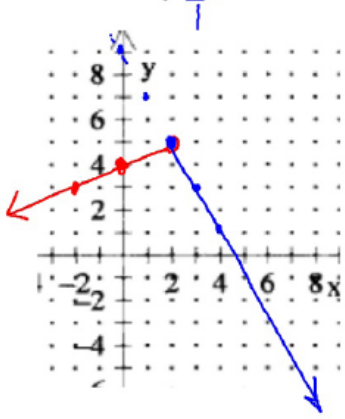
a) $f(x) = \begin{cases} x + 2, & \text{if } x > 1 \\ -x + 2, & \text{if } x \le 1 \end{cases}$

y
m=1
endpt (1,3)
(1,1)
m=-1
1/-1



b) $f(x) = \begin{cases} \frac{1}{2}x + 4, & \text{if } x < 2 \\ -2x + 9, & \text{if } x \ge 2 \end{cases}$

m=-1/2
y
(2,5)
-1/2
(2,5)



Section 2-8

General Form: $y = a|x-h| + k$

vertex: (h, k)

a determines direction of opening and $|a|$ determines the width

$a>0$ ∨ $a<0$ ∧ as $|a|$ increases, graph gets narrower

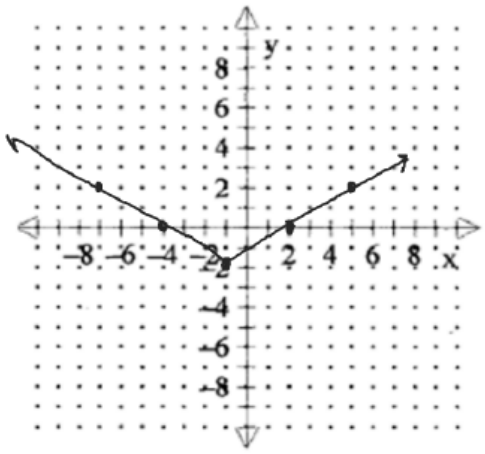
a is the slope of the ray that rises to the right from the vertex

$-a$ is the slope of the ray that rises to the left from the vertex

Ex. 1 graph $y = \frac{2}{3}|x+1|-2$

$y = \frac{2}{3}|x-(-1)|-2$ (h, k)

vertex $(-1, -2)$



3.6 Solving Systems of Equations in 3 Variables

1 **Solve for (x, y, z):**

$(1, -2, 3)$

① $2x + y + 3z = 9$

② $x - 2y + z = 8$

③ $-4x + 3y + 2z = -4$

② $x + 4 + 3 = 8$ $x = 1$

(2)① $4x + 2y + 6z = 18$

② $-4x + 3y + 2z = -4$

$5y + 8z = 14$

$-5y - z = 7$

$7z = 21$

$z = 3$, $y = -2$

① $2x + y + 3z = 9$

(-2)② $-2x + 4y - 2z = -16$

$-(5y + z = -7)$

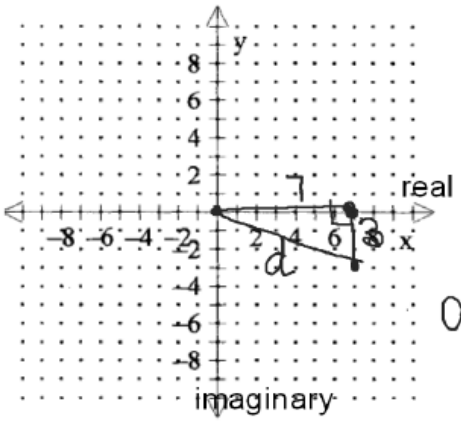
$5y + 3 = -7$

$5y = -10$

$y = -2$

Section 5-4

ex. 5 **The *absolute value of a complex number* is its distance from the origin in the complex plane.**



$$|a + bi| = \sqrt{a^2 + b^2}$$

$$|7 - 3i| = \sqrt{7^2 + (-3)^2}$$

$$\sqrt{58}$$

$$d = \sqrt{7^2 + 3^2}$$

Completing the square

ex. 4 $4x^2 - 12x + 4 = 0$

$$\frac{4}{4}$$

$$x^2 - 3x + 1 = 0$$

$$\left(-\frac{3}{2}\right)^2$$

$$x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{5}{4}}$$

$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2} \text{ or } \frac{3 \pm \sqrt{5}}{2}$$

