

Volume of Revolution Worksheet  
Disk and Washer Methods

*(Integrate by hand and double check you work--also practice integrating)*

$$\text{Disks: } V = \int_a^b \pi r^2 dx \text{ or } V = \int_c^d \pi r^2 dy \quad \text{Washers: } V = \int_a^b \pi(R^2 - r^2) dx \text{ or } V = \int_c^d \pi(R^2 - r^2) dy$$

Remember "r" or "R" is a function minus the axis!

1. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = 6$ ,  $y = 0$ ,  $x = 0$ , and  $x = 4$  about: (a) the  $x$ -axis (**452.389**) and (b)  $y$ -axis (**301.593**)
2. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = x$ ,  $y = 0$ , and  $x = 2$  about: (a) the  $x$ -axis (**8.378**) and (b)  $y$ -axis (**16.755**)
3. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = \sqrt{1 - x^2}$  and  $y = 0$  about the  $x$ -axis. (**4.189**)
4. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = x^2$  and  $y = 4$  about the  $x$ -axis. (**160.850**)
5. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = 1 - x$ ,  $y = 0$ , and  $x = 0$  about: (a) the  $x$ -axis (**1.047**), (b) the  $y$ -axis (**1.047**), and (c) the line  $y = -1$ . (**4.189**)
6. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = x$ ,  $y = 0$ , and  $2 \leq x \leq 4$  about: (a) the  $x$ -axis (**58.643**), (b) the  $y$ -axis (**117.286**), and (c) the line  $x = 4$ . (**33.510**)
7. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$  about: (a) the  $x$ -axis (**25.133**), (b) the  $y$ -axis (**80.425**), (c) the line  $x = 4$  (**53.617**), and (d) the line  $x = 6$ . (**120.637**)
8. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = 2x^2$ ,  $y = 0$ , and  $x = 2$  about: (a) the  $y$ -axis (**50.265**), (b) the  $x$ -axis (**80.425**), and (c) the line  $y = 8$  (**187.658**),
9. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = x^2$  and  $y = 4x - x^2$  about: (a) the  $x$ -axis (**33.510**) and (b) the line  $y = 6$  (**67.021**).
10. Find the volume of the solid of revolution generated by revolving the region bounded by  $y = 6 - 2x - x^2$  and  $y = x + 6$  about: (a) the  $x$ -axis (**152.681**) and (b) the line  $y = 3$  (**67.858**).
11. The region bounded by the parabola  $y = 4x - x^2$  and the  $x$ -axis is revolved about the  $x$ -axis. Find the volume of the solid. (**107.233**)

**Additional Practice if Needed--Disks and Washers**

*For problems 1 through 5, find the volume of the solid obtained by revolving about the  $x$ -axis the region with the given boundaries:*

1)  $f(x) = 2x + 1, y = 0, 1 \leq x \leq 4$  (367.566)

2)  $f(x) = \sin x, y = 0, 1 \leq x \leq \pi$  (4.935)

3)  $f(x) = \tan x, y = 0, 1 \leq x \leq \pi/4$  (0.674)

4)  $f(x) = (1/4)x^2, g(x) = x$  (26.808)

5)  $f(x) = \sin x, g(x) = \cos x, 0 \leq x \leq \pi/4$  (1.571)

*For problems 6 through 8, find the volume of the solid obtained by revolving about the  $y$ -axis the region with the given boundaries:*

6)  $y = x^2, y = 0, 0 \leq x \leq 2$  (25.133)

7)  $y = x^3, x = 2, y = 0$  (40.212)

8)  $y = 1/x, y = 0, 1/4 \leq x \leq 1$  (4.712)

9) Find the volume of the solid obtained by rotating the region bounded by the  $x$ -axis and the graph of  $y = 1 - x^2$  about the line  $y = -3$ . (28.484)

10) Find the volume of the solid obtained when the region bounded by the graphs of  $y = \sqrt{x}, y = 0$ , and  $x = 9$  is rotated about the line  $y = -2$ . (353.429)