

## **2-6** Geometric Proof

### *Objectives*

Write two-column proofs.

Prove geometric theorems by using deductive reasoning.

## **2-6** Geometric Proof

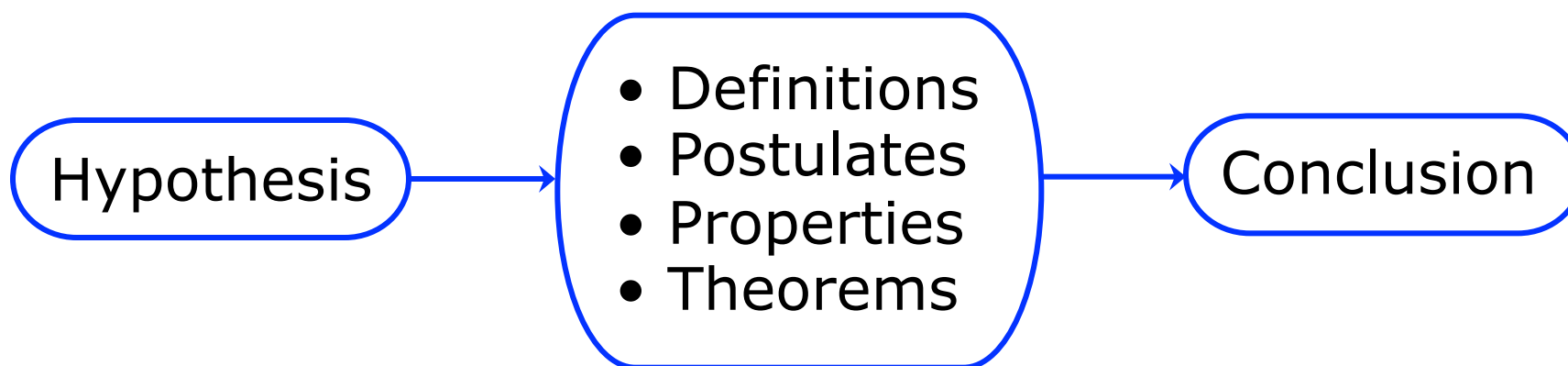
### ***Vocabulary***

theorem

two-column proof

## 2-6 Geometric Proof

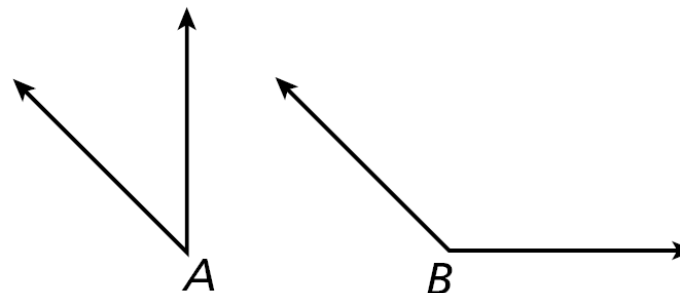
When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.



## 2-6 Geometric Proof

### Example 1: Writing Justifications

Write a justification for each step, given that  $\angle A$  and  $\angle B$  are supplementary and  $m\angle A = 45^\circ$ .

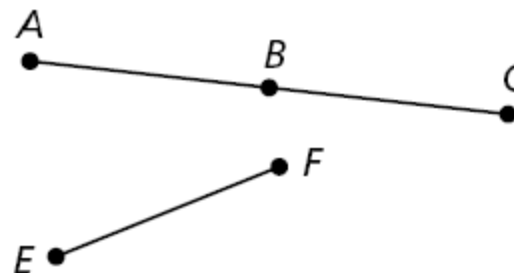


- |   |                                       |
|---|---------------------------------------|
| 1. $\angle A$ and $\angle B$ are supplementary.<br>$m\angle A = 45^\circ$ | Given information                     |
| 2. $m\angle A + m\angle B = 180^\circ$                                    | Def. of supp $\angle$ s               |
| 3. $45^\circ + m\angle B = 180^\circ$                                     | Subst. Prop of =<br><i>Steps 1, 2</i> |
| 4. $m\angle B = 135^\circ$  | Subtr. Prop of =                      |

## 2-6 Geometric Proof

### Check It Out! Example 1

Write a justification for each step, given that  $B$  is the midpoint of  $\overline{AC}$  and  $\overline{AB} \cong \overline{EF}$ .



1.  $B$  is the midpoint of  $\overline{AC}$ .

Given information

2.  $\overline{AB} \cong \overline{BC}$

Def. of mdpt.

3.  $\overline{AB} \cong \overline{EF}$

Given information

4.  $\overline{BC} \cong \overline{EF}$

Trans. Prop. of  $\cong$

## 2-6 Geometric Proof

A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.

## 2-6 Geometric Proof

### Theorem

THEOREM	HYPOTHESIS	CONCLUSION
<b>2-6-1 Linear Pair Theorem</b> If two angles form a linear pair, then they are supplementary.	$\angle A$ and $\angle B$ form a linear pair.	$\angle A$ and $\angle B$ are supplementary.

## 2-6 Geometric Proof

### Theorem

THEOREM	HYPOTHESIS	CONCLUSION
<p><b>2-6-2 Congruent Supplements Theorem</b> If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.</p>	<p><math>\angle 1</math> and <math>\angle 2</math> are supplementary. <math>\angle 2</math> and <math>\angle 3</math> are supplementary.</p>	<p><math>\angle 1 \cong \angle 3</math></p>

## 2-6 Geometric Proof

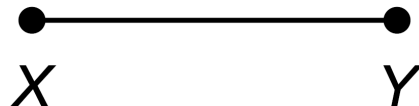
A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.