ex. 1 **Write a quadratic function in vertex form $y = a(x - h)^2 + k$**

use CTS

a) if $y = x^2 + 4x + 16$

$$y - 16 = x^2 + 4x + 4$$

$$+4$$

$$y - 12 = (x + 2)^2$$

$$y = (x + 2)^2 + 12$$

b) if $y = -2x^2 + 4x + 5$

$$y - 5 = -2(x^2 - 2x + 1)$$

$$-2$$

$$y - 7 = -2(x - 1)^2$$

$$y = -2(x - 1)^2 + 7$$

Section 5-7

Ex. 2 Solve the inequalities:

$x \le -4$ OR $0 \le x \le \frac{1}{3}$

a) $2x^2 - x > 3$

$2x^2 - x - 3 > 0$

$(2x - 3)(x + 1) = 0$

$x = \frac{3}{2}, -1$ critical points

T F T

$-1 \quad 0 \quad \frac{3}{2} \quad 2$

$x < -1$ OR $x > \frac{3}{2}$

Try 0: $0 - 0 > 3$ F

-2: $8 + 2 > 3$ T

2: $8 - 2 > 3$ T

b) $3x^3 + 11x^2 - 4x \le 0$

$x(3x^2 + 11x - 4) \le 0$

$x(3x - 1)(x + 4) = 0$

$x = 0, \frac{1}{3}, -4$ critical pts

T F T F

$-4 \quad 0 \quad \frac{1}{3}$

Try 1: $3 + 11 - 4 \le 0$ F

Try: -5 $3(-125) + 11(25) + 20 \le 0$ T

-1 $-3 + 11 + 4 \le 0$ F

$\frac{1}{5}$ $3\left(\frac{1}{125}\right) + 11\left(\frac{1}{25}\right) - \frac{4}{5} \le 0$

$\frac{3 + 55 - 100}{125} \le 0$

Section 6-4

Factoring Cubes

(sum of cube roots)(sq. 1st root − prod. of roots + sq. of 2nd root)

sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

prime

ex. 6 $x^3 + 64$

$(x)^3 + (4)^3 = (x + 4)(x^2 - 4x + 16)$

4^2

ex. 7 $512x^6 - y^3$

$(8x^2)^3 - (y)^3 = (8x^2 - y)(64x^4 + 8x^2y + y^2)$

$(8x^2)^2$

cube root x x any root

$\sqrt[3]{x}$ $\sqrt[x]{y}$

x^3 y^x

$x=3$ $y=5/2$

Factoring by completing the square

ex. 2 $x^4 + 64$

CTS → $(x^4 + 16x^2 + 64) - 16x^2$ ($16x^2 = 2(x^2)(8)$)

