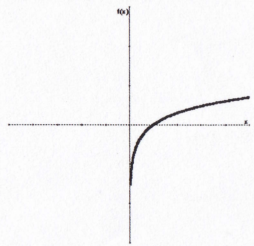
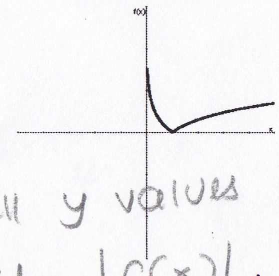
Worksheet 4-3B

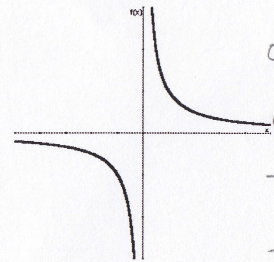
#123/1



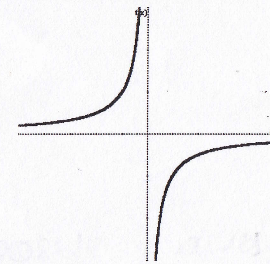


all y values are positive

$y = |f(x)|$



x	y
0.0..	∞
1	1
∞	0.0..
-∞	-0..
-1	-1
-0..	-∞

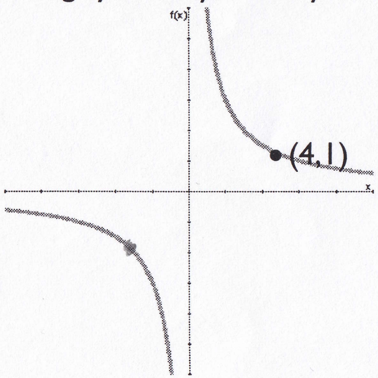


x	y
-∞	0...
-1	1
-0..	∞
0.0..	-∞
1	-1
∞	0.0..

