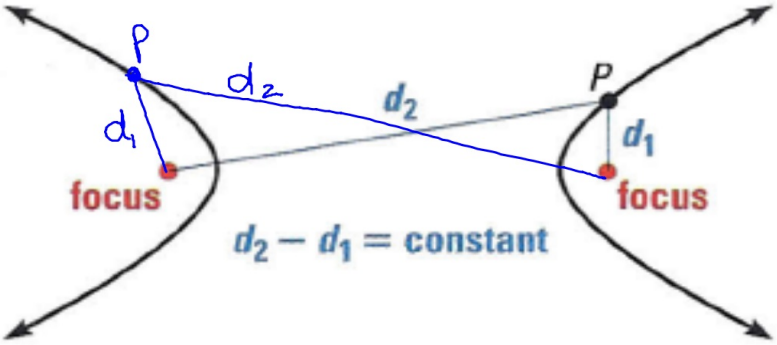
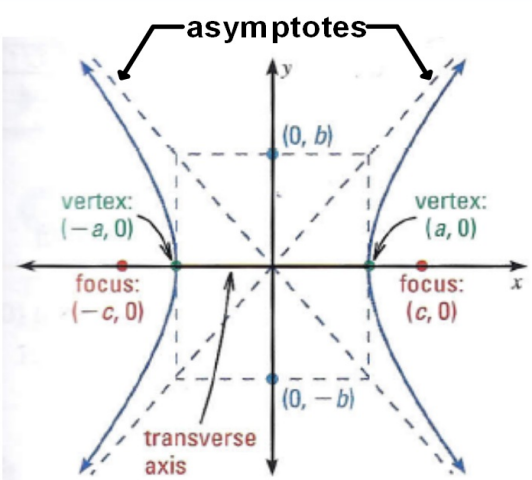
10.5 Hyperbolas

A **hyperbola** is the set of all points P in a plane such that the difference of the distances from P to 2 fixed points (**foci**) is constant.

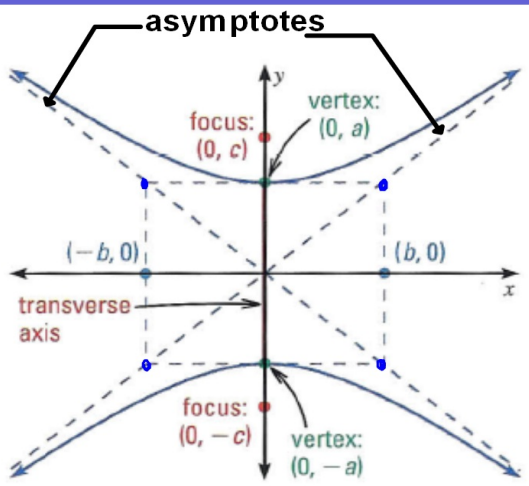
A hyperbola has 2 branches.





Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Hyperbola with vertical transverse axis

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$c^2 = a^2 + b^2$

center (0, 0)

Ex. 1

Graph $81x^2 - 9y^2 = 729$

$\frac{729}{729} \quad \frac{729}{729} \quad \frac{729}{729}$

$\frac{x^2}{9} - \frac{y^2}{81} = 1$

$a^2 = 9 \quad b = \pm 9$

$a = \pm 3$

center: $(0,0)$

vertices: $(\pm 3, 0)$

foci: $c^2 = a^2 + b^2$

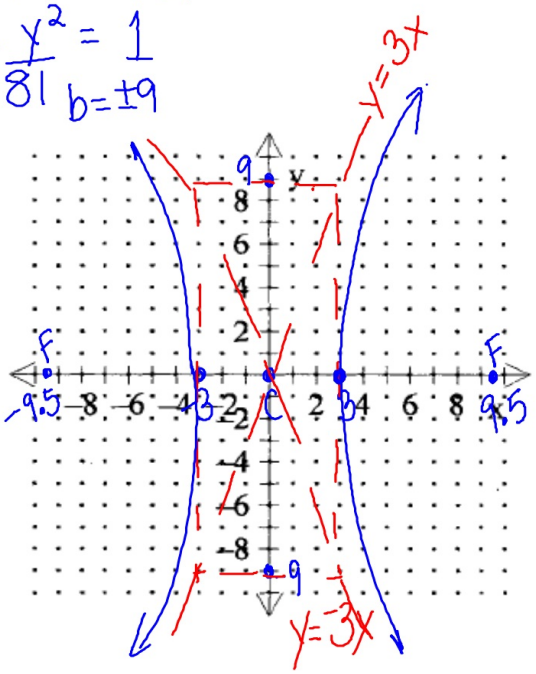
$c^2 = 9 + 81$

$\sqrt{c^2} = \sqrt{90} = \pm 3\sqrt{10}$

$(\pm 3\sqrt{10}, 0)$

$(3\sqrt{10}, 0) \quad (-3\sqrt{10}, 0)$

asymptotes: $y = \pm 3x$



Ex. 2

Graph $4(y-1)^2 - 16(x+2)^2 = 64$

$\frac{(y-1)^2}{16} - \frac{(x+2)^2}{4} = 1$

$a^2 = \pm 4 \quad b = \pm 2$

center: $(-2, 1)$

vertices: $(-2, 5) \quad (-2, -3)$

foci: $c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{20} = \pm 2\sqrt{5}$

$(-2, 1+2\sqrt{5}) \quad (-2, 1-2\sqrt{5})$

asymptotes: $y = 2x + 5 \quad y = -2x - 3$