
#3.3a Homework Solutions

1. $f(x) = 186.5$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.
2. $f(x) = \sqrt{30}$ is a constant function, so its derivative is 0, that is, $f'(x) = 0$.
3. $f(t) = 2 - \frac{2}{3}t \Rightarrow f'(t) = 0 - \frac{2}{3} = -\frac{2}{3}$
4. $F(x) = \frac{3}{4}x^8 \Rightarrow F'(x) = \frac{3}{4}(8x^7) = 6x^7$
5. $f(x) = x^3 - 4x + 6 \Rightarrow f'(x) = 3x^2 - 4(1) + 0 = 3x^2 - 4$
6. $h(x) = (x - 2)(2x + 3) = 2x^2 - x - 6 \Rightarrow h'(x) = 2(2x) - 1 - 0 = 4x - 1$
7. $f(t) = \frac{1}{4}(t^4 + 8) \Rightarrow f'(t) = \frac{1}{4}(t^4 + 8)' = \frac{1}{4}(4t^3 + 0) = t^3$
8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t \Rightarrow f'(t) = \frac{1}{2}(6t^5) - 3(4t^3) + 1 = 3t^5 - 12t^3 + 1$
9. $V(r) = \frac{4}{3}\pi r^3 \Rightarrow V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2$
10. $R(t) = 5t^{-3/5} \Rightarrow R'(t) = 5\left[-\frac{3}{5}t^{(-3/5)-1}\right] = -3t^{-8/5}$
11. $Y(t) = 6t^{-9} \Rightarrow Y'(t) = 6(-9)t^{-10} = -54t^{-10}$
12. $R(x) = \frac{\sqrt{10}}{x^7} = \sqrt{10}x^{-7} \Rightarrow R'(x) = -7\sqrt{10}x^{-8} = -\frac{7\sqrt{10}}{x^8}$
13. $F(x) = \left(\frac{1}{2}x\right)^5 = \left(\frac{1}{2}\right)^5 x^5 = \frac{1}{32}x^5 \Rightarrow F'(x) = \frac{1}{32}(5x^4) = \frac{5}{32}x^4$
14. $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2} \Rightarrow f'(t) = \frac{1}{2}t^{-1/2} - \left(-\frac{1}{2}t^{-3/2}\right) = \frac{1}{2\sqrt{t}} + \frac{1}{2t\sqrt{t}} \left[\text{or } \frac{t+1}{2t\sqrt{t}}\right]$
15. $A(s) = -\frac{12}{s^5} = -12s^{-5} \Rightarrow A'(s) = -12(-5s^{-6}) = 60s^{-6}$ or $60/s^6$
16. $B(y) = cy^{-6} \Rightarrow B'(y) = c(-6y^{-7}) = -6cy^{-7}$
17. $y = 4\pi^2 \Rightarrow y' = 0$ since $4\pi^2$ is a constant.
23. $V(x) = (2x^3 + 3)(x^4 - 2x) \xrightarrow{\text{PR}} V'(x) = (2x^3 + 3)(4x^3 - 2) + (x^4 - 2x)(6x^2) = (8x^6 + 8x^3 - 6) + (6x^6 - 12x^3) = 14x^6 - 4x^3 - 6$
27. $g(x) = \frac{3x-1}{2x+1} \xrightarrow{\text{QR}} g'(x) = \frac{(2x+1)(3) - (3x-1)(2)}{(2x+1)^2} = \frac{6x+3-6x+2}{(2x+1)^2} = \frac{5}{(2x+1)^2}$

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$$28. f(t) = \frac{2t}{4+t^2} \stackrel{\text{QR}}{\Rightarrow} f'(t) = \frac{(4+t^2)(2) - (2t)(2t)}{(4+t^2)^2} = \frac{8+2t^2-4t^2}{(4+t^2)^2} = \frac{8-2t^2}{(4+t^2)^2}$$

$$29. y = \frac{x^3}{1-x^2} \stackrel{\text{QR}}{\Rightarrow} y' = \frac{(1-x^2)(3x^2) - x^3(-2x)}{(1-x^2)^2} = \frac{x^2(3-3x^2+2x^2)}{(1-x^2)^2} = \frac{x^2(3-x^2)}{(1-x^2)^2}$$

$$30. y = \frac{x+1}{x^3+x-2} \stackrel{\text{QR}}{\Rightarrow}$$

$$y' = \frac{(x^3+x-2)(1) - (x+1)(3x^2+1)}{(x^3+x-2)^2} = \frac{x^3+x-2-3x^3-3x^2-x-1}{(x^3+x-2)^2} = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