opposite y values: $y = -f(x)$

opposite x values $y = f(-x)$

Using symmetry, identify as many

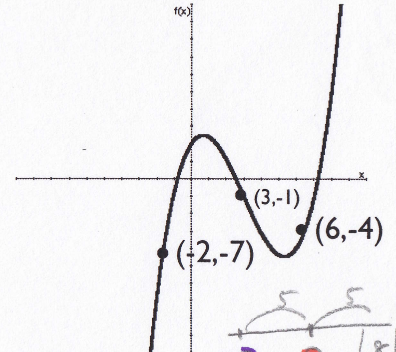


symmetric about the origin

$\rightarrow x \rightarrow -x$

$y \rightarrow -y$

$(4,1) \rightarrow (-4,-1)$



point of symmetry $(3,-1)$

$(6,-4)$

3 3

$\boxed{0}$ 3 6

$\boxed{2}$ -1 -4

$(6,-4) \rightarrow (0,2)$

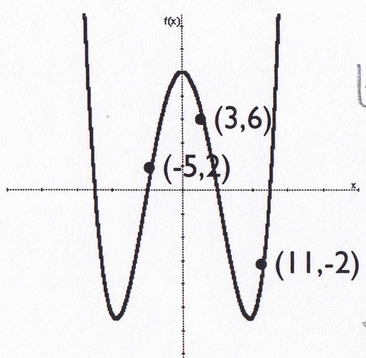
5 5

-2 3 $\boxed{8}$

$\boxed{5}$ -1 -7

6 6

$(-2,-7) \rightarrow (8,5)$



line of symmetry: $x=0$

3

$\boxed{-3}$ 0 3

$(3,6) \rightarrow (-3,6)$

y doesn't change

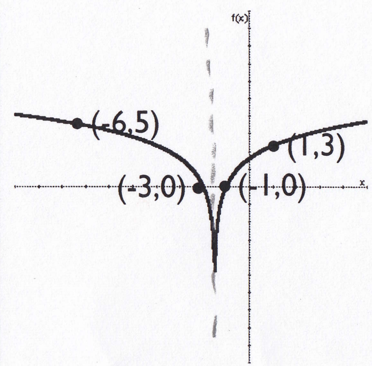
5 5

-5 0 $\boxed{5}$

$(-5,2) \rightarrow (5,2)$

$\boxed{-11}$ 0 11

$(11,-2) \rightarrow (-11,-2)$



line of symmetry: $x=-2$

y value doesn't chg

3

$\boxed{-5}$ -2 1

$(1,3) \rightarrow (-5,3)$

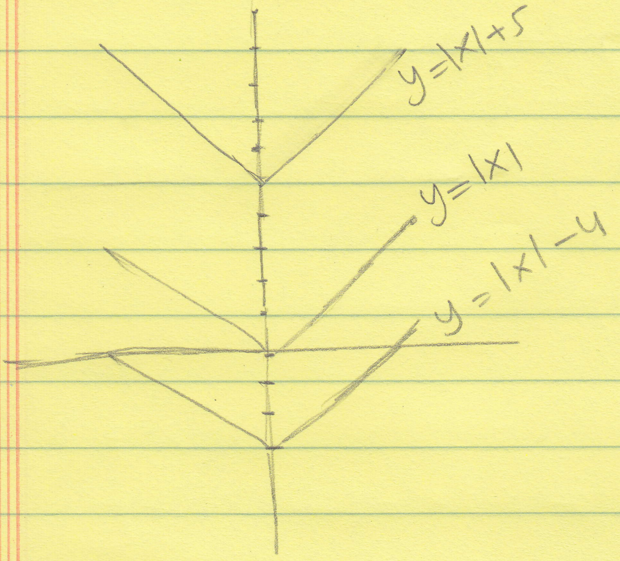
4 4

-6 -2 $\boxed{2}$

$(-6,5) \rightarrow (2,5)$

#123

p128 2 $y = |x|$, $y = |x|+5$ $y = |x|-4$



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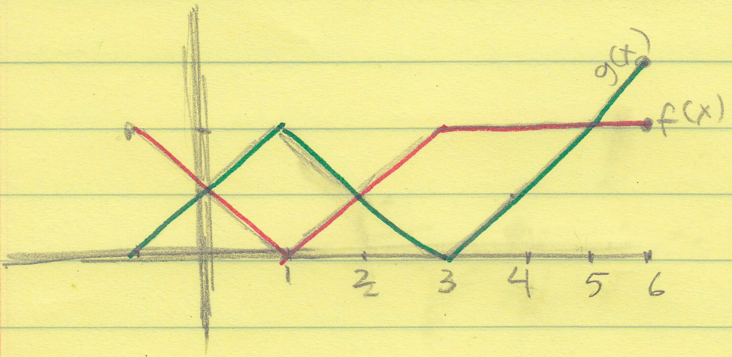

x	f(x)	g(x)	f(x)-g(x)
-1	2	0	2
0	1	1	0
1	0	2	-2
2	1	1	0
3	2	0	2
4	2	1	1
5	2	2	0
6	2	3	-1

a $f(1) - g(1) =$

$0 - 2 = -2$

b. $f(x) - g(x)$ is pos when

red graph is above green graph

$-1 \leq x < 0$; $2 < x < 5$

* $f(x) - g(x)$ is neg when red graph is below green graph

$0 < x < 2$; $5 < x \leq 6$

* $f(x) - g(x) = 0$ when red intersect green graphs

0, 2, 5

c Max $f(x) - g(x) = 2 \rightarrow$ see chart

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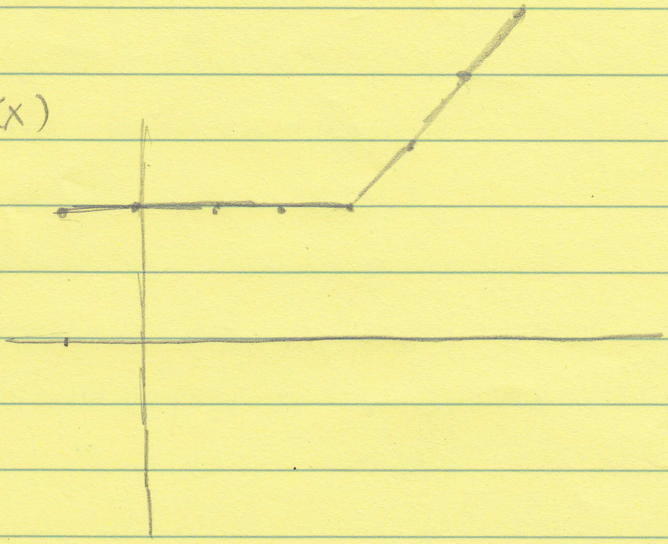
12 $f+g(x) = f(x)+g(x)$

x	$f(x)$	$g(x)$	$f(x)+g(x)$
-1	2	0	2
0	1	1	2
1	0	2	2
2	1	1	2
3	2	0	2
4	2	1	3
5	2	2	4
6	2	3	5