factor → $(x^2+8)^2 - 16x^2 = (x^2+8+4x)(x^2+8-4x)$

$A^2 - B^2$

ex. 3 $x^2 - 50x + 589$

$(x^2 - 50x + 625) + 589 - 625$

$(x-25)^2 - 36 = (x-25-6)(x-25+6)$

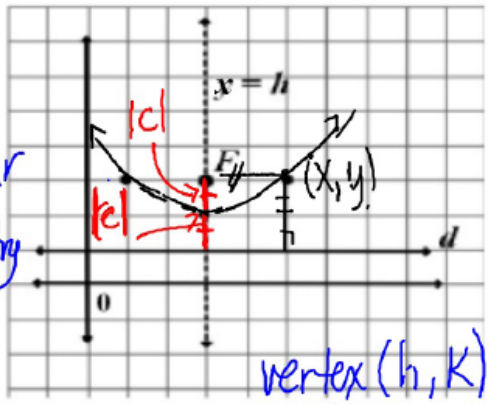
$(x-31)(x-19)$

Algebra 2H **10-2 Parabolas** **Date** _____

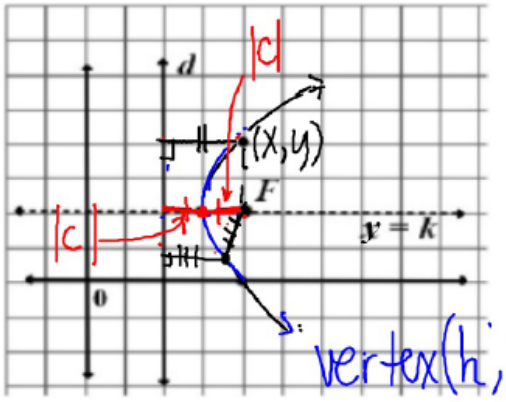
Definition of Parabola: A *parabola* is the set of points in a plane equidistant from a fixed point (focus) and a fixed line (directrix).

d = directrix
F = focus

directrix is perpendicular to axis of symmetry



vertex (h, K)



vertex (h, K)

Equation in vertex form	$y = a(x-h)^2 + K$	$x = a(y-K)^2 + h$
Axis of symmetry	$x = h$	$y = K$
Direction of opening	$a>0$ ↑ $a<0$ ↓	$a>0$ → $a<0$ ←
$\lvert c \rvert$ = distance from vertex to directrix and distance from vertex to focus; $c = \frac{1}{4a}$		

2 $x^2 - 6x - y + 10 = 0$ CTS

$x^2 - 6x + 9 = y - 10$

$+9$

$(x-3)^2 = y - 1$

$(x-3)^2 + 1 = y$

vertex (3,1)

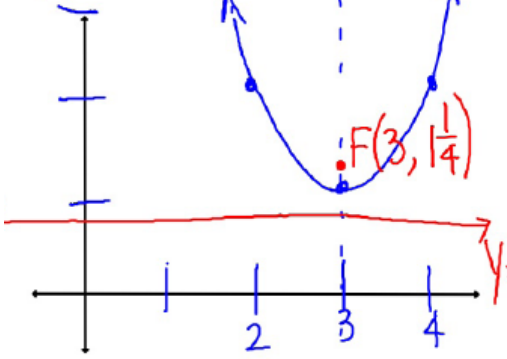
axis $x = 3$

x	y
4	2
2	2

$F\left(3, 1\frac{1}{4}\right)$

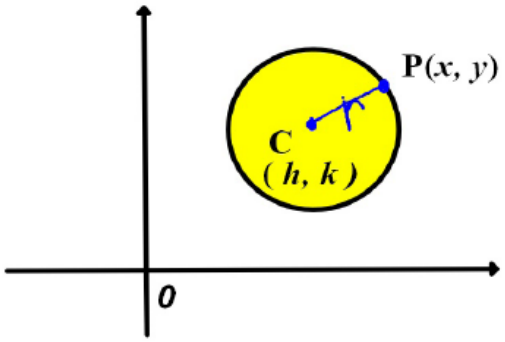
$c = \frac{1}{4 \cdot 1} = \frac{1}{4}$

$y = \frac{3}{4}$



10.3 **Equation of a Circle**

A circle is the set of all points in a plane at a fixed distance (radius) from a given point (center).



$\left(\sqrt{(x-h)^2 + (y-k)^2}\right)^2 = (r)^2$

$(x-h)^2 + (y-k)^2 = r^2$

Equation of circle in standard form.

