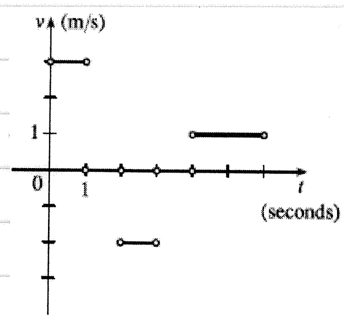
3.1b

11 (a) moving to the right: (0,1) and (4,6)
moving to the left: (2,3)
standing still: (1,2) and (3,4)

(b)




12. (a) **Runner A** runs the entire 100-meter race at the same velocity since the slope of the position function is constant. **Runner B** starts the race at a slower velocity than runner A, but finishes the race at a faster velocity.

(b) The distance between the runners is the greatest at the time when the largest vertical line segment fits between the two graphs—this appears to be somewhere between 9 and 10 seconds.

(c) The runners had the same velocity when the slopes of their respective position functions are equal—this also appears to be at about 9.5 s. Note that the answers for parts (b) and (c) must be the same for these graphs because as soon as the velocity for runner B overtakes the velocity for runner A, the distance between the runners starts to decrease.

14. $H(t) = 10t - 1.86t^2$

$$v(a) = \lim_{h \to 0} \frac{H(a+h) - H(a)}{h} = \lim_{h \to 0} \frac{[10(a+h) - 1.86(a+h)^2] - [10a - 1.86}{h}$$

$$= \lim_{h \to 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h}$$

$$= \lim_{h \to 0} \frac{-3.72ah - 1.86h^2 + 10h}{h}$$

$$= \lim_{h \to 0} (-3.72a - 1.86h + 10)$$

$$= -3.72a + 10$$

14. cont'd

ⓐ $v(1) = -3.72(1)+10 = 6.28$ m/sec

ⓑ $v(a) = -3.72a + 10$

ⓒ rock hits surface when $H(t)=0$

$0 = 10t - 1.86t^2$

$0 = t(10-1.86t)$

$t=0 \quad 10-1.86t = 0$

$t = \frac{10}{1.86} \approx 5.376$

Rock hits the surface at $t = \frac{10}{1.86} \approx 5.376$ sec

ⓓ $v\left(\frac{10}{1.86}\right) = -10$ m/sec

17. $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$

(A) 18. ⓐ $y = g(x) \quad x=5,\ g(5)=-3,\ g'(5)=4$

$g(5) = -3 \Rightarrow$ pt $(5,-3)$

$g'(5) = 4 \Rightarrow$ slope of tangent line at $x=5$ is 4

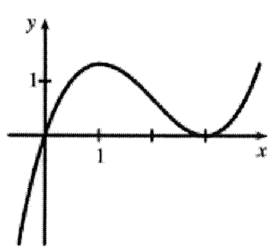
$y+3 = 4(x-5)$

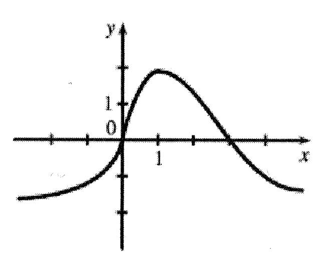
$y = 4x - 23$

(A) ⓑ $f'(4) = \frac{2-3}{0-4} = \frac{1}{4}$

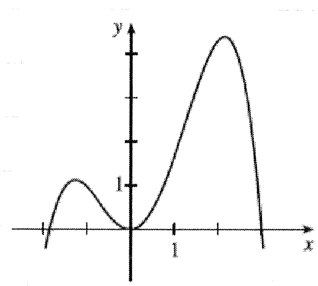
$f(4) = 3$

19. two possibilities




20.



21. $f(x) = 3x^2 - 5x$ point $(2,2)$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{[3(2+h)^2 - 5(2+h)] - 2}{h}$$

$$= \lim_{h \to 0} \frac{3(4+4h+h^2) - 10 - 5h - 2}{h}$$

$$= \lim_{h \to 0} \frac{7h + 3h^2}{h}$$

$$= \lim_{h \to 0} (7 + 3h)$$

$$= 7$$

tangent line: $y - 2 = 7(x-2)$

$y = 7x - 12$

22. $g(x) = 1 - x^3$ point $(0,1)$

$$g'(0) = \lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \to 0} \frac{[1 - h^3] - 1}{h}$$

$$= \lim_{h \to 0} \frac{h^3}{h}$$

$$= \lim_{h \to 0} h^2$$

$$= 0$$

tangent line: $y - 1 = 0(x - 0)$

$y = 1$

25. $f(x) = 3 - 2x + 4x^2$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{[3 - 2(a+h) + 4(a+h)^2] - (3 - 2a + 4a^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{2a} - 2h + 4(\cancel{a^2} + 2ah + h^2) \cancel{-3} \cancel{+2a} \cancel{-4a^2}}{h}$$