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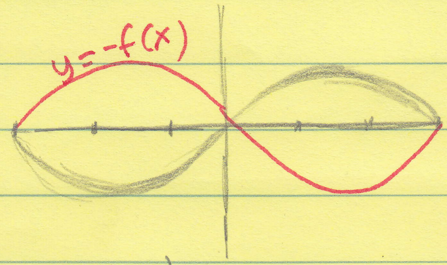


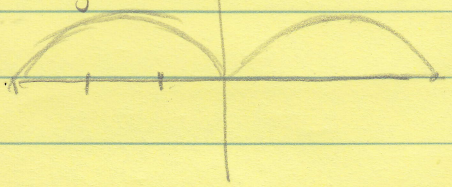
opposite y

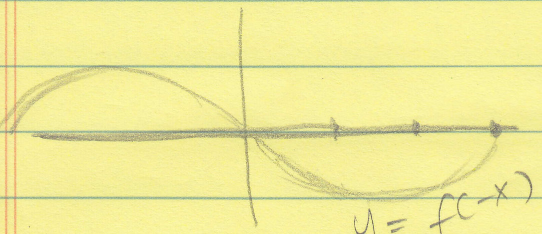
opposite x

x	$f(x)$	$-f(x)$	$\lvert f(x)\rvert$	$f(-x)$
-3	0	0	0	3 , 0
-1.5	1	-1	1	1.5 , 1
0	0	0	0	0 , 0
1.5	-1	1	1	-1.5 , -1
3	0	0	0	-3 , 0