$C(h, k)$ radius $= r$

Ellipses section 10-4

Ex . 2 graph $4(x-1)^2 + 16(y+2)^2 = 64$

$\frac{(x-1)^2}{16} + \frac{(y+2)^2}{4} = 1$

center $(1,-2)$

vertices $(5,-2),(-3,-2)$ $a=\pm 4$

co-vertices $(1,0)$ $(1,-4)$

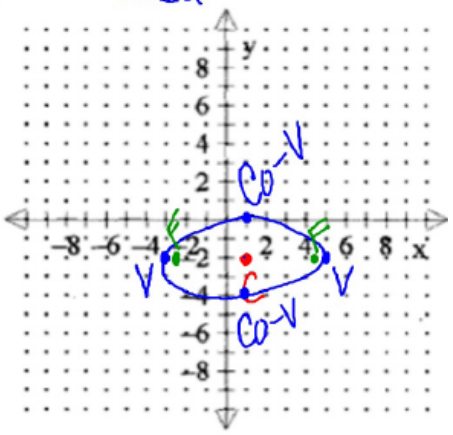
major axis horizontal

foci $(1+2\sqrt{3},-2)(1-2\sqrt{3},-2)$

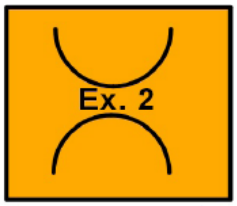
$c^2 = a^2 - b^2$

$\sqrt{c^2} = \sqrt{12}$

$c = \pm 2\sqrt{3}$



Section 10-5 Hyperbolas



Graph $4(y-1)^2 - 16(x+2)^2 = 64$

$\frac{(y-1)^2}{16} - \frac{(x+2)^2}{4} = 1$

center: $(-2,1)$ (h,k)

vertices: $(-2,5)(-2,-3)$

foci: $(-2,1+\sqrt{25})(-2,1-\sqrt{25})$

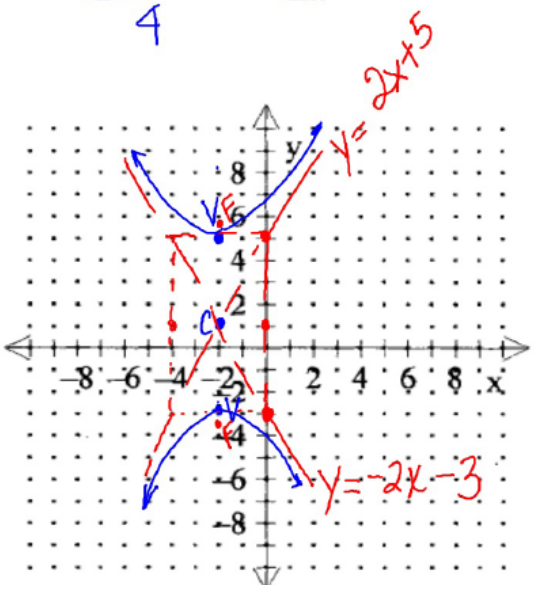
$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{20}$

$\pm 2\sqrt{5}$

asymptotes: $y = -2x - 3$

$y = 2x + 5$



General Conic Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Standard Forms

Parabola: $x = a(y-k)^2 + h$ $\quad$ $y = a(x-h)^2 + k$

$a>0$ ↱ $a<0$ ↰ $\quad$ $a>0$ ↑ $a<0$ ↓

Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ $\quad$ $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ $\quad a^2 > b^2$ $\quad$ $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Example 2: Classify and graph. Label EVERYTHING!

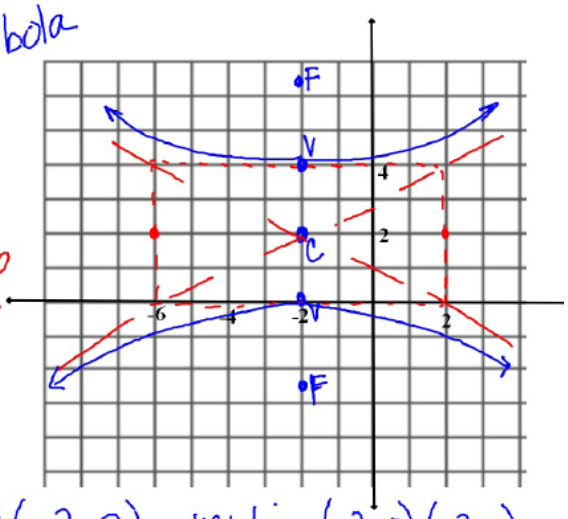
$4y^2 - x^2 - 16y - 4x - 4 = 0$ hyperbola

$4y^2 - 16y \quad -x^2 - 4x \quad = 4$

$4(y^2 - 4y + 4) - (x^2 + 4x + 4) = 4 + 16 - 4$

$4(y-2)^2 - (x+2)^2 = 16$

$\dfrac{(y-2)^2}{4} - \dfrac{(x+2)^2}{16} = 1$



$C(-2, 2)$ vertices $(-2, 0)(-2, 4)$

$c^2 = a^2 + b^2 = 20 \quad c = \pm\sqrt{20}$

$c \sim \pm 4.5$