$$= \lim_{h\to 0} \frac{-2h + 8ah + 4h^2}{h}$$

$$= \lim_{h\to 0} (-2 + 8a + 4h)$$

$$= -2 + 8a$$

27. $f(t) = \frac{2t+1}{t+3}$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\frac{2(a+h)+1}{(a+h)+3} - \frac{2a+1}{a+3}}{h}$$

$$= \lim_{h\to 0} \left(\frac{2a+2h+1}{a+h+3} - \frac{2a+1}{a+3}\right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(2a+2h+1)(a+3) - (2a+1)(a+h+3)}{(a+h+3)(a+3)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{2a^2 + 6a + 2ah + 6h + a + 3 - (2a^2 + 2ah + 6a + a + h + 3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h\to 0} \frac{5h}{h(a+h+3)(a+3)}$$

$$= \frac{5}{(a+3)(a+3)}$$

$$= \frac{5}{(a+3)^2}$$

29. $f(x) = \dfrac{1}{\sqrt{x+2}}$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{a+h+2}} - \dfrac{1}{\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \left( \frac{\sqrt{a+2} - \sqrt{a+h+2}}{\sqrt{a+2} \cdot \sqrt{a+h+2}} \right) \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{a+2} - \sqrt{a+h+2}}{h\sqrt{a+2} \cdot \sqrt{a+h+2}} \cdot \frac{\sqrt{a+2} + \sqrt{a+h+2}}{\sqrt{a+2} + \sqrt{a+h+2}}$$

$$= \lim_{h\to 0} \frac{(a+2) - (a+h+2)}{h\sqrt{a+2} \cdot \sqrt{a+h+2}\,(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{h}{h\sqrt{a+2} \cdot \sqrt{a+h+2}\,(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{a+2}\,\sqrt{a+h+2}\,(\sqrt{a+2} + \sqrt{a+h+2})}$$

$$= \frac{1}{\sqrt{a+2} \cdot \sqrt{a+2}\,(\sqrt{a+2} + \sqrt{a+2})} = \frac{1}{(a+2)(2\sqrt{a+2})}$$

$$= \frac{1}{2(a+2)^{3/2}}$$

31. $\displaystyle\lim_{h\to 0} \frac{(1+h)^{10} - 1}{h}$

compare to our equation $\displaystyle f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$a = 1$

$f(x) = x^{10}$

33. $\lim\limits_{x \to 5} \dfrac{2^x - 32}{x - 5}$

compare to $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$a = 5$

$f(x) = 2^x$

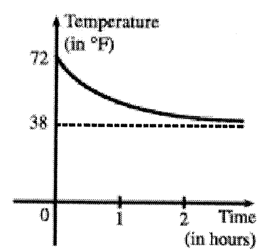
35. $\lim\limits_{h \to 0} \dfrac{\cos(\pi + h) + 1}{h}$

compare to $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

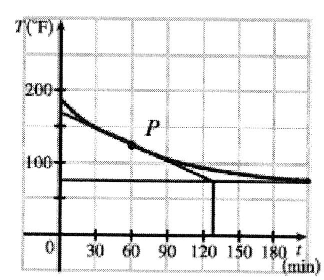
$a = \pi$

$f(x) = \cos(x)$

39. The sketch shows the graph for a room temperature of 72° and a refrigerator temperature of 38°. The initial rate of change is greater in magnitude than the rate of change after an hour.




40. The slope of the tangent (that is, the rate of change of temperature with respect to time) at $t = 1$ h seems to be about $\dfrac{75 - 168}{132 - 0} \approx -0.7\,°\mathrm{F/min}$.

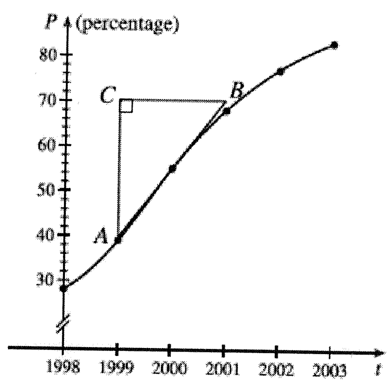


41. (a)

| time span | avg rate of cell phone growth (%/yr) |
|---|---|
| (i) 2000 to 2002 | $\frac{P(2002)-P(2000)}{2002-2000} = \frac{77-55}{2} = 11$ |
| (ii) 2000 to 2001 | $\frac{P(2001)-P(2000)}{2001-2000} = \frac{68-55}{1} = 13$ |
| (iii) 1999 to 2000 | $\frac{P(2000)-P(1999)}{2000-1999} = \frac{55-39}{1} = 16$ |

(b) averaging (ii) and (iii) we get $\frac{13+16}{2} = \frac{14.5\%}{\text{year}}$

(c)




46. (a) $f'(5)$ is the rate of growth of the bacteria population when $t = 5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, $f'$ should increase as $t$ increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.