-y

$y = \lvert f(x)\rvert$

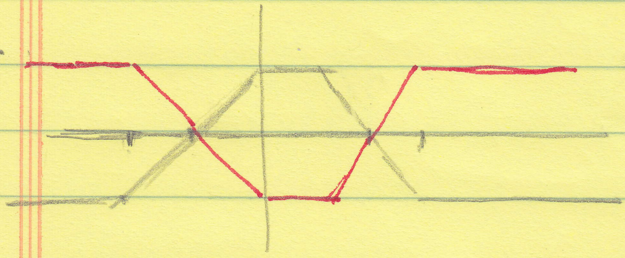


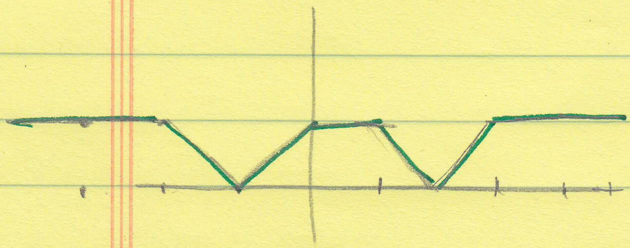


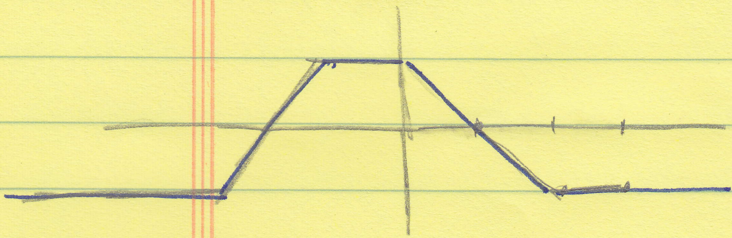
$y = f(-x)$

#123|3

2

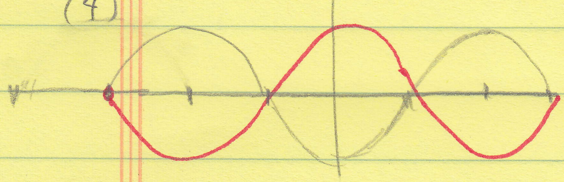


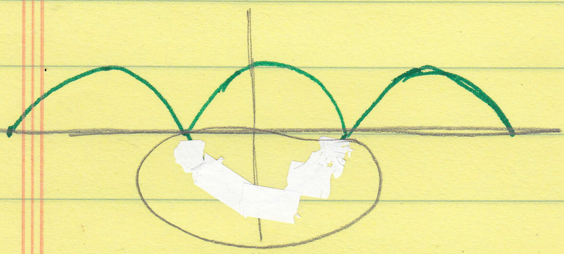


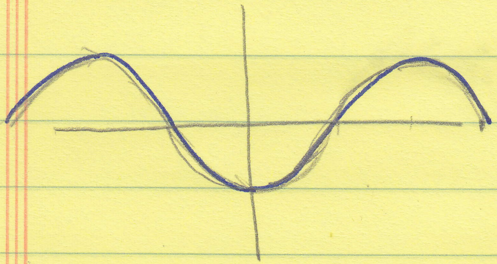


x	f(x)	−f(x)	|f(x)|	oppx f(−x)
−3	−1	1	1	(3, −1)
−2	−1	1	1	(2, −1)
−1	0	0	0	(1, 0)
0	1	−1	1	(0, 1)
1	1	−1	1	(−1, 1)
2	0	0	0	(−2, 0)
3	−1	1	1	(−3, −1)
4	−1	1	1	(−4, −1)

(4)







x	f(x)	−f(x)	|f(x)|	f(−x)
−3	0	0	0	(3, 0)
−2	1	−1	1	(2, 1)
−1	0	0	0	(1, 0)
0	−1	1	1	(0, −1)
1	0	0	0	(−1, 0)
2	1	−1	1	(−2, 1)
3	0	0	0	(−3, 0)

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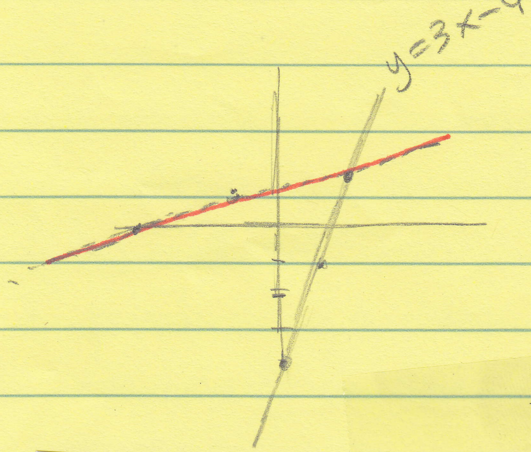
7. $y = 3x - 4$