$$\text{or } -\frac{2x^3+3x^2+3}{(x-1)^2(x^2+x+2)^2}$$

$$31. y = \frac{v^3-2v\sqrt{v}}{v} = v^2-2\sqrt{v} = v^2-2v^{1/2} \Rightarrow y' = 2v-2\left(\frac{1}{2}\right)v^{-1/2} = 2v-v^{-1/2}.$$

$$\text{We can change the form of the answer as follows: } 2v-v^{-1/2} = 2v-\frac{1}{\sqrt{v}} = \frac{2v\sqrt{v}-1}{\sqrt{v}} = \frac{2v^{3/2}-1}{\sqrt{v}}$$

$$32. y = \frac{t}{(t-1)^2} = \frac{t}{t^2-2t+1} \stackrel{\text{QR}}{\Rightarrow}$$

$$y' = \frac{(t^2-2t+1)(1) - t(2t-2)}{[(t-1)^2]^2} = \frac{(t-1)^2-2t(t-1)}{(t-1)^4} = \frac{(t-1)[(t-1)-2t]}{(t-1)^4} = \frac{-t-1}{(t-1)^3}$$

$$33. y = \frac{t^2+2}{t^4-3t^2+1} \stackrel{\text{QR}}{\Rightarrow}$$

$$y' = \frac{(t^4-3t^2+1)(2t) - (t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2} = \frac{2t[(t^4-3t^2+1) - (t^2+2)(2t^2-3)]}{(t^4-3t^2+1)^2}$$
$$= \frac{2t(t^4-3t^2+1-2t^4-4t^2+3t^2+6)}{(t^4-3t^2+1)^2} = \frac{2t(-t^4-4t^2+7)}{(t^4-3t^2+1)^2}$$

$$34. g(t) = \frac{t-\sqrt{t}}{t^{1/3}} = \frac{t}{t^{1/3}} - \frac{t^{1/2}}{t^{1/3}} = t^{2/3} - t^{1/6} \Rightarrow g'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{6}t^{-5/6}$$

$$61. \text{ (a) } s = t^3 - 3t \Rightarrow v(t) = s'(t) = 3t^2 - 3 \Rightarrow a(t) = v'(t) = 6t$$

$$\text{ (b) } a(2) = 6(2) = 12 \text{ m/s}^2$$

$$\text{ (c) } v(t) = 3t^2 - 3 = 0 \text{ when } t^2 = 1, \text{ that is, } t = 1 \text{ and } a(1) = 6 \text{ m/s}^2.$$

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63. We are given that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$.

(a) $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16$

(b) $\left(\frac{f}{g}\right)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{(-3)(6) - (1)(2)}{(-3)^2} = -\frac{20}{9}$

(c) $\left(\frac{g}{f}\right)'(5) = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \frac{(1)(2) - (-3)(6)}{(1)^2} = 20$

65. $f(x) = \sqrt{x}g(x) \Rightarrow f'(x) = \sqrt{x}g'(x) + g(x) \cdot \frac{1}{2}x^{-1/2}$, so $f'(4) = \sqrt{4}g'(4) + g(4) \cdot \frac{1}{2\sqrt{4}} = 2 \cdot 7 + 8 \cdot \frac{1}{4} = 16$.

70. (a) $y = x^2 f(x) \Rightarrow y' = x^2 f'(x) + f(x)(2x)$

(b) $y = \frac{f(x)}{x^2} \Rightarrow y' = \frac{x^2 f'(x) - f(x)(2x)}{(x^2)^2} = \frac{x f'(x) - 2f(x)}{x^3}$

(c) $y = \frac{x^2}{f(x)} \Rightarrow y' = \frac{f(x)(2x) - x^2 f'(x)}{[f(x)]^2}$

(d) $y = \frac{1 + xf(x)}{\sqrt{x}} \Rightarrow$

$$y' = \frac{\sqrt{x}[xf'(x) + f(x)] - [1 + xf(x)] \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$
$$= \frac{x^{3/2} f'(x) + x^{1/2} f(x) - \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{1/2} f(x)}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}} = \frac{xf(x) + 2x^2 f'(x) - 1}{2x^{3/2}}$$

71. The curve $y = 2x^3 + 3x^2 - 12x + 1$ has a horizontal tangent when $y' = 6x^2 + 6x - 12 = 0 \Leftrightarrow 6(x^2 + x - 2) = 0 \Leftrightarrow 6(x+2)(x-1) = 0 \Leftrightarrow x = -2$ or $x = 1$. The points on the curve are $(-2, 21)$ and $(1, -6)$.