Worksheet 2: Solutions and Teacher Notes

Note to instructors: This is a great worksheet to have students work on in pairs. The discussions can get quite heated! There is certainly more than one counterexample to each false statement and often more than one way to rewrite the false statements; the solutions below provide you with one idea for each. Students might have other ideas that make for good discussion items. To make the work go faster, I often require students only to provide a counterexample or to rewrite the false statement so that it is true.

1. **False**

   Counterexample: Note that $x = 0$ is a critical point for the function $f(x) = x^3$, but that $x = 0$ corresponds to neither a relative maximum nor a relative minimum value of f.

   Rewrite: A critical point is a POSSIBLE location for a relative maximum or minimum value of a function.

2. **True.** Note that the maximum value can occur at more than one x-value but that the maximum value itself is unique. See Problem 8 for an example.

3. **False**

   Counterexample: For $f(x) = e^x$, $f''(x)$ is always positive, but the function $f(x) = e^x$ has no relative extrema.

   Rewrite: If $f'(c) = 0$ and $f''(x)$ is always positive, then the function must have a relative minimum at $x = c$.

4. **False**

   Counterexample: The function $f(x) = |x|$ has a local minimum at $x = 0$, but $f'(0)$ is not defined.

   Rewrite: If a DIFFERENTIABLE function has a local minimum value at $x = c$, then $f'(c) = 0$.

5. **True**

6. **False**

   Counterexample: For the function $f(x) = x^4$, $f''(0) = 0$, but $x = 0$ is not a point of inflection. Note that $x = 0$ does correspond to a relative and absolute minimum value of f.

   Rewrite: If $f''(c) = 0$ for a function f, then $x = c$ may or may not be an inflection point for f and $x = c$ may or may not correspond to a relative minimum or maximum value of f.

7. **True**

8. **False**

   Counterexample: The function $f(x) = \sin(x)$ takes on its minimum value of -1 at the points $x = \frac{3\pi}{2}$, $x = \frac{7\pi}{2}$, and $x = \frac{11\pi}{2}$ in the interval $0 < x < 6\pi$.

   Rewrite: There is exactly one absolute minimum value of a continuous function on a closed interval, but this minimum value can occur at more than one point in the interval. See Problem 2.

9. **True**

10. **False**

    Counterexample: The function $f(x) = x^2$ on the interval $2 \le x \le 6$ has its absolute minimum value at $x = 2$ and its absolute maximum value at $x = 6$. Neither $x = 2$ nor $x = 6$ is a critical point of the function.

    Rewrite: To locate the absolute extrema of a continuous function on a closed interval, you must compare the y-values of all critical points AND ENDPOINTS.

11. **False**

    Counterexample: For $f(x) = 7 - x^2$, $f'(0) = 0$, $f'(x) = -2x$ decreases through $x = 0$, and f has a local (and global) maximum value at $x = 0$.

    Rewrite: If $f'(c) = 0$ and $f'(x)$ decreases through $x = c$, then $x = c$ locates a local MAXIMUM value for the function. Or, if $f'(c) = 0$ and $f'(x)$ INCREASES through $x = c$, then $x = c$ locates a local minimum value for the function.

12. **True**