$x = 3y - 4$

$3y = x + 4$

$y = \frac{x+4}{3} = \frac{1}{3}x + \frac{4}{3}$



x	y
0	-4
1	-1
2	2

x	y
-4	0
-1	1
2	2

9. $y = x^2 - 2x$

$x = y^2 - 2y$

$y^2 - 2y + 1 = x + 1$

$(y-1)^2 = x + 1$

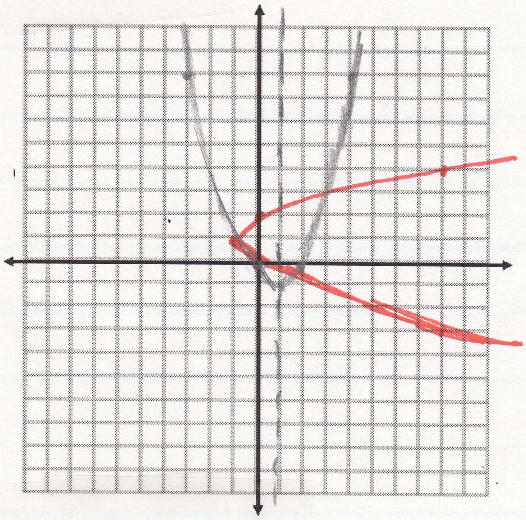
$y - 1 = \pm\sqrt{x+1}$

$y = 1 \pm \sqrt{x+1}$

$y = x(x-2)$

x	y
0	0
2	0
1	-1
4	8
-3	8

x	y
0	0
0	2
-1	1
8	4
8	-3



11. $y = x^3$

$x = y^3$

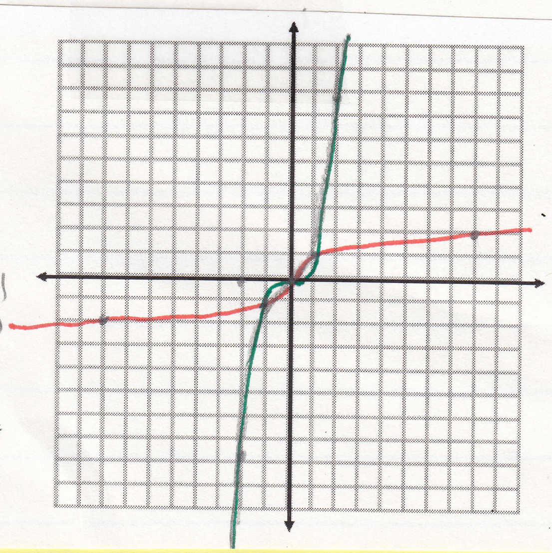
$y = \sqrt[3]{x}$

11

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

x	y
-8	-2
-1	-1
0	0
1	1
8	2



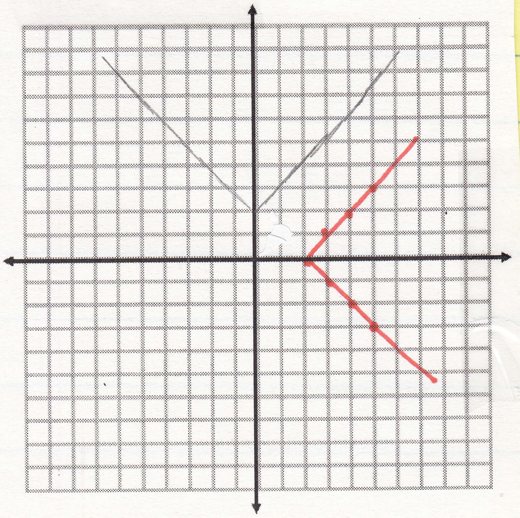
13. $y = |x| + 2$

$x = |y| + 2$

13

x	y
-3	5
-2	4
-1	3
0	2
1	3
2	4
3	5

x	y
5	-3
4	-2
3	-1
2	0
3	1
4	2
5	3



16 original $x^2+xy=4$

x-axis $y \to -y$: $x^2+x(-y)=4 \to x^2-xy=4$ no

y-axis $x \to -x$: $(-x)^2+(-x)y=4 \to x^2-xy=4$ no

$y=x$ switch x & y : $y^2+yx=4 \to y^2+xy=4$ no

origin $x \to -x$; $y \to -y$: $(-x)^2+(-x)(-y)=4 \to x^2+xy=4$ ✓

b. original : $|x|+|y|=1$

x-axis $y \to -y$: $|x|+|-y|=1 \to |x|+|y|=1$ ✓

y-axis $x \to -x$: $|-x|+|y|=1 \to |x|+|y|=1$ ✓

$y=x$ switch x & y : $|y|+|x|=1 \to |x|+|y|=1$ ✓

origin $x \to -x$; $y \to -y$: $|-x|+|-y|=1 \to |x|+|y|=1$ ✓

c $y = \frac{x}{|x|}$

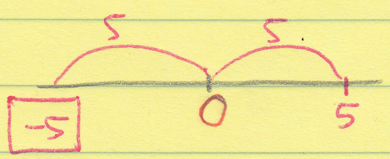
x-axis $y \to -y \to -y = \frac{x}{|x|} \to y = \frac{-x}{|x|}$ no

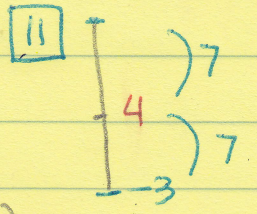
y-axis $x \to -x \to y = \frac{-x}{|-x|} \to y = \frac{-x}{|x|}$ no

$y=x$ switch x & y : $\frac{x}{1} = \frac{y}{|y|} \to y = x|y|$ no

origin $x \to -x$; $y \to -y$: $-y = \frac{-x}{|-x|} \to y = \frac{x}{|x|}$ ✓

27 $(5,-3)$ Point of symmetry $(0,4)$





$(5,-3) \to (-5,11)$

28. $f(x) = -x^3 + 15x^2 - 48x + 45$

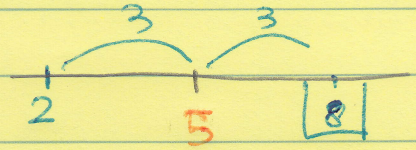
Point of symmetry: $x = \frac{-b}{3a} = \frac{-15}{3(-1)} = 5$