## 2-6 Geometric Proof

### Example 2: Completing a Two-Column Proof

Fill in the blanks to complete the two-column proof.

Given:  $\overline{XY}$



Prove:  $\overline{XY} \cong \overline{XY}$

Statements	Reasons
1. $\overline{XY}$	1. Given
2. $XY = XY$	2. Reflex. Prop. of =
3. $\overline{XY} \cong \overline{XY}$	3. Def. of $\cong$ segs.

## 2-6 Geometric Proof

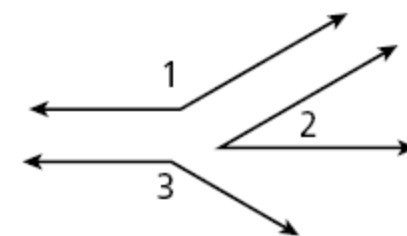
### Check It Out! Example 2

Fill in the blanks to complete a two-column proof of one case of the Congruent Supplements Theorem.

**Given:**  $\angle 1$  and  $\angle 2$  are supplementary, and  
 $\angle 2$  and  $\angle 3$  are supplementary.

**Prove:**  $\angle 1 \cong \angle 3$

**Proof:**



Statements	Reasons
1. a. _____?	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	2. Def. of supp. $\sphericalangle$
3. b. _____?	3. Subst.
4. $m\angle 2 = m\angle 2$	4. Reflex. Prop. of =
5. $m\angle 1 = m\angle 3$	5. c. _____?
6. d. _____?	6. Def. of $\cong \sphericalangle$

- $\angle 1$  and  $\angle 2$  are supp., and  $\angle 2$  and  $\angle 3$  are supp.
- $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
- Subtr. Prop. of =
- $\angle 1 \cong \angle 3$

## 2-6 Geometric Proof

### Theorems

THEOREM	HYPOTHESIS	CONCLUSION
<b>2-6-3 Right Angle Congruence Theorem</b> All right angles are congruent.	$\angle A$ and $\angle B$ are right angles.	$\angle A \cong \angle B$
<b>2-6-4 Congruent Complements Theorem</b> If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.	$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	$\angle 1 \cong \angle 3$

## 2-6 Geometric Proof

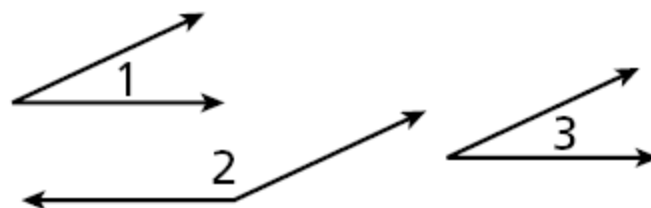
### Example 3: Writing a Two-Column Proof from a Plan

Use the given plan to write a two-column proof.

**Given:**  $\angle 1$  and  $\angle 2$  are supplementary, and

$$\angle 1 \cong \angle 3$$

**Prove:**  $\angle 3$  and  $\angle 2$  are supplementary.



**Plan:** Use the definitions of supplementary and congruent angles and substitution to show that  $m\angle 3 + m\angle 2 = 180^\circ$ . By the definition of supplementary angles,  $\angle 3$  and  $\angle 2$  are supplementary.

## 2-6 Geometric Proof

### Example 3 Continued

Statements	Reasons
<b>1.</b> $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 3$	<b>1.</b> Given
<b>2.</b> $m\angle 1 + m\angle 2 = 180^\circ$	<b>2.</b> Def. of supp. $\angle$ s
<b>3.</b> $m\angle 1 = m\angle 3$	<b>3.</b> Def. of $\cong \angle$ s
<b>4.</b> $m\angle 3 + m\angle 2 = 180^\circ$	<b>4.</b> Subst.
<b>5.</b> $\angle 3$ and $\angle 2$ are supplementary	<b>5.</b> Def. of supp. $\angle$ s

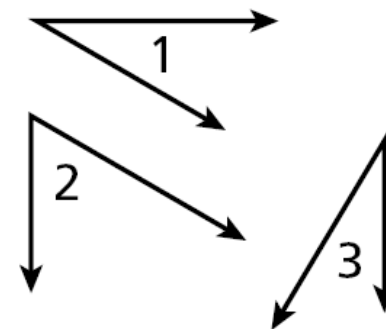
## 2-6 Geometric Proof

### Check It Out! Example 3

Use the given plan to write a two-column proof if one case of Congruent Complements Theorem.

**Given:**  $\angle 1$  and  $\angle 2$  are complementary, and  
 $\angle 2$  and  $\angle 3$  are complementary.

**Prove:**  $\angle 1 \cong \angle 3$



**Plan:** The measures of complementary angles add to  $90^\circ$  by definition. Use substitution to show that the sums of both pairs are equal. Use the Subtraction Property and the definition of congruent angles to conclude that  $\angle 1 \cong \angle 3$ .

## 2-6 Geometric Proof

### Check It Out! Example 3 Continued

Statements	Reasons
<b>1.</b> $\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	<b>1.</b> Given
<b>2.</b> $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 2 + m\angle 3 = 90^\circ$	<b>2.</b> Def. of comp. $\angle$ s
<b>3.</b> $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	<b>3.</b> Subst.
<b>4.</b> $m\angle 2 = m\angle 2$	<b>4.</b> Reflex. Prop. of =
<b>5.</b> $m\angle 1 = m\angle 3$	<b>5.</b> Subtr. Prop. of =
<b>6.</b> $\angle 1 \cong \angle 3$	<b>6.</b> Def. of $\cong \angle$ s