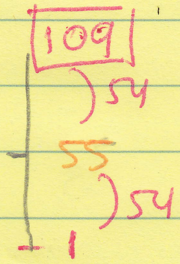
$f(5) =$

	-1	15	-48	45
5		-5	50	10
	-1	10	2	55

Point of symmetry $(5, 55)$

$(2, 1)$ = local min





$(2, 1) \rightarrow (8, 109)$

29. $y = 3x^2 - x^3$

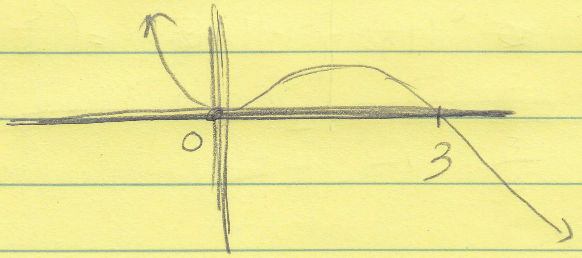
$y = -x^3 + 3x^2 \rightarrow y = -x^2(x-3)$

Quick sketch:

end behavior

zeros: 0 → dbl

3 → single

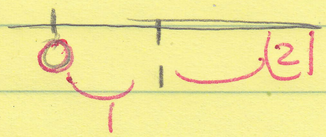


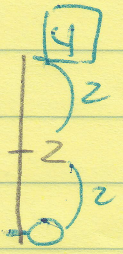
Local min = $(0, 0)$

b. Point of symmetry: $x = \frac{-b}{3a} = \frac{-3}{3(-1)} = 1$

$f(1) = 3(1)^2 - (1)^3 = 3 - 1 = 2$

Pt of symmetry $(1, 2)$





local min $= (0, 0)$

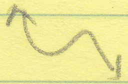
local max $= (2, 4)$

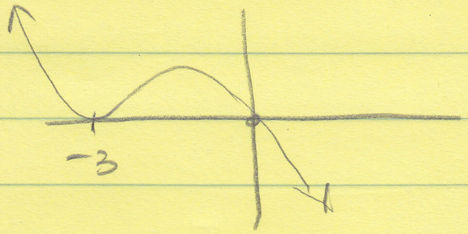
30 Graph: $y = -x^3 - 6x^2 - 9x$

$= -x\,(x^2 + 6x + 9)$

$= -x\,(x+3)^2$

zeros: 0 ; -3 (dbl)

end behavior 



Local min: $(-3, 0)$

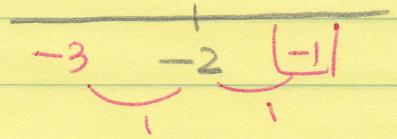
Point of symmetry:

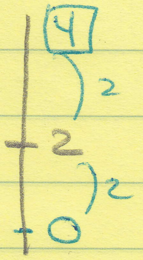
$$x = \frac{-b}{3a} = \frac{-(-6)}{3(-1)} = -2$$

-2	-1	-6	-9	0
		2	8	2
	-1	-4	-1	2

Point of symmetry $(-2, 2)$

Local min $(-3, 0)$





Local max $= (-1, 4)$

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$f(x) = x^2 \qquad g(x) = \sqrt{16-x^2}$

$f(g(x)) = f(\sqrt{16-x^2}) \rightarrow D: 16-x^2 \geq 0$

$(4+x)(4-x) \geq 0$

5: $+ - \rightarrow -$

0: $+ + \rightarrow +$

-5: $- + \rightarrow -$

$-$ $+$ $-$

-4 4

$-4 \leq x \leq 4$

$= (\sqrt{16-x^2})^2 = 16-x^2 \rightarrow$ D: all real #s

Final domain: $-4 \leq x \leq 4$

$g(f(x)) = g(x^2) \rightarrow$ D: all real #s

$= \sqrt{16-(x^2)^2}$

$= \sqrt{16-x^4} \rightarrow D: 16-x^4 \geq 0$

$(4+x^2)(4-x^2) \geq 0$

imaginary root

ignore

$(2+x)(2-x) \geq 0$

$-$ $+$ $-$

-2 2

3: $+ - \rightarrow -$

0: $+ + \rightarrow +$

-3: $- + \rightarrow -$

$-2 \leq x \leq 2$

Final domain: $-2 \leq x \leq 2$